Actor-Critic Methods

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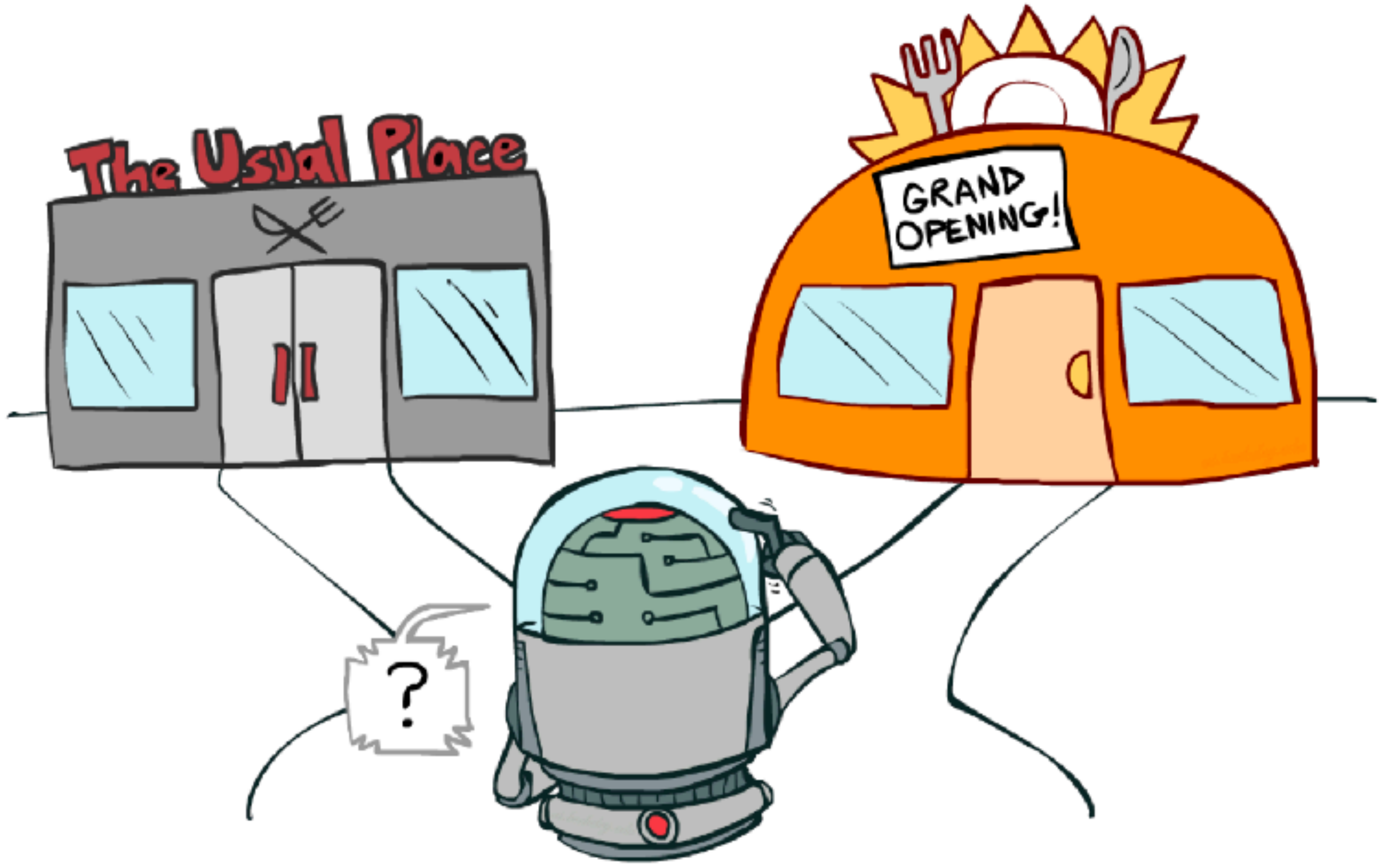
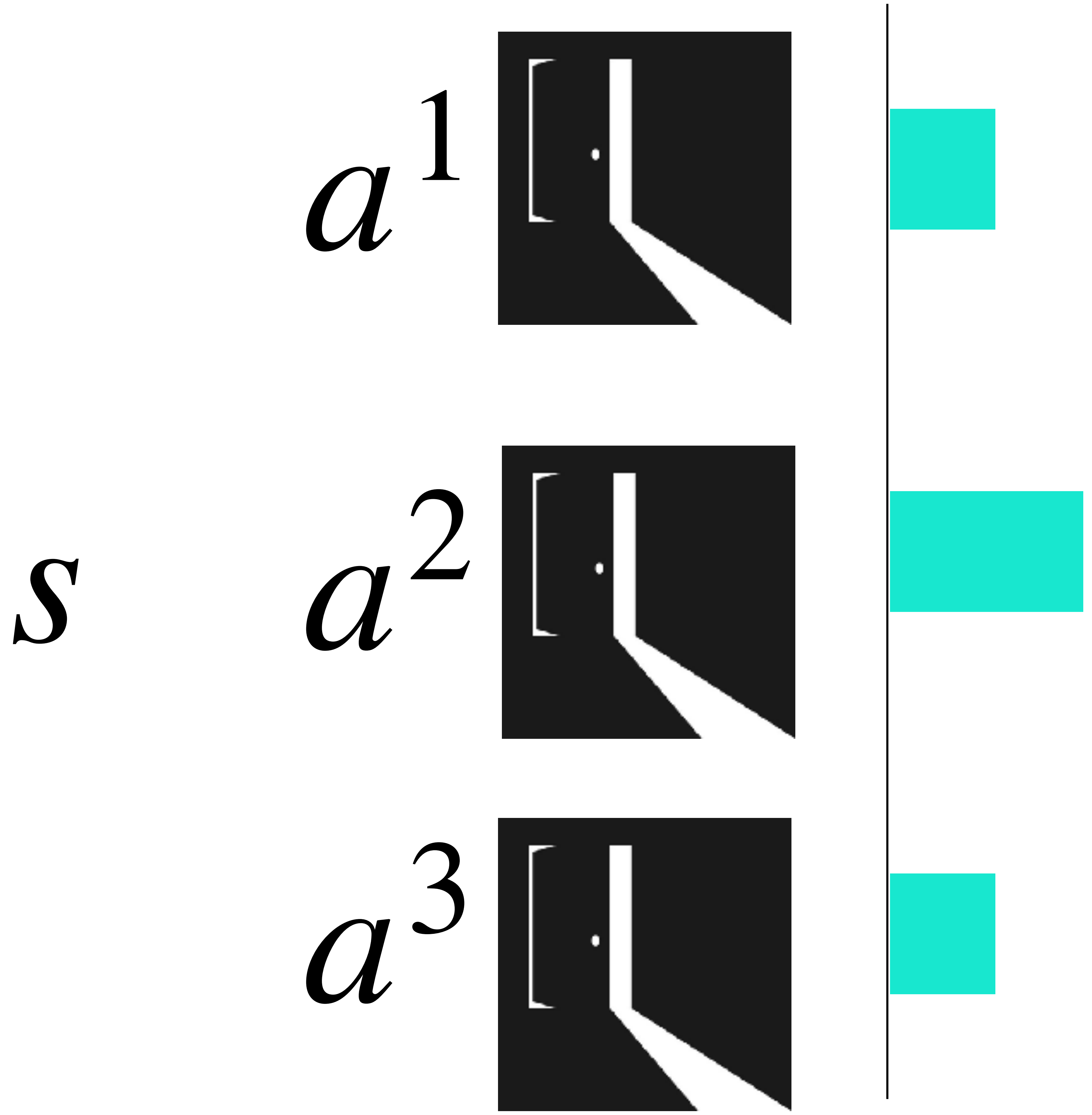
Computer Science

