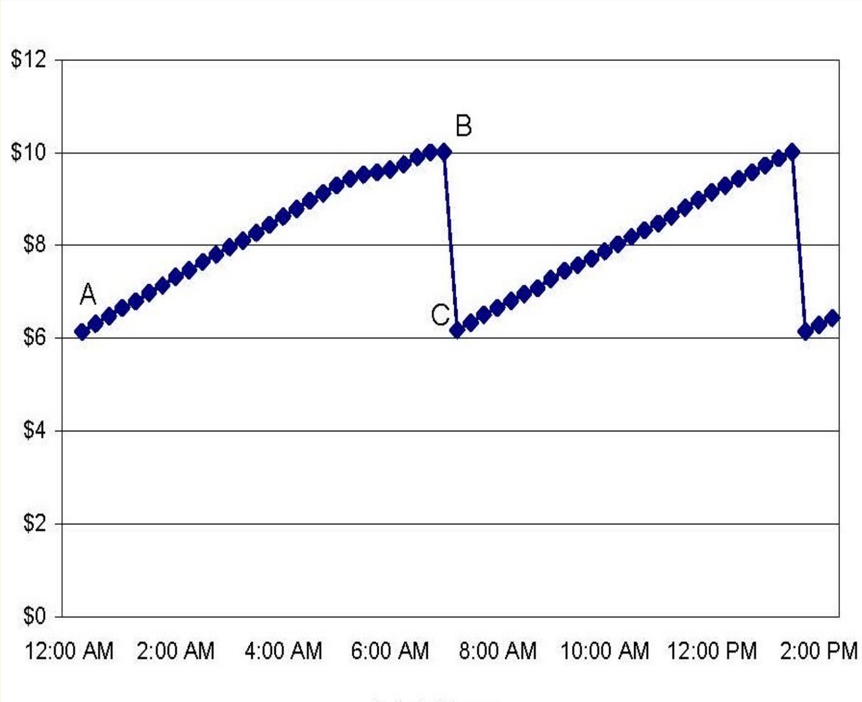
$12
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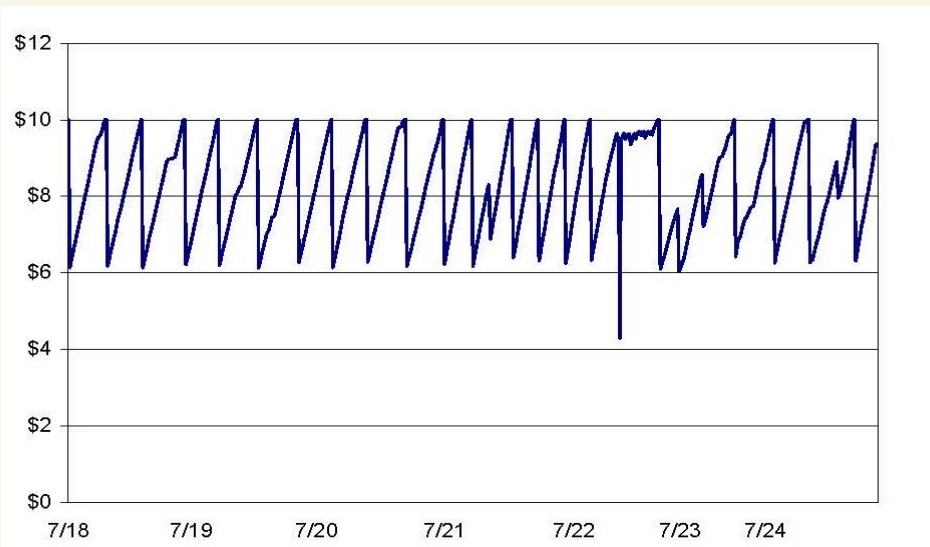
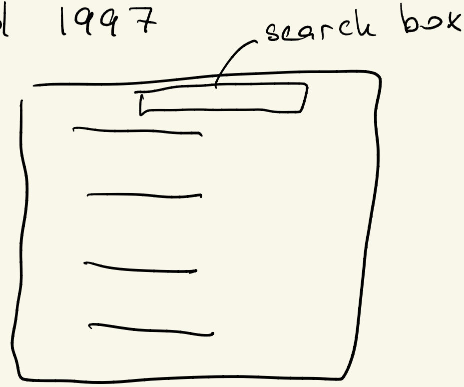

