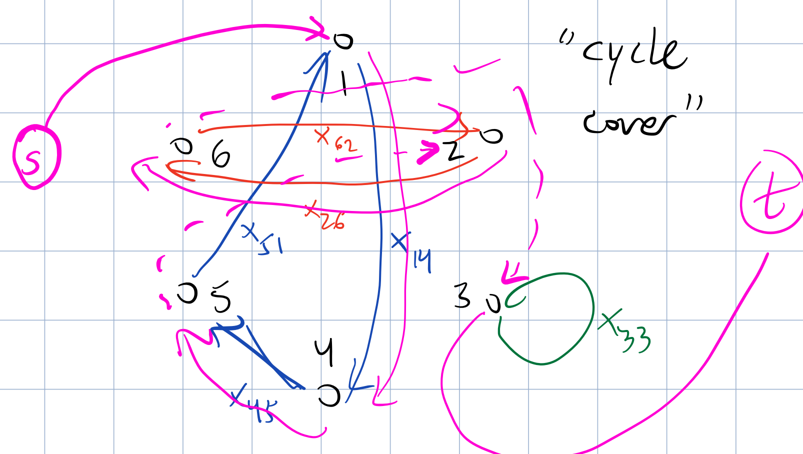
$\boxed{\text{24 Sep 2021}}$ Finish Parallel Bipartite Matching
Start Network Flow

Reducing determinant to ABP.

Thinking about permutations using cycle diagrams.

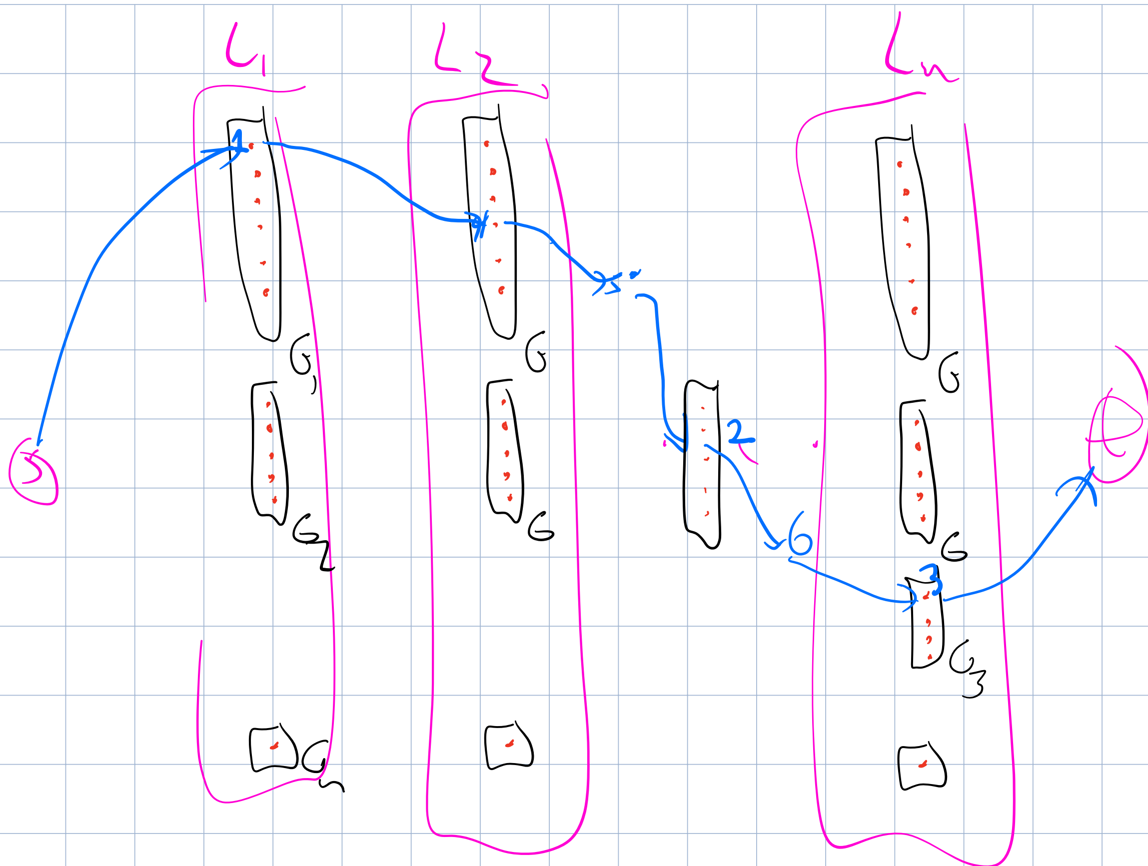
$1 \mapsto 4$
$2 \mapsto 6$
$3 \mapsto 3$
$4 \mapsto 5$
$5 \mapsto 1$
$6 \mapsto 2$



"cycle cover"

$$\{\text{permutations of } [n]\} \longleftrightarrow \left\{\begin{array}{l}\text{cycle covers} \\ \text{of } K_n\end{array}\right\}$$

monomial corresp. to σ in the determ. poly. $\longmapsto (-1)^{n-(\#\text{ cycles})} \cdot \left(\text{product of edge labels}\right)$



vertices in L_k are denoted

$v[i,j,k]$ grouped into gadgets G_j.

with $i \geq j$.