Recap in
60 seconds!

Recap: Two Ingredients of RL



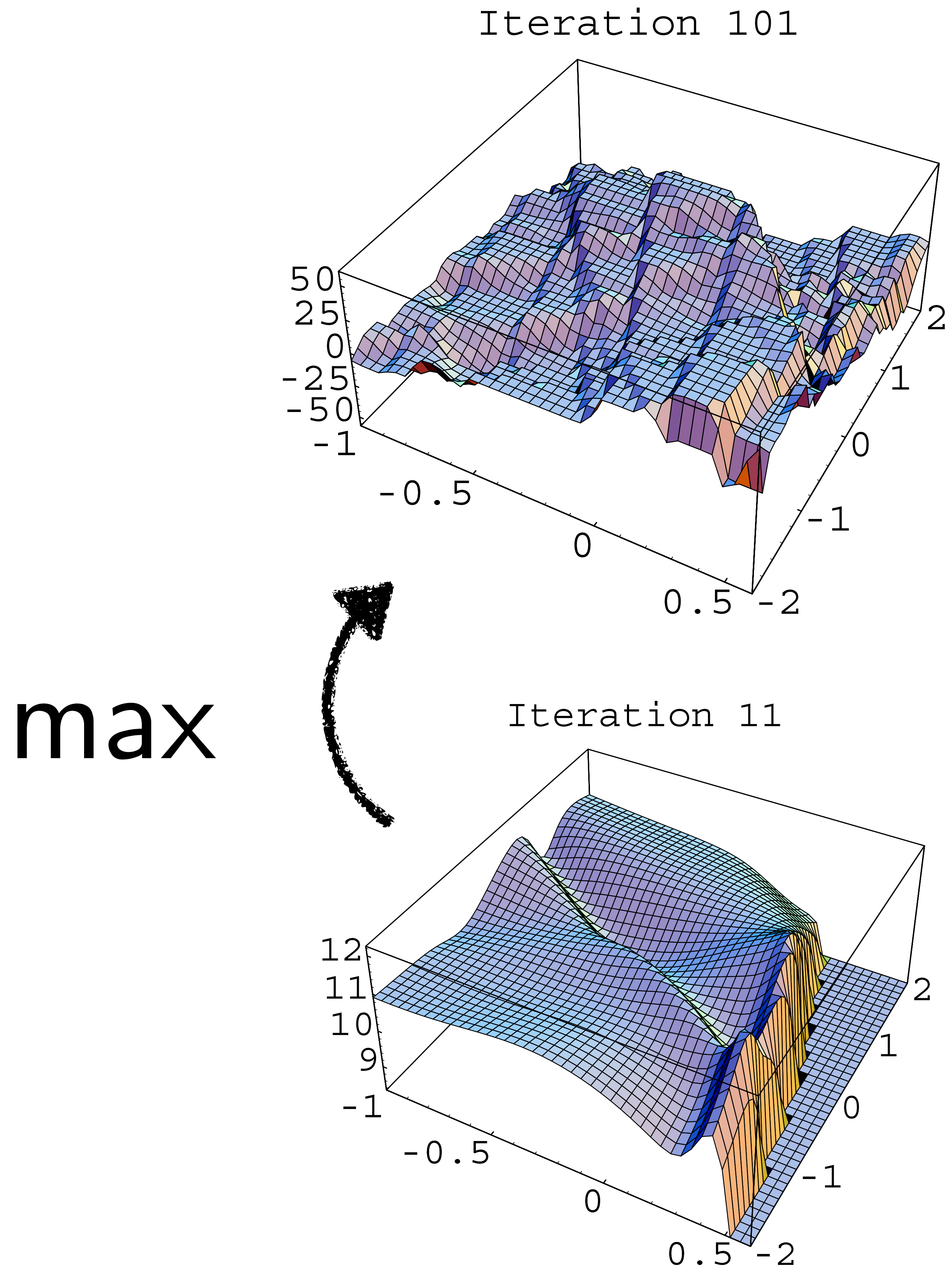
Exploration Exploitation



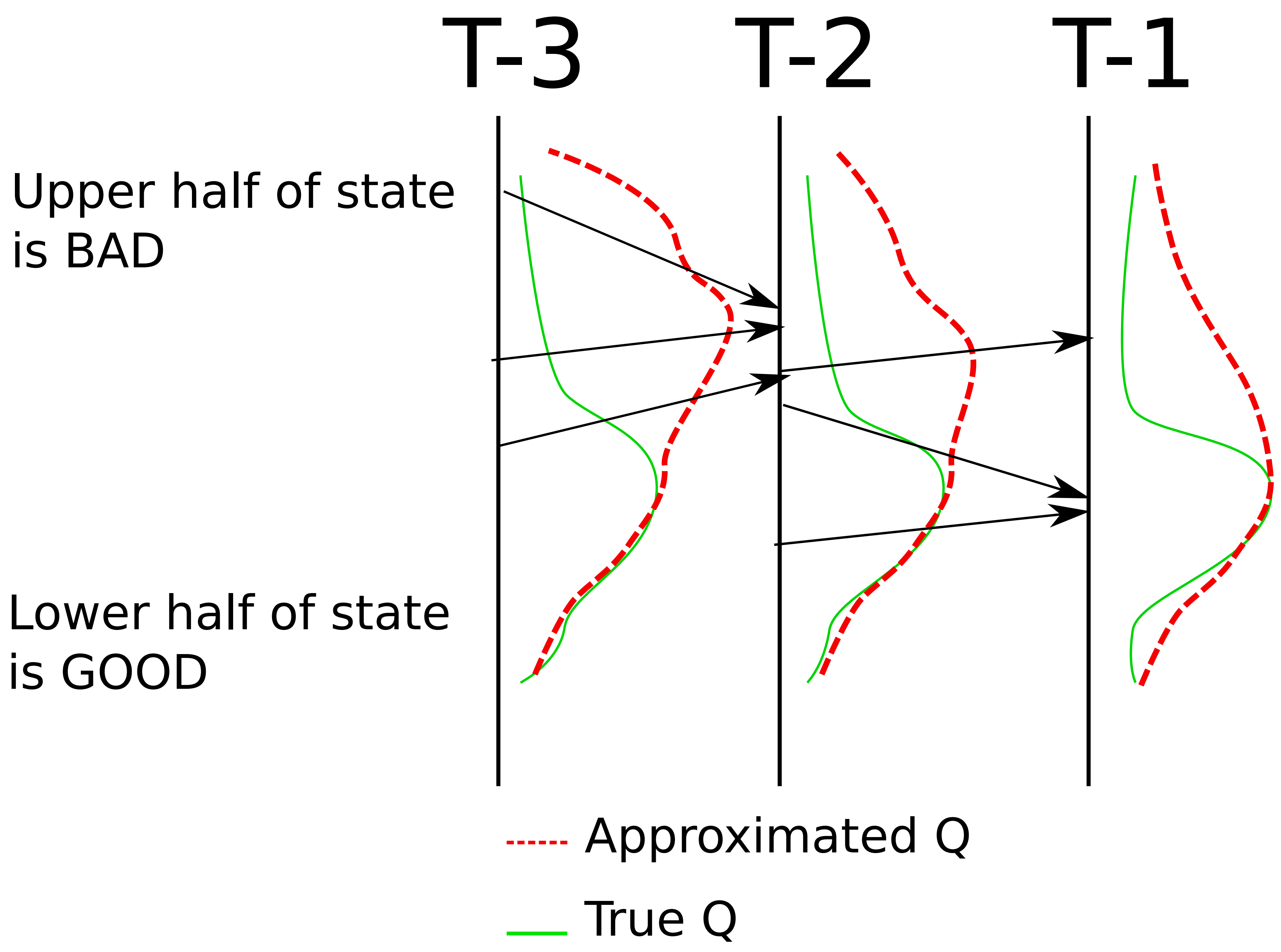
Estimate Values $Q(s, a)$

Curses of Function Approximation

Value Iteration: Bootstrapping



Policy Iteration: Distribution Shift



The Power of a Policy!

All we need at the end of the day is a good policy.

Black box: Try different policies and pick the best one

Gray box: Be smarter, push probability mass on actions that lead to high values

$$\nabla_\theta J = E_{p(\xi|\theta)}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t|s_t)\, Q^{\pi_\theta}(s_t, a_t)\right]$$

The Three Nightmares of Policy Optimization

Nightmare 1: High Variance

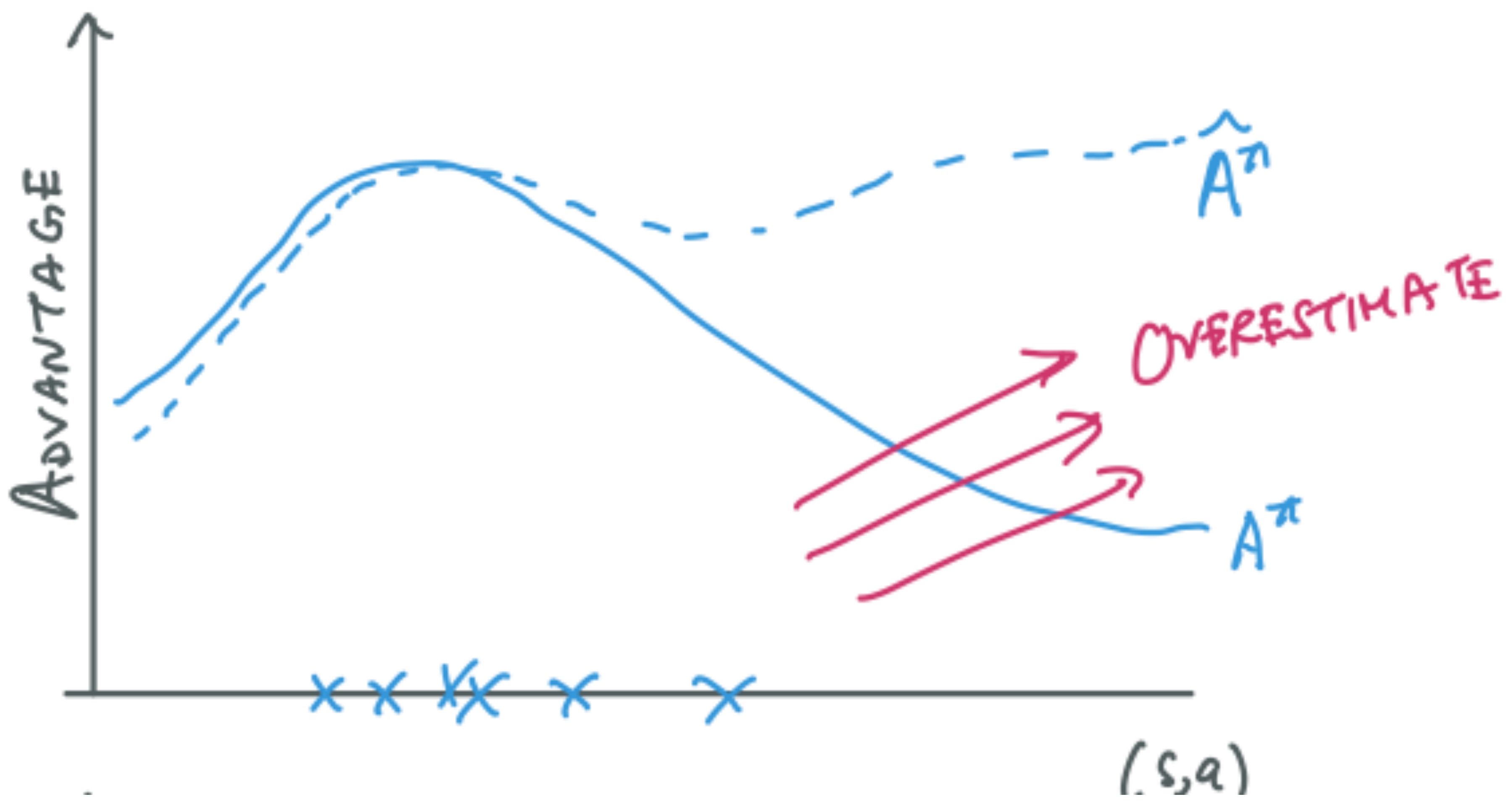
$$\nabla_\theta J = E_{s \sim d^{\pi_\theta}(s), a \sim \pi_\theta(a|s)} [\nabla_\theta \log \pi_\theta(a|s) \boxed{Q^{\pi_\theta}(s,a)}]$$

