Reducing determinant to ABF.

Thinking about permutations using cycle diagrams.

\[ \begin{align*}
1 & \rightarrow 4 \\
2 & \rightarrow 6 \\
3 & \rightarrow 3 \\
4 & \rightarrow 5 \\
5 & \rightarrow 1 \\
6 & \rightarrow 2 \\
\end{align*} \]

\[ \text{5 permutations of } [\pi]^2 \leftrightarrow \text{cycle cover of } K_n \]

monomial correspond to \( \pi \) in the \( \text{poly} \cdot \) (product of edge labels)

\[ \text{to } \pi \in \text{poly} \leftrightarrow (-1)^{(n-(d \text{ cycles})} \]
Vertices in $L_k$ are denoted $v[i, j, k]$ grouped into gadgets $G_j$ with $i \geq j$.

Edge set: $v[j, j, k]$ has incoming edges from $v[i, j', k-1]$ for $j' < j$.

$v[i, i+1, k]$, for $i > j$, has incoming edges from $v[i', i, k-1]$ for $i' \neq i$. 
Label edge \((i', j, k'] \rightarrow [i, j, k]\)

with \(-X_{i', i}\)

Label edge\
\[
\begin{align*}
\nu[i, j', k'] & \rightarrow \nu[j, j, k] \\
\nu[i, j', n] & \rightarrow \varepsilon
\end{align*}
\]

with \(+X_{i, j'}\)

Label \(S \rightarrow \nu[i, i, i']\) with 1.

All paths that don't arise from cycle covers contribute 0 in total to the ABF value.
To find a bipartite perfect match:

1. Assign random costs
   \[ c(i,j) \in \{ 0, 1, \ldots, m/2 \} \]

2. Assign values to ABP variables according to
   \[ x_{ij} \leftarrow x(i,j) \]

3. Monomial \[ \pm \prod_{(i,j) \in M} x_{ij} \]
   becomes \[ \pm 2^{\text{cost}(M)} \]
MWV Isolation Lemma:
If \( \mathcal{F}_0 \) is any family of subsets of \([m]\) and \( c(i) \) for \( i \in [m] \)
we define RVs \( u \) in
\[ u_0, \ldots, u_m \text{ s.t.} \]
then with probability \( \geq 1 - \delta \) there is a unique element of \( \mathcal{F}_0 \)
of minimum total cost.

Proof. Call \( i \in [m] \) "confused" if
min-cost set containing \( i \) and
min-cost set excluding \( i \)
have same cost.

Min-cost set is non-unique \( \iff \exists \) confused element.

\[
\Pr[\text{min-cost non-uniq}] \\
\leq \Pr(\exists \text{ confused element}) \\
\leq \mathbb{E}[\# \text{ confused}] \\
= \sum_{i=1}^{m} \Pr(i \text{ confused}).
\]

Fix all costs except \( u_i \),
\( S_0 = \min \text{ cost set excluding } i \)
\[ S_i = \quad \text{containing } i \]
\[ \text{cost}(S_0) \]
\[ \text{cost}(S_i \setminus \{i\}) \]
are all predetermined.

\( i \) confused if
\[ \text{cost}(i) = \text{cost}(S_i) - \text{cost}(S_i \setminus \{i\}) \]
\[ \Pr \left( \text{cost}(i) = \text{RHS} \right) \leq \frac{5}{m}. \]