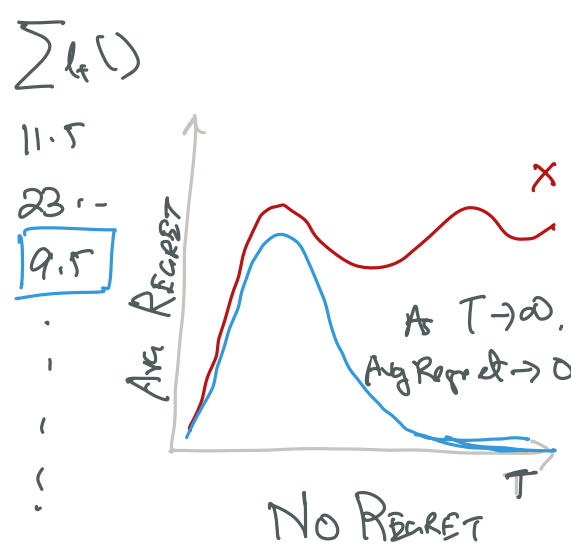


N EXPERTS

	$l_1()$	$l_2()$	$l_3()$			
π^1	0.1	π_2 1.0	0.1	-	-	-
π^2	π_1 1.0	0.1	0.1			
π^3	0.1	0.1	0.5 π_3			
\vdots	\vdots	\vdots	\vdots			
\vdots	\vdots	\vdots	\vdots			
π^N	\vdots	\vdots	\vdots			



$\pi_1 = \pi^2$
 Avg. REGRET = $\frac{1}{T} \left[\sum_{t=1}^T l_t(\pi_t) - \min_{\pi^* \in \{\pi^1, \dots, \pi^N\}} \sum_{t=1}^T l_t(\pi^*) \right]$
Loss of LEARNER BEST EXPERT IN HINDSIGHT

BOUND THE REGRET FOR FTL

$$FTL \quad \pi_t = \underset{\pi}{\operatorname{argmin}} \sum_{i=1}^{t-1} l_i(\pi) \quad \text{REGRET} = \sum_{t=1}^T l_t(\pi_t) - \min_{\pi^*} \sum_{t=1}^T l_t(\pi^*)$$

$$\pi_2 = \underset{\pi}{\operatorname{argmin}} l_1(\pi) \quad \Rightarrow \quad \underbrace{l_1(\pi_2)}_{\leq l_1(\pi^*)} \leq l_1(\text{of any policy } \pi)$$

$$\pi_3 = \underset{\pi}{\operatorname{argmin}} l_1(\pi) + l_2(\pi) \quad \Rightarrow \quad \underbrace{l_1(\pi_3)}_{\leq l_1(\pi^*)} + l_2(\pi_3) \leq l_1(\pi^*) + l_2(\pi^*)$$

$$\underbrace{l_1(\pi_2)}_{\leq l_1(\pi^*)} + l_2(\pi_3) \leq \underbrace{l_1(\pi_3)}_{\leq l_1(\pi^*)} + l_2(\pi_3) \leq l_1(\pi^*) + l_2(\pi^*)$$

$$l_1(\pi_2) \leq l_1(\pi^*)$$

$$l_1(\pi_2) + l_2(\pi_3) \leq l_1(\pi^*) + l_2(\pi^*)$$

$$l_1(\pi_2) + l_2(\pi_3) + l_3(\pi_4) + l_4(\pi_5) \leq l_1(\pi^*) + l_2(\pi^*) + l_3(\pi^*) + l_4(\pi^*)$$

$$\sum_{t=1}^T l_t(\pi_{t+1}) \leq \sum_{t=1}^T l_t(\pi^*)$$

$$\text{REGRET} = \sum_{t=1}^T l_t(\pi_t) - \min_{\pi^*} \sum_{t=1}^T l_t(\pi^*)$$

$$\text{If } \pi_t \rightarrow \pi_{\text{fix}} \text{ as } t \rightarrow \infty$$

$$\text{Avg Regret} \rightarrow 0$$

$$\lesssim \sum_{t=1}^T \left(l_t(\pi_t) - l_t(\pi_{t+1}) \right)$$