CS 6840 Algorithmic Game Theory Dec 6, 2024

Lecture 40: Auctions with Budgets

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Last lecture we saw that in sequential budgeted first-price auctions, if $U_i \geq \max_{\lambda \in [0,1]} \left\{ \hat{U}_i(\lambda) \right\} - Reg \ \forall i$, then we could bound the optimal liquid welfare by $\frac{3+\sqrt{5}}{2} LW + O(n)Reg$. Being more careful, this factor can be taken down from the ~ 2.61 factor to ~ 2.41.
However, it is not know how to achieve this condition, so to fix it we need a new parameter $\gamma \geq 1$, giving the new bound:
If $U_i \geq \frac{1}{\gamma} \max_{\lambda \in [0,1]} \left\{ \hat{U}_i(\lambda) \right\} - Reg \ \forall i$, then $LW^* \leq \frac{2+\gamma+\sqrt{4+\gamma^2}}{2} LW + O(n)Reg$.

The other case that is logical to look at is sequential budgeted second-price auctions. But there is no possible bound here (with the same assumptions as before). An example:
$n = 2$, $v_{1t} = v_{2t} = 1$, $B_1 = \varepsilon T$, $B_2 = T$.
Clearly $LW^* = T$.
However, if $b_{1t} = 1$ and $b_{2t} = 0$ for all t, neither has regret (player 1 optimizes his utility, wins every item and pays 0; player 2 could only win an item bidding > 1 and paying 1, getting utility 0 anyway).
In this case, $LW = \min\{\varepsilon T, T\} \leq \varepsilon T$.
An observation for this example is that player 1 is (grossly) overbidding respective to his budget. We could achieve a good bound if we assume $\sum_{t=1}^{T} b_{it} \leq B_i$ for all i, but this seems like an extremely unreasonable assumption: why would a person not bid high at time t if they have not yet spent much of their budget.