Solution: Subtract off a baseline!

$$\nabla_\theta J = E_{d^{\pi_\theta}(s)} E_{\pi_\theta(a|s)} [\nabla_\theta \log(\pi_\theta(a|s) \, (Q^{\pi_\theta}(s,a) - V^{\pi_\theta}(s))].$$

$$\nabla_\theta J = E_{d^{\pi_\theta}(s)} E_{\pi_\theta(a|s)} [\nabla_\theta \log(\pi_\theta(a|s) A^{\pi_\theta}(s,a)]$$

Nightmare 2: Distribution Shift



Solution: Take small steps!

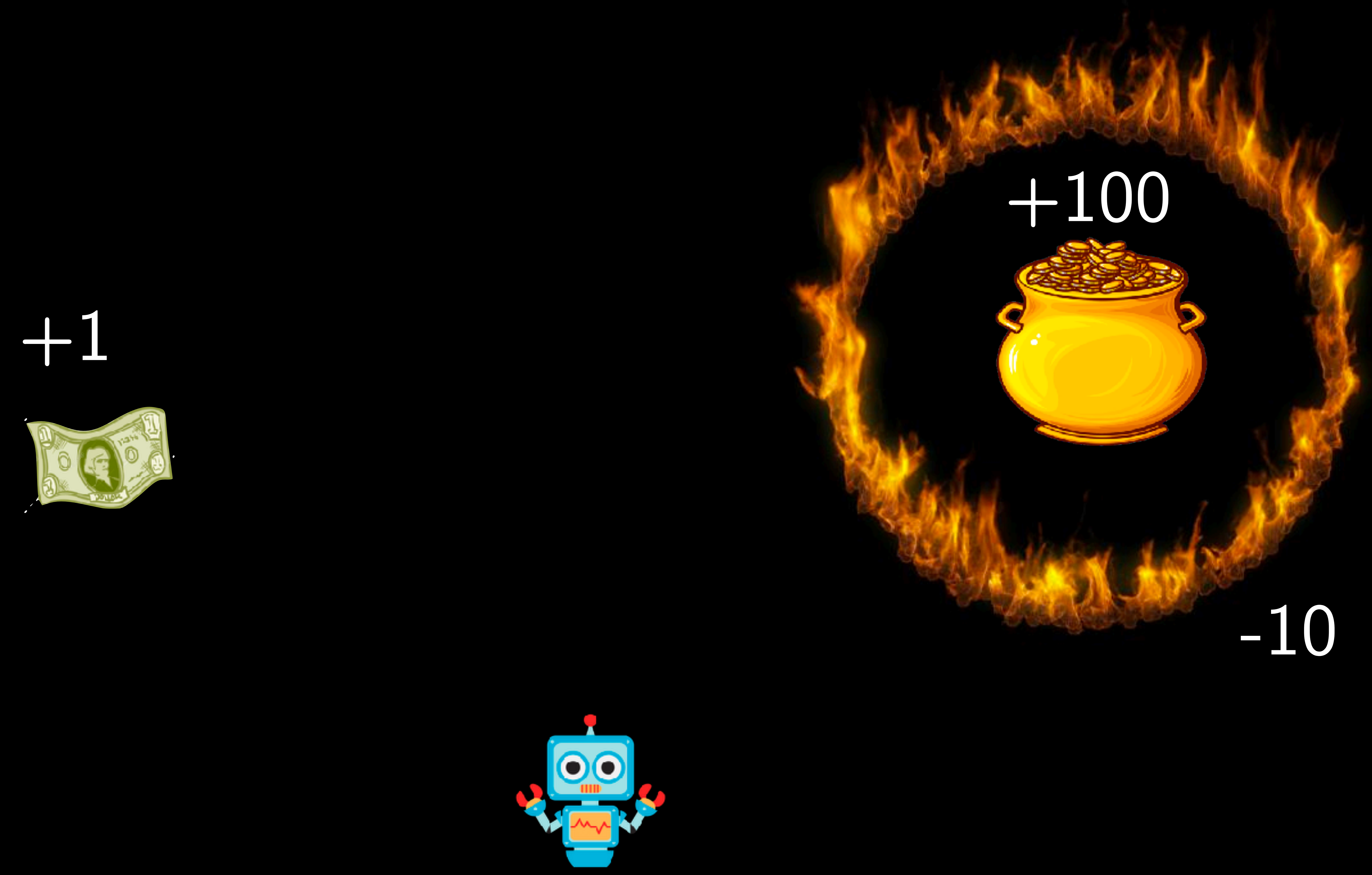
$$\max_{\Delta\theta} J(\theta + \Delta\theta)$$

