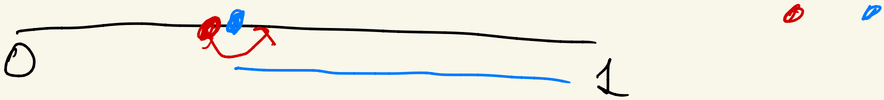
Hotelling games

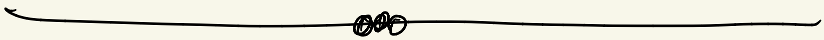


Nash : both in center.

~~all~~ both on center is unique Nash

Question 3 players:

is all three in center Nash?

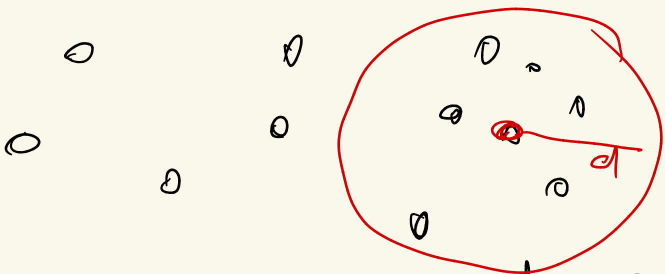


Finite version

N location

node v

w_v customers

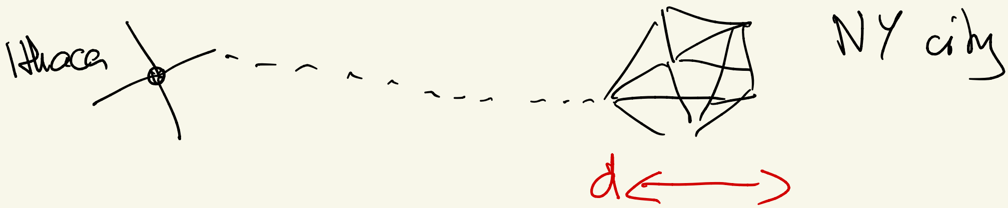


location v & w $d(v,w) \geq 0$

customers choose closest location i

distance $\leq d$

two player: N location



two providers both in city

social opt (= max customers served)

one NYcity one in Ithaca

u_i = # customers dist $\leq d$

where i's is closest

assume: if equal distance

~~use~~ customers divide equally

social welfare $SW = \sum_i u_i$

= # customers served

Price of anarchy:

$$\max \frac{\text{SW optimal solution}}{\text{SW Nash equilibrium}}$$

over possible

Nash equilibria

k players   N location w_v customers per location v

max dist v

$(s_1, \dots, s_k) = s$ a Nash equilibrium

$u_i(s_i^*, s_{-i})$

$s^* = (s_1^*, \dots, s_k^*)$ ~~op~~ social optimum

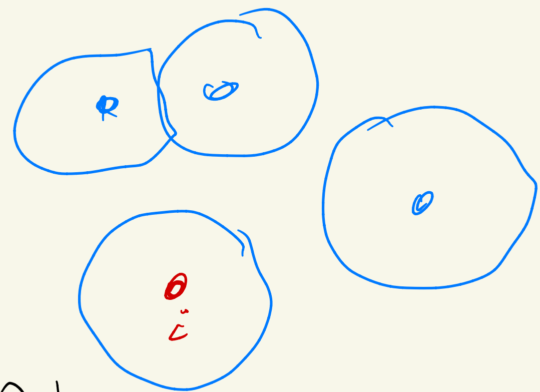
We know: $u_i(s) \geq u_i(s_i^*, s_{-i})$

We need

$u_i(s_i^*, s_{-i})$

s^*

all customers within dist $\leq d$ to s_i^* are being served



X_i^* i serves in Opt

X_i i serves at Nash

Claim: $u_i(s_i^*, s_{-i}) \geq |X_i^* \setminus \bigcup_j X_j|$

$$\Longrightarrow \sum_i u_i(s_i^*, s_{-i}) \geq \sum_i \Big| X_i^* \setminus \bigcup_j X_j \Big|$$

$\parallel$

people served in opt but not served at Nash

$$\Longrightarrow \quad \sum u_i(s_i^*, s_{-i}) \geq SW(s^*) - SW(s)$$

Putting it together

$$SW(s) = \sum_i u_i(s) \geq \sum u_i(s_i^*, s_{-i})$$

$$\geq SW(s^*) - SW(s)$$

$$\Longrightarrow SW(s) \geq \frac{1}{2} SW(s^*)$$

<u>Thm</u> : Price of anarchy for pure Nash ≤ 2