Edge set: $v[j,j,k]$ has incoming edges from $v[i,j',k-1]$ $\forall j' < j$.

$v[i,j,k]$ for $i>j$ has incoming edges from $v[i',j,k-1]$ $\forall i' \neq i$.

Label edge $(i', j, k-1] \longrightarrow [i, j, k]$

with $-x_{i', i}$

Label edge $v[i, j', k-1] \longrightarrow v(j, j, k]$

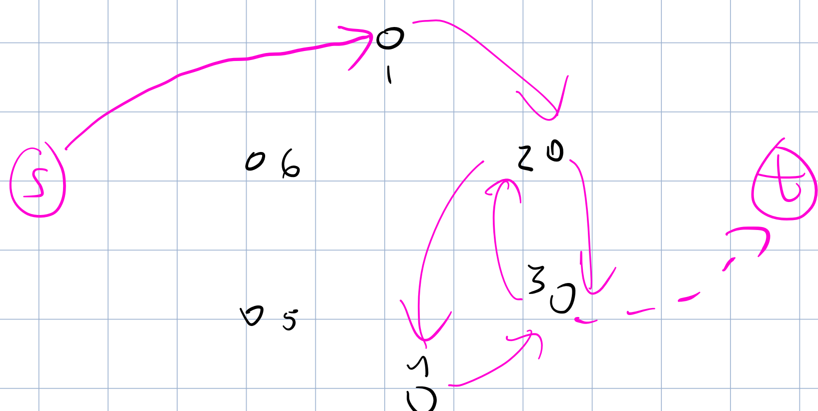
or $v[i, j', n] \longrightarrow t$

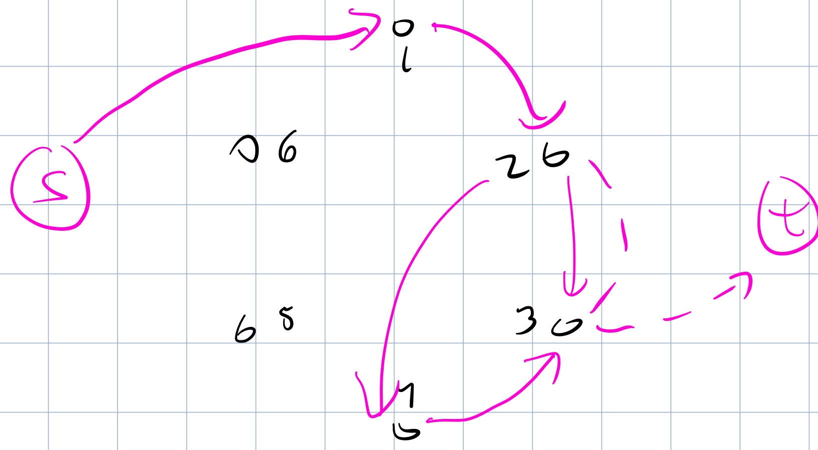
with

$+x_{i, j'}$

Label $s \longrightarrow v[i, i, 1]$ with 1.

All paths that don't arise from cycle covers contribute $\emptyset$ in ~~total~~ to the ABP value.





To find a bipartite perf match:

1. Assign random costs
   $$c(i,j) \in \{0, 1, \ldots, m/\delta\}$$

2. Assign values to ABP variables according to
   $$x_{ij} \leftarrow 2^{c(i,j)}.$$

3. Monomial $\pm \prod_{(i,j)\in M} x_{ij}$

   becomes
   $$\pm 2^{\mathrm{cost}(M)}.$$

MVV Isolation Lemma:

If $\mathcal{F}$ is any family of subsets of $[m]$, and $c(i)$ for $i \in [m]$ are indep RVs unif in $\{0, ---, m/\delta\}$ then with probability $\geq 1-\delta$ there is a unique element of $\mathcal{F}$ of minimum ~~total~~ cost.

Proof. Call $i \in [m]$ "confused" if min-cost set containing i and min-cost set excluding i have same cost.

Min-cost set is non-unique $\iff \exists$ confused element.

$$\begin{aligned}
\Pr[\text{min-cost non-uniq}] &= \Pr(\exists \text{ confused element}] \\
&\leq \mathbb{E}[\# \text{ confused}] \\
&= \sum_{i=1}^{m} \Pr(i \text{ confused}).
\end{aligned}$$

Fix all costs except i,

S_0 = min cost set excluding i

S_1 = - - - - containing i

$\text{cost}(S_0)$

$\text{cost}(S_1 \setminus \{i\})$

are all predetermined.

i confused iff $\text{cost}(i) = \text{cost}(S_0) - \text{cost}(S_1 \setminus \{i\})$.

$\Pr(\text{cost}(i) = \text{RHS}) \leq \frac{\delta}{m}$.