s.t. $KL(\pi(\theta + \Delta\theta) || \pi(\theta)) \leq \epsilon$

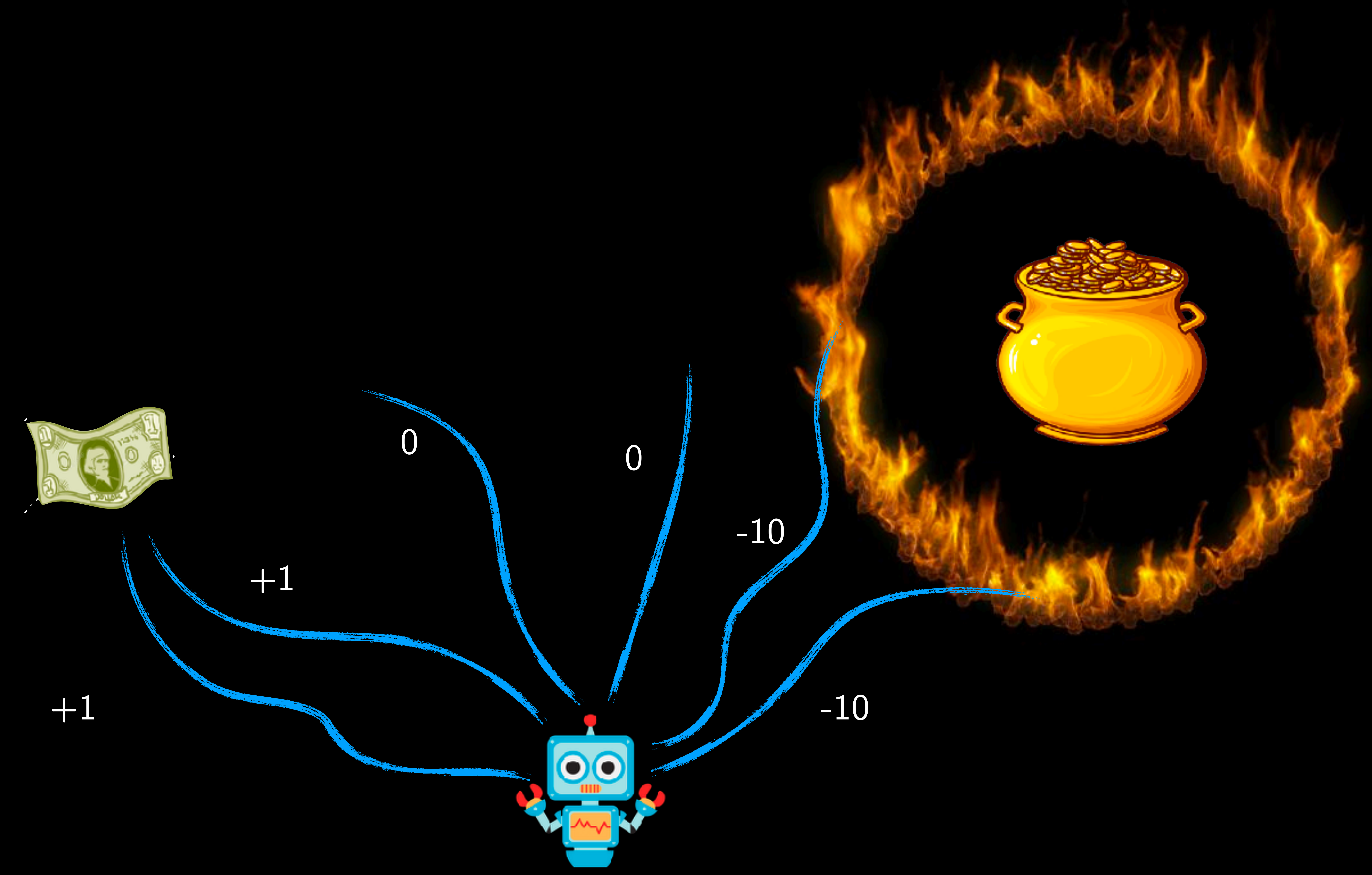
Nightmare 3:

