19 Sep 2025 LP Algorithms

Problem (Vertex Cover)

Given undirected graph $G = (V, E)$.
Find a set of vertices, S, such that every edge has at least one endpoint belonging to S.
Minimize number of vertices in S.

Decision variables $\{x_v\}_{v \in V}$ where

$x_v = 1$ meant to encode $v \in S$.
$x_v = 0$ " " " " $v \notin S$.

$$\min \sum_{v \in V} x_v$$

(P) s.t. (y_e) $x_u + x_v \geq 1$ $\forall$ edge $(u,v) \in E$.

$x_v \geq 0$ $\forall$ vertex v.

$$\max \sum_{e \in E} y_e$$

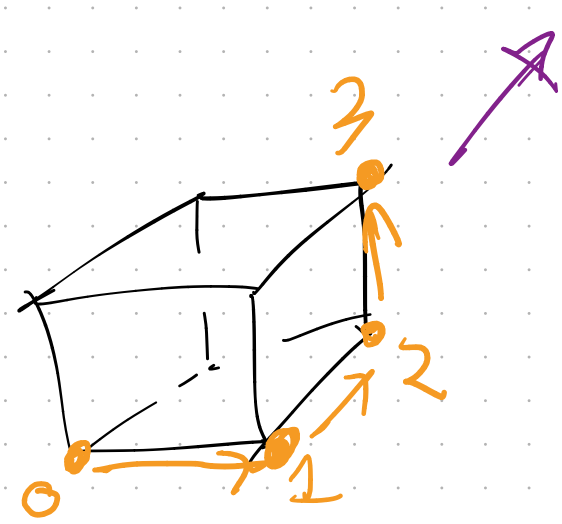
(D) s.t. $\sum_{e:\ v \text{ an endpoint of } e} y_e \leq 1$ $\forall$ vertex v

$y_e \geq 0$ $\forall$ edge e

1. Simplex algorithm

The constraints defining the LP determine a high-dimensional polyhedron.

Simplex alg performs local search on the vertices, moving to a neighboring vertex that improves the optimization objective until no further improvement is possible.



Klee-Minty Cube: $\max \quad x_n$

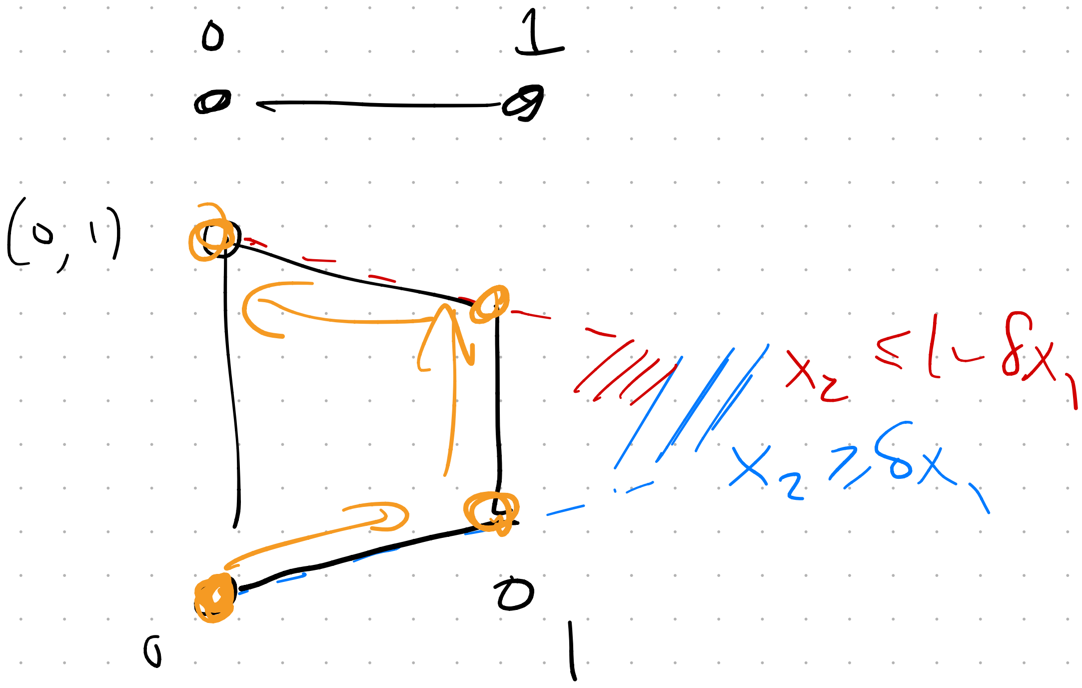
$$\text{s.t.} \quad 0 \le x_1 \le 1$$

$$\delta x_1 \le x_2 \le 1 - \delta x_1$$

$$\delta x_2 \le x_3 \le 1 - \delta x_2$$

$$\vdots$$

$$\delta x_{n-1} \le x_n \le 1 - \delta x_{n-1}$$


0
1
(0, 1)
$x_2 \le 1 - \delta x_1$
$x_2 \ge \delta x_1$
0
0
1