CS 681 Fall 2001 – Problem Set 6

Due in class Monday, December 3

(1) Consider the following simple model of gambling in the presence of bad odds. At the beginning, your net profit is 0. You play for a sequence of n rounds; and in each round, your net profit increases by 1 with probability 1/3, and decreases by 1 with probability 2/3.

Using the Chernoff bounds (4.1) and/or (4.2) from the handout, show that the expected number of steps in which your net profit is positive can be upper-bounded by an absolute constant, independent of the value of n.

(2) Suppose you are presented with a very large set S of real numbers, and you'd like to approximate the median of these numbers by sampling. You may assume all the numbers in S are distinct. Let $n = |S|$; we will say that a number x is an *ε-approximate median* of S if at least $(\frac{1}{2} - \varepsilon)n$ numbers in S are less than x, and at least $(\frac{1}{2} - \varepsilon)n$ numbers in S are greater than x.

Consider an algorithm that works as follows. You select a subset $S' \subseteq S$ uniformly at random, compute the median of S', and return this as an approximate median of S. Show that there is an absolute constant c, independent of n, so that if you apply this algorithm with a sample S' of size c, then with probability at least .99, the number returned will be a (.05)-approximate median of S. (You may consider either the version of the algorithm that constructs S' by sampling with replacement, so that an element of S can be selected multiple times, or without replacement.)