today: recall auction multiple items

$$V_i(A) = \max_{j \in A} V_{ij}$$

Last time: getting close to no-regret is NP-hard

& even running step follow the leader is also NP-hard.

No envy (no-regret) auction v_{ij} & price paid.

$$(*) \quad \sum_{t=1}^{T} u_i^t(s^t) \geq \max_j \left(\lambda \sum_{t=1}^{T} v_{ij}^t - \sum_{t=1}^{T} p_j^t \right) - \text{Regret}$$

price of item j as happened at time t

assume $v_{ij}^t = v_{ij}$

(1) No-envy $\Longrightarrow$ PoA bound

All player have no envy

$$SW = \sum_t \sum_i u_i(s^t) + Rev \geq \lambda T \cdot OPT - n\,Reg$$

$n = \#$ players

Let max Matching M

use i use $*$ with j s.t $(i,j) \in M$

& $i \notin M$ use ≥ 0

$$\sum_{i \in M} \sum_t u_i^t(s^t) \geq \sum_{(i,j) \in M} \left(\lambda \; T v_{ij} - \sum_{t=1}^{T} p_j^t - Reg\right)$$

$$SW = \sum_i \sum_t u_i^t(s^t) + \sum_j \sum_t p_j^t$$

$$\geq \lambda T \sum_{(i,j) \in M} v_{ij} - u \cdot Reg$$

$$= \lambda \underbrace{T \; Opt}_{\text{Opt over T times}} - u \; Reg$$

Z con well enough to get this

be the leader for envy!

try to find: set of item S

~~...~~ max value:

$$\max_S \left(\lambda v_i(S) - \sum_{j \in S} \frac{1}{T} \sum_{t=1}^{T} p_j^t\right)$$

Note: best to choose single item

Step 1: be the leader ⟹ no an vy

by induction: $t=1$ by choice

$$u_i^1(s^1) \geq \lambda v_{ij} - p_j^1 \qquad j \text{ choice}$$

bid $v_{ij}/2$ on j & $\lambda = 1/2$

old proof

$$u_i^1(v_{ij}/2, s_{-i}^1) \geq \frac{1}{2} v_{ij} - p_j^1 \geq \max_{j'} \ldots$$

induction step

$$\sum_{z=1}^{t-1} u^z(s^z) + u^t(s^t) \overset{i.h}{\geq} \frac{1}{2}(t-1)v_{ij} - \sum_{z=1}^{t-1} p_j^z$$

let j choice at time t

$$+\left(\frac{1}{2} v_{ij} - p_j^t\right)$$

by bidding $\frac{1}{2} v_{ij}$

$$\geq \max_{j'} \frac{1}{2} t v_{ij'} - \sum_{z=1}^{t} p_{j'}^z \quad \text{all } j'$$

(2) Add noise but keep "cheating"

t included in choice

$$\sum_{\tau=1}^{t} u_i^{\tau}(s^{\tau}) + \max_{j\in} \xi_j \geq \max_j \lambda t v_{ij} - \sum_{\tau=1}^{t} p_j^{\tau} + \xi_\tau$$

(3) if ξ choosen randomly & indep

s^t then

$$\Pr(j^t = j^{t-1}) \geq 1 - \varepsilon$$

(4) Randomization every step

same expectation.