CS 6840: Algorithmic Game Theory **Spring 2017**

Lecture 38: May 3

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38.1 Administrative details

- No lecture on Wednesday, May 10.
- Final exam is cumulative, takes 72h and can be taken any time starting from May 10 until the end of the exam period. Should send email at least 24h in advance (even sooner if very early start).

38.2 Remaining topics

- Role of reserve price in maximizing welfare
- Pricing as a simple mechanism and its connection to smoothness

38.3 Reserve prices and maximal revenue

38.3.1 Introduction

Until now, we have only been analyzing utilities and social welfare for auctions. We now move to the seller's perspective, who wants to maximize revenue. A scenario that motivates this is one in which there is only one buyer, since there is no lower bound on how much they pay in both first-price and second-price auction. A solution is setting a **reserve price**, which is the minimum price that the seller accepts.

38.3.2 Single buyer case

The buyer either pays price p or doesn't get the item at all. We are interested in the optimal p.

Given the buyer's value distribution, we define v(q) to be the value so that $P(v \leq v(q)) = q$. It is fair to assume that v is continuous and strictly increasing. This means that for any p between the min and max value, there is a q so that $v(q) = p$. Then the probability that the item is bought with price $p = v(q)$ is $P(v \geq v(q)) = 1 - q$. The expected revenue is thus $R(q) = v(q)(1 - q)$, which is maximized in a point where the derivative is 0.

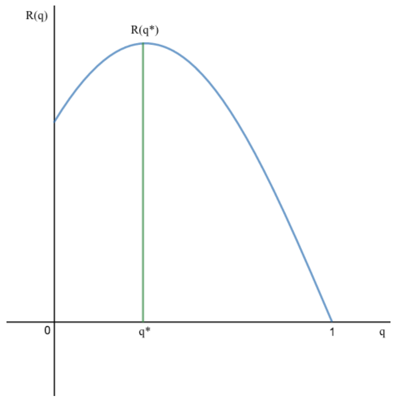

Figure 38.1: Graph of $R(q)$ with the maximum in q^*

Another frequently made assumption is that R is concave, in which case there is exactly one maximum, $R(q^*)$. The optimal reserve price will be $v(q^*)$. A distribution that generates such an R is called **regular**.

38.3.3 Multiple buyers case

Theorem 38.1 *Adding another buyer and doing a second price auction increases the revenue more than adding a reserve price. Here we assume the two buyers have the same value distribution.*

Proof: Let's compute the expected revenue from the first buyer. For a fixed bid $v(q_2)$ of the second buyer, buyer 1 wins with probability $(1-q_2)$ and pays $v(q_2)$, so the expected revenue is $v(q_2)(1-q_2) = R(q_2)$.

We now notice that the distribution of q_2 is uniform on $[0,1]$, so the expected value of the revenue from player 1 is $E[R(q_2)] = \int_0^1 R(q)dq$. The same is true about the expected revenue from player 2, so the overall expected revenue of the auction is $2\int_0^1 R(q)dq$. We will prove geometrically that this is larger than $R(q^*)$.

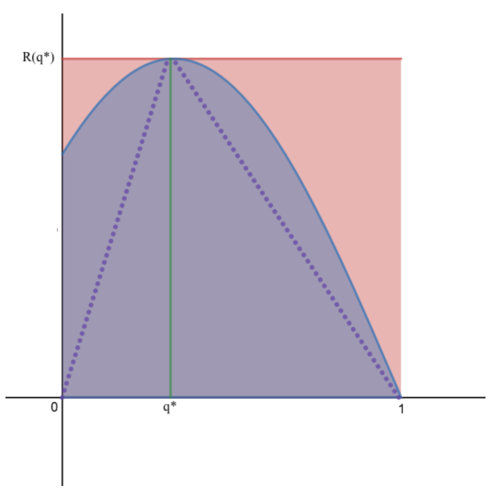

Figure 38.2: Geometrical representations of $E(R)$ and $R(q^*)$

By the definition of the integral, $\int_0^1 R(q)dq$ is the area under the graph (blue). Since the range of R is [0,1], we can geometrically represent $R(q^*)$ as the area of the red rectangle. Since R is concave, its curve is above the 2 dotted diagonals. Moreover, the sum of areas under the diagonals is half the area of the rectangle, which means $2\int_0^1 R(q)dq \geq R(q^*)$, concluding the proof.

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Observation: In practice, it is impossible to perfectly observe the distribution of values v in order to obtain q^*. However, by proving $E(R(q)) \geq R(q^*)/2$, we showed that if the seller is picking a random sample from the value distribution as the reserve price, the expected loss is at most half the maximum revenue.

38.3.4 Other results

Theorem 38.2 *(Bulow and Klemperer) In an auction with N buyers, adding an additional buyer increases revenue more than adding reserve prices*

Theorem 38.3 *(Myerson) If all values are from the same distribution, the maximum revenue is obtained with second price auction and setting reserve to q^*.*

References

[1] Jeremy Bulow; Paul Klemperer. Auctions Versus Negotiations. The American Economic Review, Vol. 86, No. 1. (Mar., 1996), pp. 180-194

[2] Roger B. Myerson, Optimal Auction Design Mathematics of Operations Research Vol. 6, No. 1. (Feb,. 1981), pp 58-73.