game: n players

player i : strategies S_i

players choose strategies

$s = (s_1 \ldots s_n)$

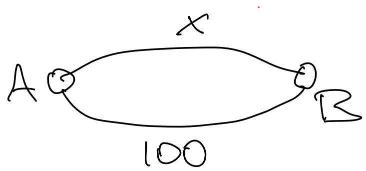
value or cost for each player

$u_i(s)$ = value for player i

[$c_i(s)$ = or cost]

Recall

$n = 100$



delay – cost

$S_i = \{u, d\}$

i chooses down: $c_i = 100$

i chooses up c_i = # people up

Nash equilibrium

pure equilibrium: $s = (s_1 \ldots s_n)$

all players i $\quad c_i(s) \leq c_i(s_1 \ldots s_i' \ldots s_n) = c_i(s_i', s_{-i})$

all $s_i' \in S_i$

One Nash eq.

- all players up
- 99 people up & 1 down

Rock-Paper Scissor

	S	R	P
S	0	−1	+1
R	+1	0	−1
P	−1	+1	0

payoffs for row player in utilities

No pure Nash:

Claim: $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ each Nash: ← probabilities

each i:

select dist σ_i over S_i

$\sigma = \sigma_1 \times \cdots \times \sigma_k$

(*) $\underset{s\sim\sigma}{E}(u_i(s)) \geq \underset{s\sim\sigma}{E}(u_i(s_i', s_{-i}))$

all players i & all $s_i' \in S_i$

Claim: if (*) true for all $s_i' \in S_i$,

$\Rightarrow$ also true add s_i' chooseu at random

$E_{s_i'}\{E_{s\sim\sigma}(u_i(s_i', s_{-i}))$

Rock-Paper Scissor

	S	R	P
S	0	−1	+1
R	+1	0	−1
P	−1	+1	0

payoffs for row player in utilities

No pure Nash:

Claim: $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ each Nash: ← probabilities

each i:

select dist σ_i over S_i

$\sigma = \sigma_1 \times \cdots \times \sigma_k$

(*) $\underset{s\sim\sigma}{E}(u_i(s)) \geq \underset{s\sim\sigma}{E}(u_i(s_i', s_{-i}))$

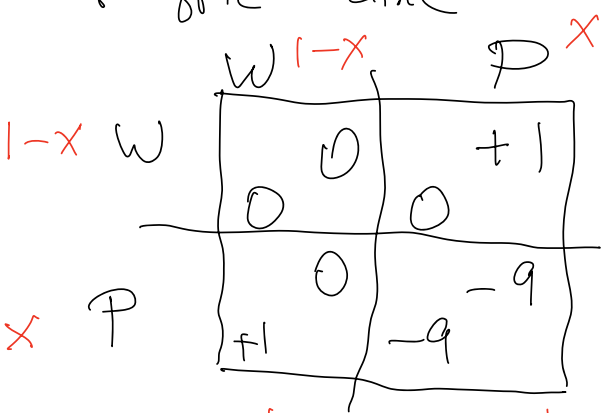
all players i & all $s_i' \in S_i$

Claim: if (*) true for all $s_i' \in S_i$,

$\Rightarrow$ also true add s_i' chooseu at random

$E_{s_i'}\{\underset{s\sim\sigma}{E}(u_i(s_i', s_{-i}))$

traffic game



Nash eq.

(W, P)

(P, W)

prob of choosing outcome:

row player payoffs W: 0

row — " — P:

$+1(1-x) - 9x$

Observation:

Observation:

if W & P have same payoff

$\Rightarrow$ any mixing satisfies Nash condition

if either is better payoff

$\Rightarrow$ mixed strategy not Nash

Nash required

$$(1-x) - 9x = 0 \Rightarrow x = \frac{1}{10}$$