CS 6840 – Algorithmic Game Theory (3 pages) Spring 2012

Lecture 27 Scribe Notes

Instructor: Eva Tardos *Chaoxu Tong(ct423)*

Review Last time, we discussed greedy algorithm as mechanism and generalized single item auction to matroid case when greedy is still optimal.

Today We'll look at combinatorial auctions, in which there are a set of items S on sale and player i has value $v_i(A) \geq 0$ for all subset $A \subseteq S$.

We can do VCG on this setting, but there are some troubles(within brackets) related to the procedure of VCG

1) Ask players to report fns $v_i(A)$ [If S big, then too many values to report, e.g. $2^{|S|}$]

2) Find allocation A_i s.t. $A_i \cap A_j = \emptyset$ which,

$$\max \sum v_i(A_i)$$

[NP-hard to compute the "set-packing" allocation problem]

3) Compute payment

Today we are going to focus on *single minded bidders* where players i has value v_i and set A_i, and player i gets value v_i if he receives any set containing A_i, and gets value 0 otherwise.

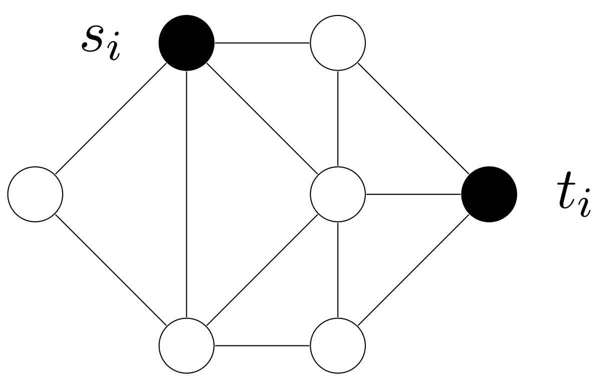
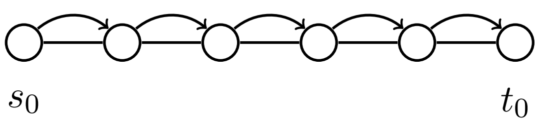


Figure 1: The routing example

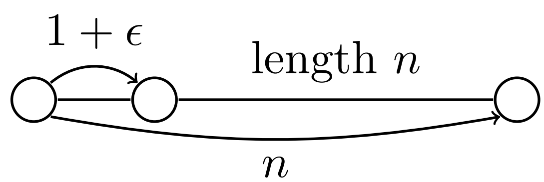
A similar example is the routing case, where S = edges in graph G, player i has value v_i and some source-sink pair $s_i - t_i$, player i gets value v_i if A_i contains $s_i - t_i$ path, and 0 otherwise.

Algorithm 1 The greedy algorithm framework

Start with $I = \emptyset$
while not all players have been processed **do**
  Select v_i with max $v_i/\sqrt{|A_i|}$ or $v_i/\sqrt{d(s_i,t_i)}$ ($d(s_i,t_i)$ is the length of shortest path from s_i to t_i using only edges unassigned)
  Add to I if possible(A_i disjoint from sets in I)
end while



(a) Bad example if we choose the max v_i, here $v_0 = 1+\epsilon$ and $v_i = 1$ for other i



(b) Bad example if we choose the max $v_i/|A_i|$

Greedy algorithm Is there a payment scheme making greedy truthful? When reporting v_i is equilibrium? We need

- Results for players monotone in value
- Reporting higher v_i cannot cause player to lose
- Reporting true A_i is also best for winning.

payment that makes this truthful: *critical value*.

The algorithm assume sorted order and price determined by first set after i in sorted order that is not allocated due to A_i.

Theorem 1. Greedy with $v_i/\sqrt{|A_i|}$ and critical value payment is truthful and $\sqrt{n}$-approximation for $|S| = n$.

Proof. To show this is truthful, first, the players have better report a superset of A_i. If reporting a set not containing A_i, then even if they win, they will get no value. If they report any set larger than A_i, this will only decrease their likelihood of winning. Thus, it's truthful to report the true set A_i. The truthfulness of reporting value v_i follows the same argument of second price's truthfulness.

Now let's show the mechanism is $\sqrt{n}$-approximation. Suppose the algorithm took $A_i, i \in I$ and the Opt took $A_j, j \in O$. Let C_i be the set in Opt not taken due to A_i. Then we have $v_i/\sqrt{|A_i|} \geq v_j/\sqrt{|A_j|}$ for any $j \in C_i$,

$$\sum_{j\in C_i} v_j \leq \frac{\sum_{j\in C_i}\sqrt{|A_j|}}{\sqrt{|A_i|}} v_i$$

Since $|C_i| \leq |A_i|$ and $\sum_{j\in C_i}|A_j| \leq n$,

$$\frac{\sum_{j\in C_i}\sqrt{|A_j|}}{\sqrt{|A_i|}} v_i \leq \frac{v_i}{\sqrt{|A_i|}}(|A_i|\sqrt{\frac{n}{|A_i|}}) = \sqrt{n} v_i$$

Sum over all $i \in I$, we get

$$\sum_{i \in I} \sum_{j \in C_i} v_j \leq \sqrt{n} \sum_{i \in I} v_i$$

which concludes our proof. $\square$