Algorithmic Game Theory	Problem Set 3
CS 6840 Spring 2010	Due Monday, April 26th

We will maintain a FAQ for the problem set on the course Web page. You may use any fact we proved in class without proving the proof or reference, and may read the relevant chapters of the book. However, you may not use other published papers, or the Web to find your answer.

A full solution for each problem includes proving that your answer is correct. Please start by explaining what is the high-level idea of the solution (the main insights/nontrivial things necessaries to solve them problem). If you think its useful you may add also pseudocode for details. Do not submit a code only. It can make the solutions more readable if you introduce convenient notation and use it.

If you cannot solve a problem, write down how far you got, and why are you stuck. You may solve the problems with a partner, and may use a different partner for each problem set (though not a different partner for each individual problem). Please hand in a shared problem set with both your names on the solutions. We are happy to help finding partners, let us (Renato or Hu) know if you don't have a partner, and would like one.

Solutions can be submitted on CMS, or handed in at Renato or Hu's office. Please type your solution, to make it easier to read.

(1) Solve problem 1.3 in the book.

(2) Consider a 2-person 0-sum game given by an $n \times n$ game matrix A. We can think of the strategy sets for both players as $X = \{1, \ldots, n\}$. Now consider an n person game where the n players all have strategy sets X, and effectively the players are playing the game A with their neighbors around the cycle. That is, if player i is using strategy x_i then the payoff for each player is $a(x_{i-1}, x_i) - a(x_i, x_{i+1})$, where i is understood mod n (i.e., $0 = n$ and $n + 1 = 1$).

(a) Show that this game has a Nash equilibrium where the expected payoff of each player is 0.

(b) Is this statement also true on a path, i.e., where players 1 and n only participate in one game (with players 2 and $n - 1$ respectively).

(3) Solve problem 1.4 in the book.

(4) Solve problem 1.6 in the book.

(5) Consider a 2 person game, with strategy sets X and Y for the two players. We say that the game has opposite interest if for strategies $x_1, x_2 \in X$ and $y_1, y_2 \in Y$, we have $u_1(x_1, y_1) < u_1(x_2, y_2)$ if and only if $u_2(x_1, y_1) > u_2(x_2, y_2)$. Note that 0-sum (or constant sum) games are games of opposite interest, but there are opposite interest game that are not of constant sum.

(a) We have seen that in 0-sum games for each of the two players all Nash equilibria have the same value for the player. Is it also true in opposite interest games that a player i has the same utility in every Nash? show or give a counter example.

(b) In zero sum games we considered the best safe strategy, i.e, the strategy for a player that guarantees as high a payoff as possible no matter what the opponent does. Let vg_i be this guaranteed value for player i. In zero-sum games the best safe strategy was a Nash equilibrium. Is that guaranteed to be the case for opposite interest games also. Prove or give a counter example. Can the Nash value be smaller than vg_i? Can it be bigger?

(6) Consider a symmetric 2 person game, with strategy sets $X = X$ for both players. Let $u_i(x, y)$ denote the utility for player $i = 1$ or 2 when player 1 plays x and player 2 plays y. We say that the game is symmetric if we have that $u_1(x, y) = u_2(y, x)$ for all $x, y \in X = Y$.

(a) Can a symmetric game have a pure Nash equilibria? (even if all values $u_1(x, y)$ are different?)

(b) Do all symmetric games have pure Nash equilibria?

(c) Show that a symmetric game always have a symmetric equilibrium, i.e., a probability distribution π such that (π, π) is an equilibrium.

(d) One generalization of symmetric games to more players are symmetric anonymous games: all players have the same strategy set X, the same playoff function described as follows, if the player i is laying a strategy $x \in X$ and let n_y denote the number of other players that play strategy y, than the payoff for player i is $u(x, n)$, depends only on x and the vector n, and is the same for all players. Does item (c) above generalize to symmetric anonymous games?