Complexity of finding equilibria

We know

no-regret $\longrightarrow$ coarse corr. eq

no-swap regret $\longrightarrow$ corr. eq

Can we find solution CE or CCE?

CE

	R	P	S
R	-2, -2	$1/6$	$1/6$
P	$+1$, -1, $1/6$	-2, -2	$1/6$
S	$1/6$	$1/6$	-2, -2

find CCE as a linear program

x_s = prob for strategy vector $s = (s_1, \dots s_n)$

$x_s \geq 0$

$\sum_s x_s = 1$

** player i: $\sum_s x_s u_i(s) \geq \sum_s x_s u_i(s_i', s_{-i})$

all s_i'

for CE same as above plus

(*) for all i, all s_i' & s_i'': $\sum_{s: s_i=s_i'} x_s u_i(s) \geq \sum_{s_i=s_i'} x_s u_i(s_i'', s_{-i})$

n players & k strategies each

(*) $n \cdot k^2$ inequalities

(**) $n \cdot k$

$+ \quad 1 \qquad \sum x_s = 1$

plus $\quad x_s \geq 0$

trouble $\quad k^n$ variables

can **we optimize social welfare?**

Theorem: finding max social welfare E in NP-hard

Proof: 3-SAT $\quad x_1 \ldots x_n$

$(\quad) \wedge (\quad) \ldots (\ldots)$

$x_1 \vee \overline{x_2} \vee x_3$

n players $x_i \longrightarrow T$ & F

$c_i(S) = \#$ clauses x_i or $\overline{x}_i$ in that are not true

Claim: congestion game

x_i — $T = \{j$ clauses $\overline{x_i}$ in the clause$\}$

x_i — $F = \{j$ clause x_i in the clause$\}$

$$c_j(x) = \begin{cases} 1 & x = 3 \\ 0 & x \leq 2 \end{cases}$$

clause j

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finding social cost NP-hard

Minimizing $\emptyset$

$$\sum_e \sum_{k=1}^{x_e} c_e(x) = \#\text{unsatisfied clauses}$$

NP-hard to find

Finding a Nash: local optimization

max $m = \#$clauses updates

$\exists$ Nash player $n+1$ player strategy a
NP-complete

Proof: extra player $n+1$ — a & b

$$c_{n+1}(b, s) = 0$$

$$c_{n+1}(a, s) = \begin{cases} 0 & \text{if } s \text{ satisfies } \phi \\ 1 & \text{otherwise} \end{cases}$$

Positive side: finding CE or CCE
"doable in poly time"