Local Optima

The Ring of Fire

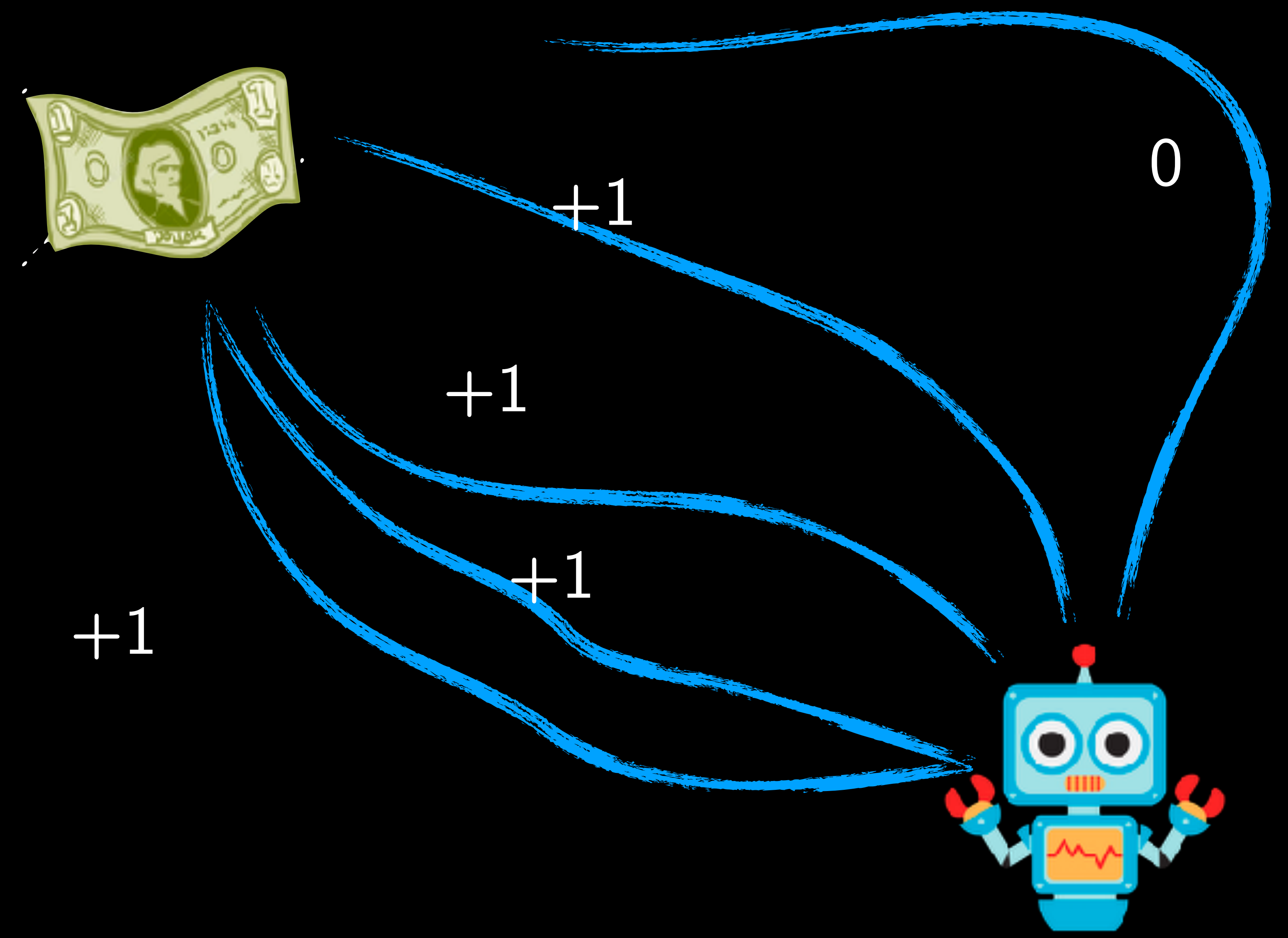


The Ring of Fire



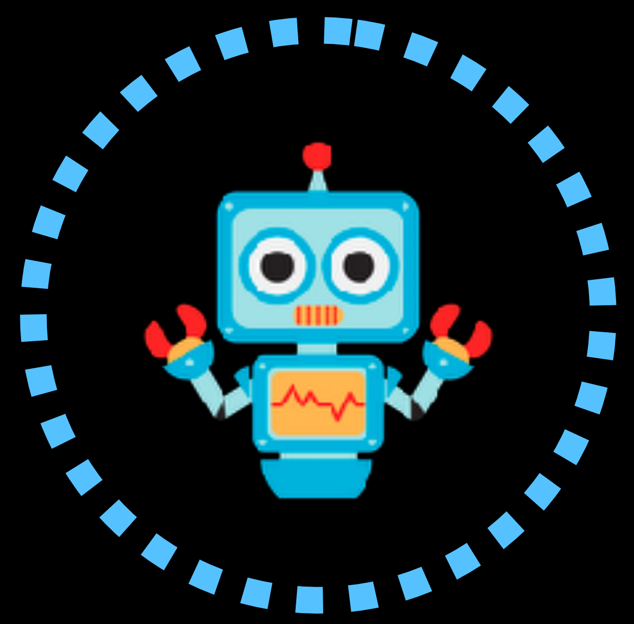
The Ring of Fire

Get's sucked into a local optima!!



