CS 682 Homework #3
Due February 21, 2001

Let H_i denote a two-head read-only finite automaton whose heads move independently from left-to-right.

1. Prove that

$$\{\ H_i \mid L(H_i) \text{ is cofinite}\}$$

   is a Σ_2-complete set.

2. Place the following sets in the lowest possible class of the Kleene Hierarchy. Give a short justification for your answer, no formal proof of the hardness is required. As usual, G_i, $\mathcal{L}_i$ and A_i denote context-free grammars, linearly-bounded automata and finite automata, respectively.

   (a) $\{H_i \mid\mid L(H_i) \mid\ = \ \infty\}$, $\{G_i \mid\mid L(G_i) \mid\ = \ \infty\}$

   (b) $\{(M_i, H_j) \mid L(M_i)\ \cap\ L\ (H_j) \text{ is regular}\}$

   (c) $\{(A_i, G_j) \mid L(A_i)\ \subseteq\ L(G_j)\}$

   (d) $\{(A_i, G_j) \mid L(G_j)\ \subseteq\ L(A_i)\}$

   (e) $\{M_i \mid L(M_i) \text{ is an infinite context-free language}\}$

   (f) $\{M_i \mid M_i$ accepts an infinite number of TM-names of TMs that accept cofinite sets$\}$.

3. Let P be a non-trivial property on the r.e. sets that is false for context-free languages. Prove that

$$\{M_i \mid P(L(M_i) = \text{True}\}$$

   is Π_2 - hard.