CS 681 Fall 2001 – Problem Set 3

Due in class Wednesday, October 17

(1) Consider the following problem. You are given a flow network with unit-capacity edges: it consists of a directed graph $G = (V, E)$, a source $s \in V$, and a sink $t \in V$; and $c_e = 1$ for every $e \in E$. You are also given a parameter k.

The goal is delete k edges so as to reduce the maxmimum s-t flow in G by as much as possible. In other words, you should find a set of edges $F \subseteq E$ so that $|F| = k$ and the maximum s-t flow in $G' = (V, E - F)$ is as small as possible subject to this.

Give a polynomial-time algorithm to solve this problem.

(2) Suppose you're managing a collection of processors and must schedule a sequence of jobs over time.

The jobs have the following characteristics. Each job j has an arrival time a_j when it is first available for processing, a length ℓ_j which indicates how much processing time it needs, and a deadline d_j by which it must be finished. (We'll assume $0 < \ell_j \leq d_j - a_j$.) Each job can be run on any of the processors, but only on one at a time; it can also be pre-empted and resumed from where it left off (possibly after a delay) on another processor.

Moreover, the collection of processors is not entirely static either: you have an overall pool of k possible processors; but for each processor i, there is an interval of time $[t_i, t_i']$ during which it is available; it is unavailable at all other times.

Given all this data about job requirements and processor availability, you'd like to decide whether the jobs can all be completed or not. Give a polynomial-time algorithm that either produces a schedule completing all jobs by their deeadlines, or reports (correctly) that no such schedule exists. You may assume that all the parameters associated with the problem are integers.

Example. Suppose we have two jobs J_1 and J_2. J_1 arrives at time 0, is due at time 4, and has length 3. J_2 arrives at time 1, is due at time 3, and has length 2. We also have two processors P_1 and P_2. P_1 is available between times 0 and 4; P_2 is available between times 2 and 3. In this case, there is a schedule that gets both jobs done:

- At time 0, we start job J_1 on processor P_1.
- At time 1, we pre-empt J_1 to start J_2 on P_1.
- At time 2, we resume J_1 on P_2. (J_2 continues processing on P_1.)
- At time 3, J_2 completes by its deadline. P_2 ceases to be available, so we move J_1 back to P_1 to finish its remaining one unit of processing there.
- At time 4, J_1 completes its processing on P_1.

Notice that there is no solution that does not involve pre-emption and moving of jobs.

(3) Given a list of n natural numbers $d_1, d_2, \ldots, d_n$, show how to decide in polynomial time whether there exists an undirected graph $G = (V, E)$ whose node degrees are precisely the numbers $d_1, d_2, \ldots, d_n$. (That is, if $V = \{v_1, v_2, \ldots, v_n\}$, then the degree of v_i should be exactly d_i.) G should not contain multiple edges between the same pair of nodes, or "loop" edges with both endpoints equal to the same node.

(Optional 4th Question.) You are given a graph $G = (V, E)$, and a set of nodes $S \subseteq V$. Give a polynomial-time algorithm to find a maximum-size set of paths in G such that (i) the ends of each path are distinct nodes in S, and (ii) no two paths have any nodes of V in common.

(Hint: Notice that if $S = V$, then G has a set of such paths of size $n/2$ if and only if it has a perfect matching. So any algorithm for this problem will immediately give a matching algorithm as well.)