

OH on Monday 11-12

2 - extra late day (final due date Wed!)

don't forget all the Sendhil talks

? presentation of project?

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Remaining topics:

- more complex item set valuations
- cooperation
- speed of reaching good social welfare
- interesting games we didn't see
  - bandwidth sharing
  - project selection
- complexity of finding equilibria
  - ? fairness
  - ? multiperiod games
  - ? mechanism design
  - ? budgeted repeated auction

2nd price multiple items

① single item bidding value  
dominant strategies

bids  $b \gg v$  & all others bid 0

this Nash & P&A is bad!

Assumption: no overbidding

e.g. multi item separate auctions

bid  $b_j$  for item  $j$

no-overbidding = all sets  $v(S) \geq \sum_{j \in S} b_j$

Bidding optimally in given random  
environment is NP-hard

Recall  $\perp T_1, \dots, T_m$

bid on  $T_i$  &  $v' \gg v \gg 1$   
on all others

Same proof, same bids

equally hard in 2nd price

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So far we had  $v_i(S) = \max_{j \in S} v_{ij}$

Example 2: 
$$v_i(s) = \begin{cases} v_{ij} & \text{if } S = \{j\} \\ \max_{\ell \neq j \in S} v_{ij} + \frac{1}{2} v_{i\ell} & \end{cases}$$

General version: (XOS valuation)

$v_{ij}^k$  person  $i$  on item  $j$ , option  $k$

$$v_i(s) = \max_k \sum_{j \in S} v_{ij}^k$$

Claim above 2 are special cases:

by  $(0, \dots, 0, v_{ik}, 0, \dots, 0) \Rightarrow \max_j v_{ij}$

add  $(0, \dots, 0, v_{ij}, 0, \dots, \frac{1}{2} v_{i\ell}, 0, \dots, 0)$

to get second example

Submodular valuation

$$A \subseteq B \text{ \& } j$$

$$v(A+j) - v(A) \geq v(B+j) - v(B)$$

Claim submodular is XOS

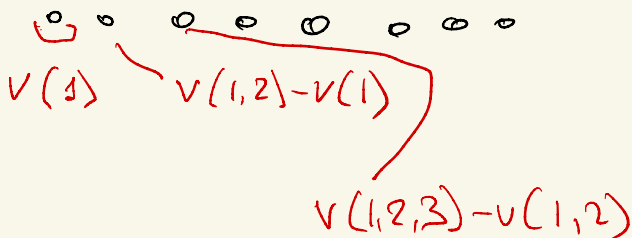
Proof:  $\pi$  permutation elements

$$A_i^\pi = \{j \mid \pi(j) < \pi(i)\}$$

define 
$$v_i^\pi = v(A_i^\pi + i) - v(A_i^\pi)$$

Claim 1 
$$v(S) \leq \max_{\pi} \sum_{j \in S} v_j^\pi$$

put  $S$  first in ordering



first  $|S|$  elements in this order

$$\sum_{i \in S} v_i^\pi = v(S)$$

Claim 2: 
$$v(S) \geq \sum_{j \in S} v_j^\pi \quad \text{all } \pi$$



pushing  $j \notin S$  later only increases the values in  $S$