CS 6840 Algorithmic Game Theory Monday, August 26, 2024

Lecture 1: Introduction

Instructor: Eva Tardos *Scribe: Cristian Palma*

1 General Information

Instructor: Eva Tardos, office hours: Monday 2:30-3:30 and Thursday 3-4 in Gates 331.

TAs:

- Shawn Ong, office hours: Wednesday 4-5 in Rhodes 657 conference room 2.
- Chido Onyeze, office hours: TBA.

Course description Algorithmic Game Theory combines algorithmic thinking with game-theoretic, or more generally, economic concepts. Designing and analyzing large-scale multi-user systems and as well as such markets, requires good understanding of tools from algorithms, game theory, and graph theory. One focus this semester will be learning in repeated games. The course will develop mathematically sophisticated techniques at the interface between algorithms and game theory, and will consider their applications to markets, auctions, networks, as well as the Internet.

More on https://www.cs.cornell.edu/courses/cs6840/2024fa/.

Topics: *Outcomes in Games* and *Price of Anarchy*:

- Games, equilibria, examples of games
- Price of anarchy in routing games
- No-regret learning, and 2 person 0-sum games
- Smooth games and learning outcomes, best response dynamic
- Best Nash and strong price of anarchy
- Auction as Games
- Matching
- Fairness

Course material: There is no required textbook for the course. The following are useful references.

- Tim Roughgarden. *Twenty Lectures on Algorithmic Game Theory*, Cambridge University Press, 2016.
- Tim Roughgarden, Vasilis Syrgkanis and Eva Tardos, *The Price of Anarchy in Auctions* (survey), Journal of Artificial Intelligence Research, 2017.
- David Easley and Jon Kleinberg, *Networks, Crowds and Markets*, Cambridge University Press 2010.

- Aaron Roth, *Learning in Games and Games in Learning* (working draft).

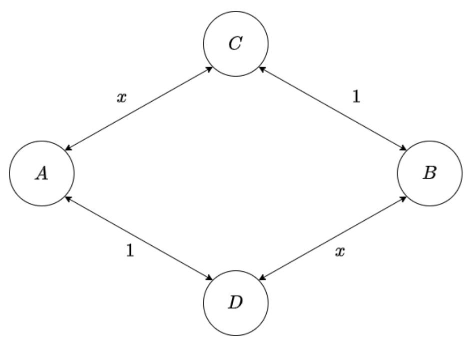
Grading: Your grade will based on 4 sets of homework sets (10% each), class participation, scribe notes (5%,), completion of a course evaluation, course project (including a proposal) (30%,), and a take-home final exam (25%).

2 Traffic Routing Examples

To motivate the topics of the course, let us take a look at the following traffic routing problem with congestion dependent edges.

2.1 Braess' Paradox

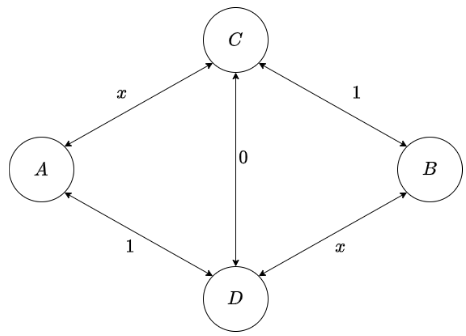
Consider the following network on 4 nodes, where 1 unit of people (a thousand or larger so that it make sense to have fractional values) needs to get from A to B.



Each edge is labeled with the time $t(x)$ it takes to go through it, which might depend on the amount of cars x are using it.

Suppose everyone takes the upper path. Then, the total time is 2 for everyone. Now consider the scenario where half of the people take the upper path, and the other half takes the bottom. In this case, the time to go through the first edge of the upper path is $1/2$, giving a total time of $3/2$ for this strategy. Similarly, the time it takes for the bottom people is also $3/2$. This scenario seems better than the first one, and in some sense (that we have not formally defined) is optimal. We say that this situation is at *equilibrium* as no individual has an incentive to change strategy because that would only increase its total time. More generally, a *Nash equilibrium* is a situation where all individuals are taking their best option given what everyone else is doing.

Now suppose a super fast highway that takes no time to cross to go from C to D.

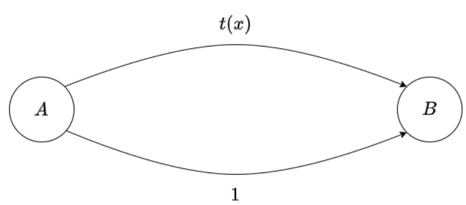


Now the case where people are divided 50-50 still takes 3/2 time for everyone. Notice that this situation is not at equilibrium, as an individual might consider going from A to C, then using the highway to D and finally arrive to B on 1 unit of time. As everyone would prefer to arrive faster to their destination, they all end up using the latter strategy. However, this impacts the congestion sensitive edges resulting on a time of 2 for everyone! This situation is now at equilibrium because any individual trying to change to another path would also take 2 units of time. How could adding a seemingly good addition make things worse? This is called *Braess' paradox*. The selfish behavior of the people induced a worse outcome.

The *price of anarchy* is defined as the cost at equilibrium divided by the social optimum cost. If we consider the cost as being the average time to get to B, then it is possible to prove that 3/2 is indeed the optimal social cost. So for the network with the highway, the price of anarchy corresponds to $2/(3/2) = 4/3$.

2.2 How expensive anarchy is

Consider the following graph:



Whenever the upper edge has a crossing time of $t(1) \leq 1$, the case where everyone is going the upper edge is an equilibrium with a cost of $t(1)$.

If $t(x) = x$, one can prove that the optimum situation is when half go upper, and half go bottom. Where the upper people take 1/2 units of time, while the bottom take 1 unit, giving an average value of 3/4, and thus a price of anarchy of 4/3.

More generally, let $t(x) = x^n$ be the crossing time of the upper edge. Let α be the fraction of people on the upper edge, whose total time is $t(\alpha) = \alpha^n$. The $1 - \alpha$ fraction of the people taking the bottom edge take 1 unit of time to get to B. This gives an average cost of $\alpha^{n+1} + 1 - \alpha$, which is minimized at $\alpha = 1/\sqrt[n]{n+1}$ with a value of:

$$\frac{1}{(n+1)\sqrt[n]{n+1}} + 1 - \frac{1}{\sqrt[n]{n+1}}$$

Notice that the optimal fraction of people on the upper part tends to 1, and its optimal cost tends to 0 when n goes to infinity. This means that the price of anarchy turn larger and larger, even though the part of people that should be on the upper part is near the equilibrium value 1. At equilibrium, for large n, having even one person choose the bottom edge would significantly improve the social cost, but no individual has an incentive to do so.