CS 682 Homework #4
Due February 28, 2001

For $A,\ B\ \subseteq\ \Sigma^*$, let

$$\text{A/B} = \{\ x \mid (\exists y)\ [xy \ \epsilon\ A \text{ and } y \ \epsilon\ B]\}.$$

1. Prove that for any $L(M_i)$ there exist two context-free grammers, G_j and G_k such that

$$L(G_j)/L(G_k) \ = \ L(M_i).$$

2. Let $I_1,\ I_2, \ldots$ be a recursive list of FA with a two-way head that can scan the input symbols and either erase them or leave them unaltered.

   Prove that

$$\{I_j \mid L(I_j) \text{ is finite}\} \text{ is } \Sigma_2\text{-complete.}$$

3. (a) Prove that there exists an infinite LBA language $L(\mathcal{L}_i)$ that is immune to all regular sets, i.e., $L(A_j)\ \subseteq\ L(\mathcal{L}_i)$ implies that $L(A_j)$ is finite.

   (b) Is there an infinite recursive set immune to all LBA languages? Prove the correctness of your answer.