Idea: What if we had a "good reset distribution?"

Start distribution



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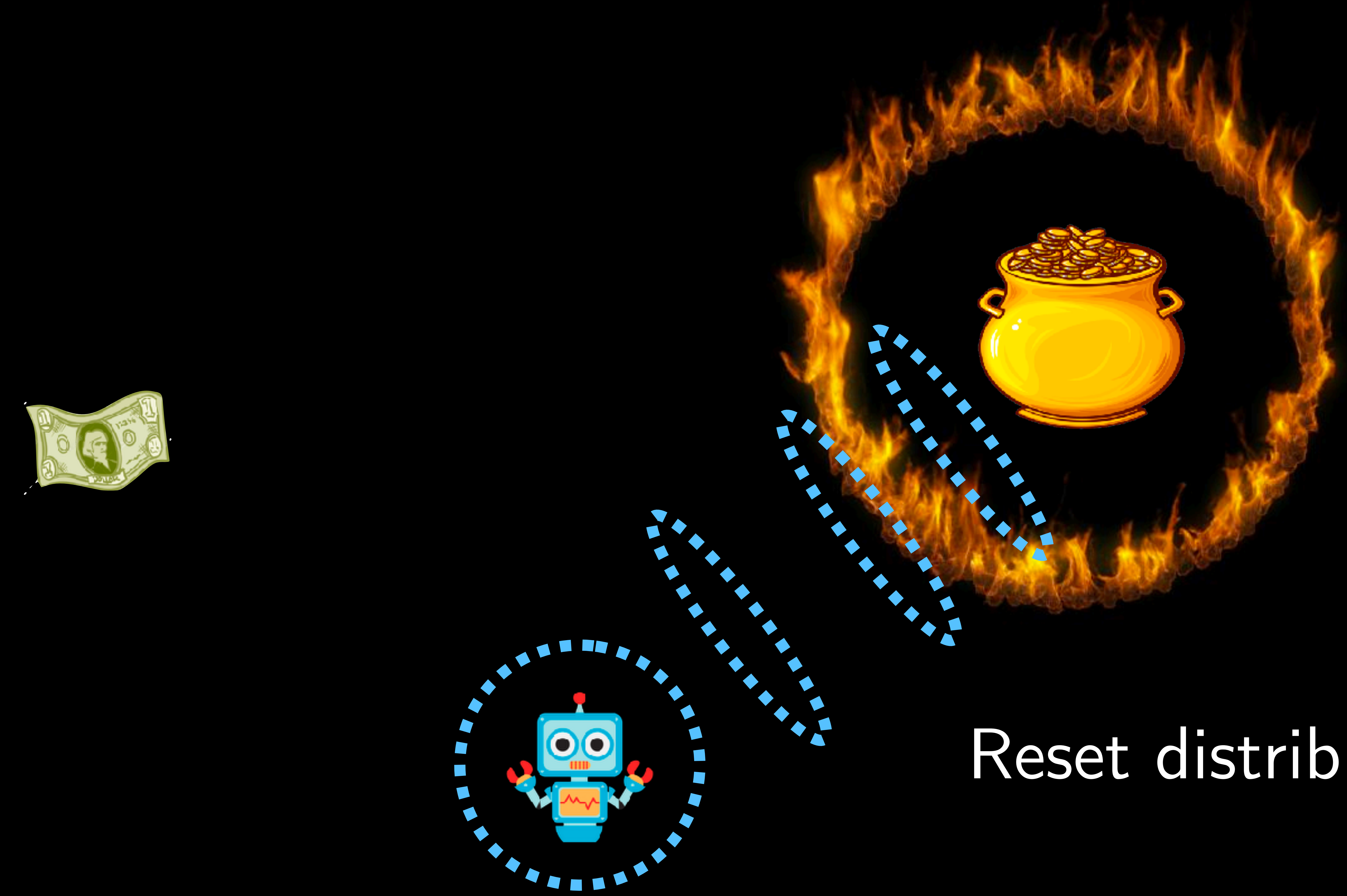
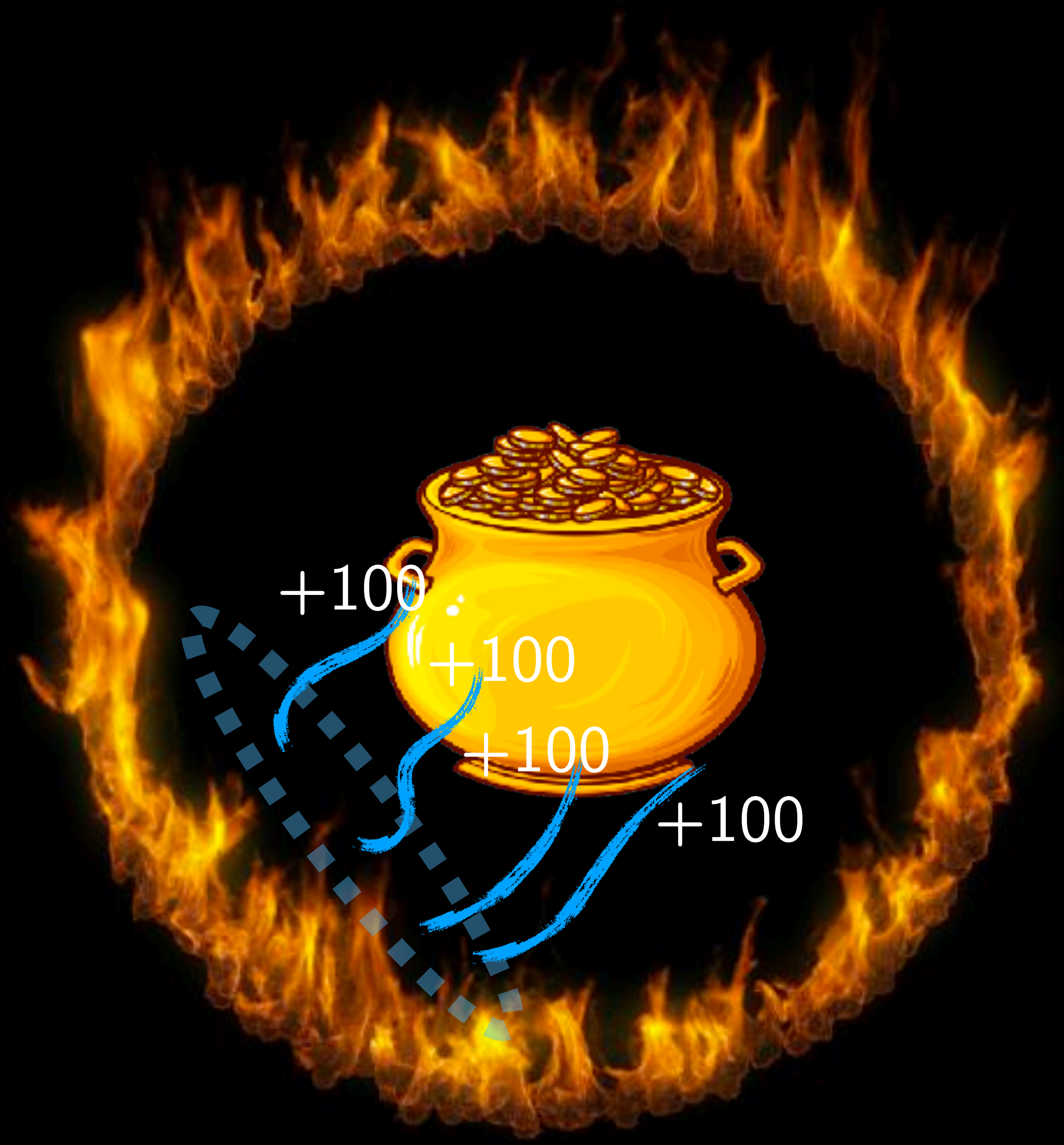


