5 Nov 2025 — Multiplicative Weights I: Hedge

Announcements

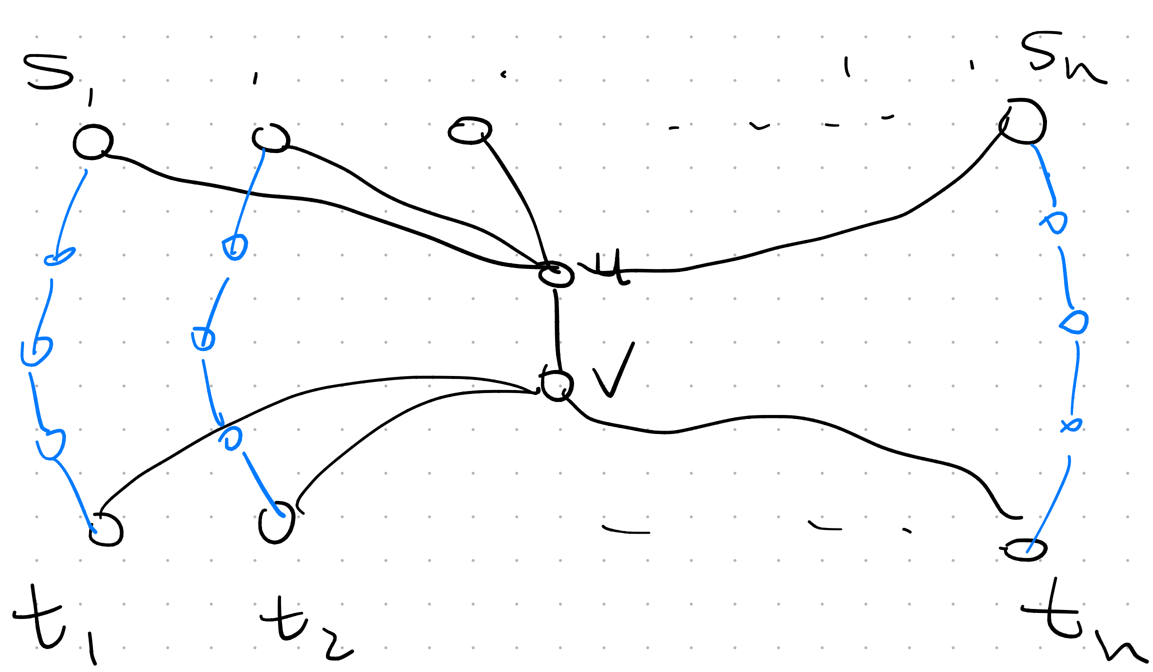
1. Prelim info: see Ed post.

Ithaca	A–O	Gates	310
	P–Z	Ives	115
Cornell Tech		Bloomberg	497

2. Pset 3 solutions: see Ed.

Solving max multicommodity flow:

Idea #1: Just send every unit from s_i to t_i on the shortest path.



Investment problem. Assets $1, \ldots, n$.

Investor with wealth $W(t)$ at end of time t.

(Initially $W(0) = 1$.)

Price of asset i during period t grows by a factor of $(1+\varepsilon)^{r_i(t)}$.

$0 \le r_i(t) \le 1$.

At start of round t:

- investor divides wealth $W(t-1)$ into shares $x_i \cdot W(t-1)$ invested in asset i.
- vector $r(t) = (r_1(t), \ldots, r_n(t))$ is revealed.
- wealth grows to

$$W(t) = \sum_{i=1}^{n} x_i W(t-1) \cdot (1+\varepsilon)^{r_i(t)}$$

Goal. Make $W(T)$ large.

<u>Prediction problem</u> Gambler invests \$1 in each of T games played in sequence.

Betting $x_i(t)$ on alternative i yields payoff $x_i(t) \cdot r_i(t)$.

Assume $0 \le r_i(t) \le 1$.

Timing:

- first gambler picks $(x_1(t), \dots, x_n(t))$
- then $(r_1(t), \dots, r_n(t))$ revealed
- payoff $\sum_{i=1}^{n} x_i(t) \cdot r_i(t)$ gained.

<u>Goal</u>: make combined payoff after T rounds large.

"Uniform buy-and-hold"

Initially take $W(0) \stackrel{?}{=} 1$ and divide into shares of size $\frac{1}{n}$.

Invest $\frac{1}{n}$ in each asset i.

time $t > 0$: reinvest whatever was in asset i at end of prev. round into asset i in this round.

Let $r_i(1:t) = \sum_{s=1}^{t} r_i(s)$.

At end of period t, asset i has grown by factor of
$$\prod_{s=1}^{t} (1+\varepsilon)^{r_i(s)} = (1+\varepsilon)^{r_i(1:t)}$$
relative to time $\emptyset$.

Buy-and-hold investor has

$w_i(t)$

$$W(t) = \sum_{i=1}^{n} \boxed{\frac{1}{n} \cdot (1+\varepsilon)^{r_i(1:t)}}$$
wealth at end of round t.

Uniform buy-and-hold means setting

$$x_i(t) = \frac{w_i(t-1)}{W(t-1)}.$$

Suppose i^* is the asset with best growth rate over T periods.

$$i^* \in \arg\max \{ r_i(1:T) \}.$$

Then investor's wealth, using UB&H, satisfies

$$W(T) \geq \frac{1}{n} \cdot (1+\varepsilon)^{r_{i^*}(1:T)}.$$

strict when $n>1$.