CS 6840: Algorithmic Game Theory **Spring 2017**

Lecture 1: January 25

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1.1 Course Information

1.1.1 Logistics

- The course website can be found here.
- Here is the Piazza page.
- Students can work in partnerships of up to two people for the following assignments:
  - Four homework assignments, which will be due roughly every two weeks.
  - One open-ended project.
- The partnerships are intended as *collaborations*, not as a means to avoid learning half the course material.
- There will also be a take-home final exam, similar in spirit to the homework assignments, to be completed individually.

1.1.2 Textbooks

Despite what the Cornell store may tell you, there are not exactly required purchases for this class. We will draw content from:

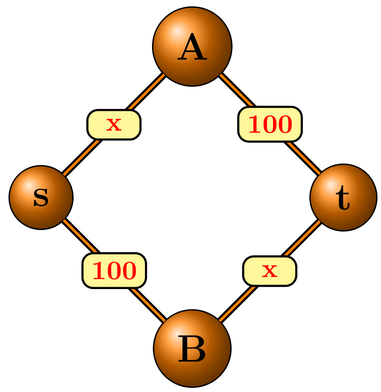
- Some of Roughgarden's excellently written *Twenty Lectures on Algorithmic Game Theory*, mostly the latter half.
- Select chapters from David Parkes' upcoming book on the subject.
- A collection of Web articles (links will be provided).

1.2 Course Outline

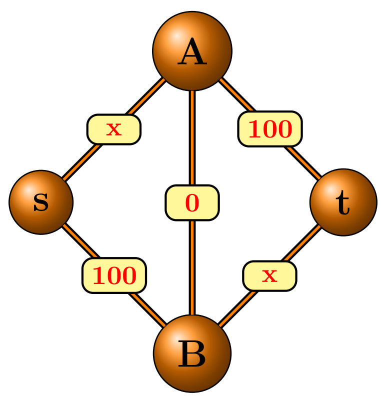
Broadly speaking, the course can be divided into three parts.

1.2.1 Part 1: Games and Outcomes

By a *game*, we mean the sort of activity in which one's choices impact the outcomes of other participants. One game with some interesting properties is the following "traffic game": 100 people are simultaneously trying to travel from node s to node t along the given roads. Here the edge labels denote travel times, and x denotes the number of travelers using the road.



Here the Nash equilibrium (i.e. the state in which no traveler can improve their outcome by changing only their own strategy) is to have half the travelers travel along the upper path, and half along the lower path. All travelers then take $50 + 100 = 150$ minutes to complete their commute.



Now Elon Musk is here to help. He's built an ultra-fast rail line between cities A and B, capable of transporting travelers and their cars in no time at all. The old Nash equilibrium is no longer an equilibrium at all; it's hard to imagine anyone would take either the routes $s \to B$ or $A \to t$, since they're weakly dominated by taking the other branch at that point and then using the high-speed rail. Instead, everyone travels along the route $s \to A \to B \to t$, and the total travel time is now $100 + 100 = 200$ minutes for everyone.[1] Thanks, Elon!

This conclusion (that adding a new road can make traffic *worse*) is known as Braess's paradox. In this case, it is clear that everyone would benefit from a central Traffic Czar who coordinated everyone's route; they would enforce the old Nash equilibrium, which is optimal in terms of average travel time. But there is no Traffic Czar. We can quantify the "price of anarchy" here as $\frac{200}{150} = \frac{4}{3}$.

[1] Since the travelers are granular here, technically the $s \to B$ and $A \to t$ routes can support up to one traveler each, in which case some travelers could get by with a mere 198-minute travel time. We'll ignore this.

A physical analog of Braess's paradox using springs can be seen here.

We've skipped over some questions of how Nash equilibria are realized. We can explore several possible models, including:

- A best-response dynamic, in which everyone picks an initial strategy and then players are allowed to switch one at a time.
- A learning algorithm approach, where travelers can switch their strategies in real time.

In the learning algorithm setting, we might be concerned with oscillations about the Nash equilibrium; first all travelers take the upper route, then simultaneously they all switch to the "faster" lower one, and so on.

1.2.2 Part 2: Simple Auctions

An *auction* is a setup where buyers submit bids for one or more goods, which are then distributed among the buyers in some accordance to their bids. Favorite auctions of theorists include second-price auctions (the highest bidder pays the second-highest price) and the VCG procedure (a generalization of the former to situations with more than one item). In the real world, Google currently uses the generalized second price (GSP) auction to sell ad slots, and some ill-advised companies even use first-price auctions. This was the case with GoTo.com, the first major online commercialized search engine; straightforward-sounding auction procedures often incentivize bizarre behaviors.

In a first-price auction, there is a general downward pressure on bids: it would be foolish to exceed one's value for a particular good by bidding up, and by bidding down it's possible to nab the same good for less (provided no other bidders are leapfrogged in the process). Consider two pizzerias bidding for ad space: one initially bids to pay 50 cents per click, the other only 30. The better ad slot goes to the higher-paying customer, of course. The higher bidder now sees that it can profit by (next time) bidding 31 cents and nabbing the same spot. The lower bidder might decide to tank its own bid, having lost the auction anyway; or now it might nudge its own bid up to grab the better slot at an additional cost of only 2 cents. Such behavior was common in the early days of ad placement: bids typically changed on the order of once a minute. It's not clear that an equilibrium exists in our example, or (if it does) how long one might expect it takes to reach it.

1.2.3 Part 3: Using Data—Inferring Parameters of and Designing Systems

Often in the real world we don't know what people want, but we do know what they do. What can we infer about the first from the second? And how can we design systems to make people behave the way we want them to? This is the highlight of the course; it's possible time will be shaved off other topics to leave enough time for this one.

Properly incentivizing system participants is an issue that has plagued competitive sports leagues for years. Some examples of perils faced include:

- In the women's Olympic Badminton event in 2012, the Chinese team, favorites to win the gold medal, suffered an upset loss in the group stage of the tournament. They advanced to the next round, but with a low seed. Teams in other groups were observed throwing their matches after securing the required berth to advance. Read Professor Robert Kleinberg's take on these events here.
- An analogous event occurred in soccer's 1998 AFF Championship.

- In what later became known as the Anschluss game, West Germany and Austria spent the vast majority of the game making no serious attempts to score, having achieved a mutually agreeable 1-0 score that allowed both to advance.
- In the 1994 Caribbean Cup, Barbados met Grenada in the final qualification round. Barbados would advance if it won by at least two goals, whereas Grenada would advance if it won, drew, or lost by a single goal. In a competition quirk, goals scored during overtime counted double. Barbados was ahead 2-0 going into the 83rd minute when Grenada scored a goal. After several fruitless minutes on the attack, Barbados chose to score an own goal in the 87th minute, planning to keep the game tied until overtime. For the next three minutes, Barbados defended both goals from Grenada's attackers, who needed only score a goal for *either* team to secure their own advancement.

Rather than berate the players involved for exercising poor sportsmanship in the pursuit of getting the best result for their own team, one can ask: can these systems be designed such that they do not encourage this type of behavior?