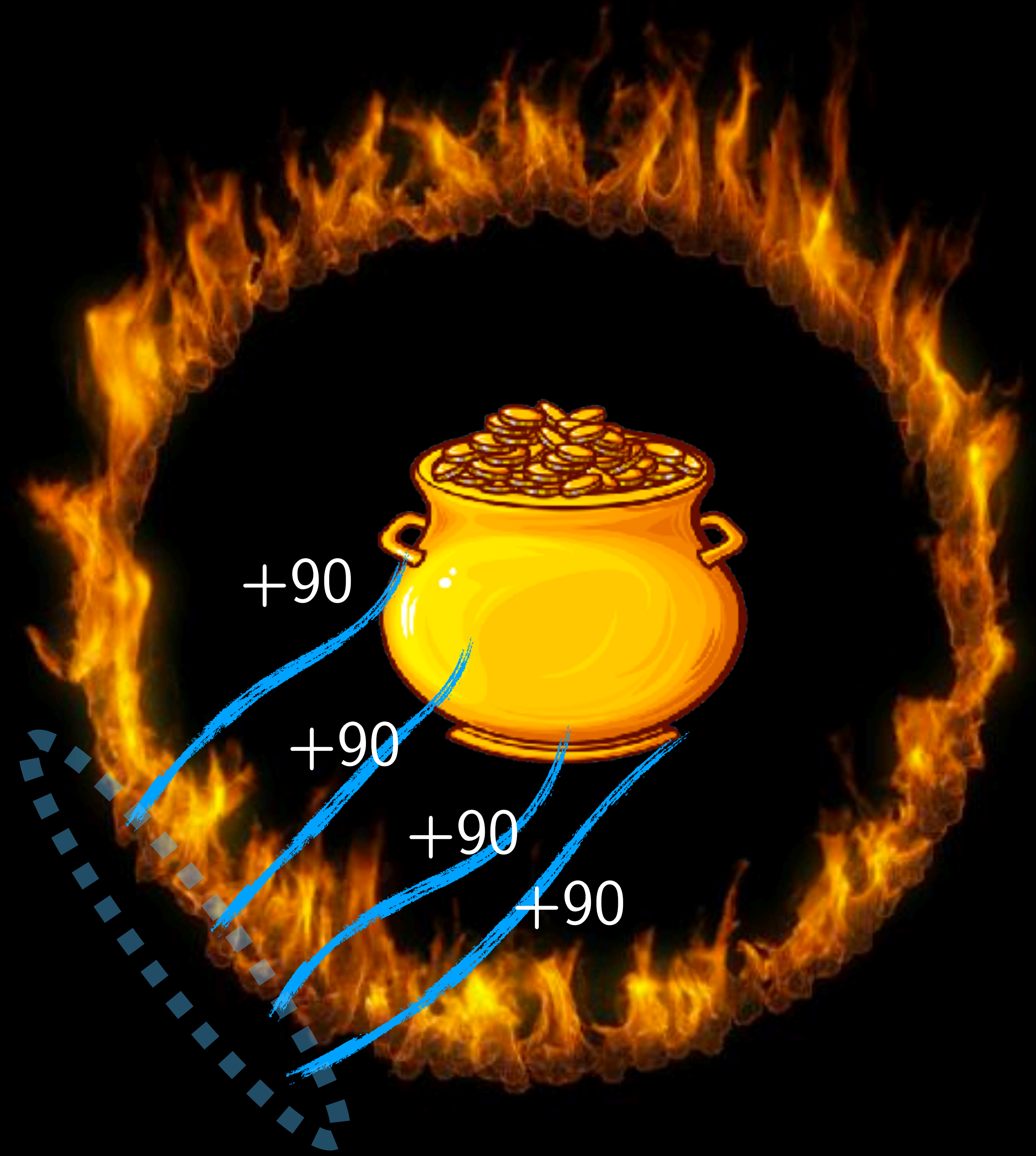
Reset distribution

Idea: What if we had a “good reset distribution?”