$12
$10
$8
$6
$4
$2
$0
7/18
7/19
7/20
7/21
7/22
7/23
7/24

Selling per click started 1997

Overture

first price

& sorted by bid



2002 generalized second price

bid sort by bid

position i pays bid of position $i+1$ as price (or reserve)

Example

value per click $\underline{10}$, 4, 2

\# clicks 200, 100 /per hour

bidding truthfully:

top: $200(10-4) = 1200$

next: $100(4-2) = 200$

Fact: not truthful

\# clicks 200, 180

top bidding 3 gets

$180(10-2) = 1440$

Price of anarchy?

assume $b_i \le v_i$ all bids

Thm: PoA (assuming no overbidding) ≤ 4

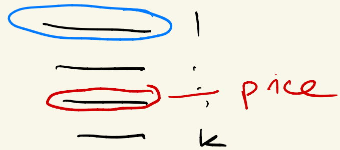
(3.16...)

Made with Goodnotes

Proof: $u_i\left(v_i/2,\ s_{-i}\right) \geq v_i/2 - \max_j b_j$

rest of proof as usual.

English auction



$p = 0$ raise it

k th price set when only k people left

assume $v_1 > \dots > v_k > \dots v_n$

k^{th} price $= \mid v_{k+1}$ — pays $\gamma_k \cdot v_{k+1}$

assume click rate $\gamma_1 > \dots > \gamma_n > 0$

next drops out if

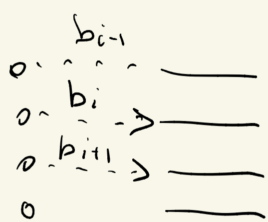
$$\gamma_k\left(v - v_{k+1}\right) = \gamma_{k-1}\left(v - p\right)$$

set p_{k-1} by v_k

$$\underbrace{\gamma_{k-1} p_{k-1}}_{\text{price paid for slot } k-1} = v_k\left(\gamma_{k-1} - \gamma_k\right) + \gamma_k v_{k+1}$$

Result

1. method truthful
2. result is an equilibrium of GSP
3. envy free $\Rightarrow$ do not envy neighboring slot



if envying below $\Rightarrow$ not equilibrium (bid just below b_{i+1})

not ~~envying~~ envying $i-1$

$$\gamma_{i-1}(v_i - b_i) \le \gamma_i \, (v_i - b_{i+1})$$

price at $i-1$

true by rule setting prices

<u>Claim</u> envy free $\Rightarrow$ $v_1 \geq \dots \geq v_k \geq \dots$

(*) hence solution max social welfare

<u>Claim 2:</u> if not envy free

$\Rightarrow$ increase b_i until

either envy free

or $i-1$ drops down

Proof of (*)

$\to \square_{i-1}$ p_{i-1}

$\to \square_i$ p_i

$$\gamma_{i-1}(v_i - p_{i-1}) \le \gamma_i (v_i - p_i)$$

i not envying slot $i-1$

$$\gamma_i (v_{i-1} - p_i) \le \gamma_{i-1}(v_{i-1} - p_{i-1})$$

$i-1$ not envying slot i

sum the two

$$\gamma_{i-1} v_i + \gamma_i v_{i-1} \le \gamma_i v_i + \gamma_{i-1} v_{i-1}$$

$$v_i(\gamma_{i-1} - \gamma_i) \le v_{i-1}(\gamma_{i-1} - \gamma_i) \Rightarrow v_i \le v_{i-1} \checkmark$$