Monday summary $\quad r_i(t) \geq 0 \qquad n = \#$ options

$$\underbrace{\sum_{t=1}^{T} x_i(t)\, r_i(t)}_{\text{you exp reward}} \geq (1-\varepsilon) \underbrace{\max_i \sum_{t=}^{T} r_i(t)}_{\text{max reward of a single arm}} - \frac{\ln n}{\varepsilon}$$

$x_i(t)$ prob dist $\qquad r_i(t)$ = reward arm i at time

you exp reward $\qquad$ max reward of a single arm

Assume $\quad r_i(t) \in [-1, 1]$

$$\geq \max_i \sum_t r_i(t) - \varepsilon T - \frac{\ln n}{\varepsilon}$$

set $\varepsilon = \sqrt{\frac{\ln n}{T}}$

$$= \max_i \sum_{t=1}^{T} r_i(t) - 2\sqrt{T \ln n}$$

Hedge / Multiplative weight

Claim: 2-person 0-sum game
playing alg with this guarantee
converges to equilibrium

Example RPS player 2

		R	S	P
1/3	R	0	-1	+1
pl 1 1/3	P	+1	0	-1
1/3	S	-1	+1	0
		1/3	1/3	1/3

player 1 win from player 2

Nash equilibrium: strategies x & y prob dist
if both player positive prob only
on current best strategy, given the other
person's strategy

A payoff matrix: player 2 pays player 1

Expected payoff $x^T A y = \sum_{ij} a_{ij} x_i y_j$

selecting a_{ij} with $x_i y_j$

best for x $\max_i \sum_j a_{ij} y_j$

best for y $\min_j \sum_i a_{ij} x_i$

Nash if $\min_j \sum a_{ij} x_i = \sum_{ij} a_{ij} x_i y_j = \max_i \sum_j a_{ij} y_j$

Step: x & y prob

$$\boxed{\min_j \sum_i a_{ij} x_i \le \sum_{ij} a_{ij} x_i y_j \le \max_i \sum_j a_{ij} y_j}$$

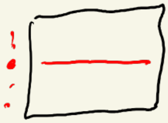
because $\min \le$ average y_j

$\max \ge$ weighted average x_i

Both player run alg above for T steps
choose one strategy i

$r_i(t)$ player 1 $= a_{i j_t}$ if player 2 choose j_t

$r_j(t)$ player 2 $= -a_{i_t j}$ if player 1 choose i_t



---

z_{ij} = frequency (i,j) pair was choosen $= \frac{n_{ij}}{T}$

n_{ij} = # times (i,j) was player

$$x_i = \sum_j z_{ij}, \qquad y_j = \sum_i z_{ij}$$

Claim x & y "almost" Nash

Player 1 guarantee

$$\sum_{i,j} n_{ij} a_{ij} \geq \max_e \sum_{ij} n_{ij} a_{ej} - 2\sqrt{T \ln |S|}$$

reward

$$= \max_e \sum_j n_j a_{ej} - 2\sqrt{T \ln |S|}$$

n_i = # times i usen

m_j = # -"- j used

Player 2:

$$-\sum_{i,j} n_{ij} a_{ij} \geq \max_k -\sum_{ij} n_{ij} a_{ik} - 2\sqrt{T \ln |S|}$$

$$= -\min_k \sum_i n_i a_{ik} - 2\sqrt{T \ln |S|}$$

divide by T

$$\max_e \sum_j y_j a_{ej} - 2\sqrt{\frac{\ln |S|}{T}} \leq \sum_{ij} z_{ij} a_{ij} \leq \min_k \sum_i x_i a_{ik} + 2\sqrt{\frac{\ln |S|}{T}}$$

Putting it together

from start:

$$\boxed{\min_j \sum_i x_i a_{ij} \le \sum_{ij} x_i a_{ij} y_j \le \max_i \sum a_{ij} y_j \le}$$

using new inequalities

$$\le \sum_{ij} z_{ij} a_{ij} + 2\sqrt{\frac{\ln|S|}{T}} \le \underset{k}{\overset{\text{min}}{\cancel{\max}}} \sum_i x_i a_{ik} + 4\sqrt{\frac{\ln|S|}{T}}$$

limit a $T \to \infty$ x & y converges to Nash

Example

	R	P	S
R	0	-1	+1
P	+1	0	-1
S	-1	+1	0

limitation $z_{ij} = \begin{cases} 0 & \text{if } i = j \\ 1/6 & i \neq j \end{cases}$