Run REINFORCE
from different start states

Idea: What if we had a "good reset distribution?"



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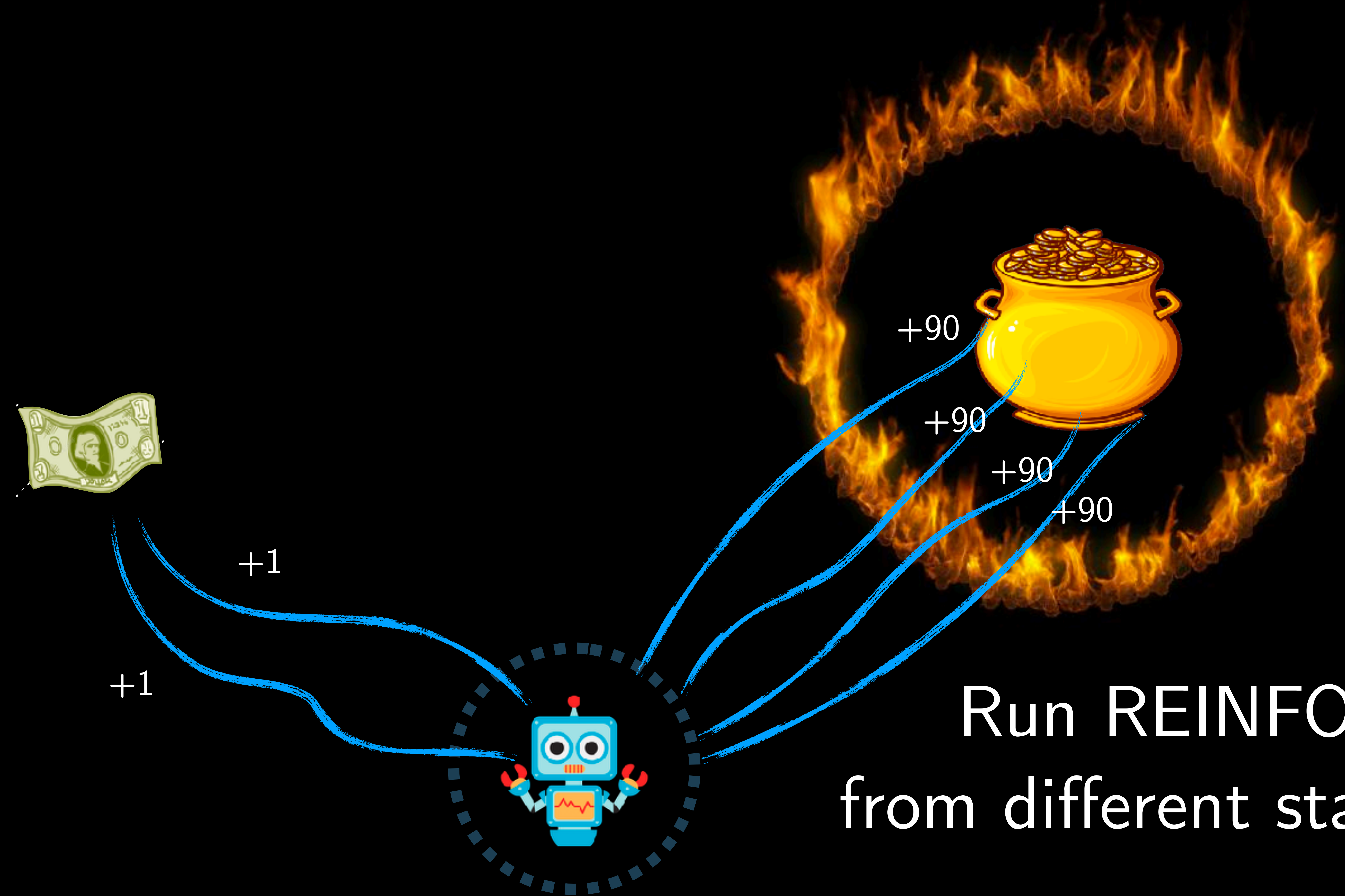

Run REINFORCE
from different start states

Solution: Use a good “reset” distribution

Choose a reset distribution $\mu(s)$ instead of start state distribution

Try your best to “cover” states the expert will visit

Justify using the PDL!

Vanilla Policy Gradient (REINFORCE)

Start with an arbitrary initial policy $\pi_\theta(a \,|\, s)$

while *not converged* **do**

Roll-out $\pi_\theta(a \,|\, s)$ to collect trajectories $D = \{s_0^i, a_0^i, r_0^i, \ldots, s_{T-1}^i, a_{T-1}^i, r_{T-1}^i\}_{i=1}^N$

Compute reward-to-go for each timestep for each trajectory

$$\hat{Q}^{\pi_\theta}(s_t^i, a_t^i) = \sum_{t'=t}^{T-1} r(s_{t'}^i, a_{t'}^i)$$

Compute gradient

$$\nabla_\theta J(\theta) = \frac{1}{N} \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^i \,|\, s_t^i) \; \hat{Q}^{\pi_\theta}(s_t^i, a_t^i) \right]$$

Update parameters $\quad \theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Let's apply the fixes to the nightmares!

Start with an arbitrary initial policy $\pi_\theta(a \mid s)$

while *not converged* **do**

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Compute gradient

$$\nabla_\theta J(\theta) = \frac{1}{N}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^i \mid s_t^i)\ \hat{Q}^{\pi_\theta}(s_t^i, a_t^i)\right]$$

Update parameters $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Fix #1: Subtract baseline

Start with an arbitrary initial policy $\pi_\theta(a \mid s)$

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Compute reward-to-go for each timestep for each trajectory $\hat{Q}^{\pi_\theta}(s_t^i, a_t^i) = \sum_{t'=t}^{T-1} r(s_{t'}^i, a_{t'}^i)$

Fit value function $\hat{V}^{\pi_\theta}(s_t^i) \approx \sum_{t'=t}^{T-1} r(s_{t'}^i, a_{t'}^i)$

How??

Compute advantage $\hat{A}^{\pi_\theta}(s_t^i, a_t^i) = \hat{Q}^{\pi_\theta}(s_t^i, a_t^i) - \hat{V}^{\pi_\theta}(s_t^i)$

Compute gradient

$$\nabla_\theta J(\theta) = \frac{1}{N}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a_t^i \mid s_t^i)\, \hat{A}^{\pi_\theta}(s_t^i, a_t^i)\right]$$

Update parameters $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Fitting values!

Monte-Carlo

$$V(s) \leftarrow V(s) + \alpha(G_t - V(s))$$

Needs full time-horizon trajectories

Temporal Difference

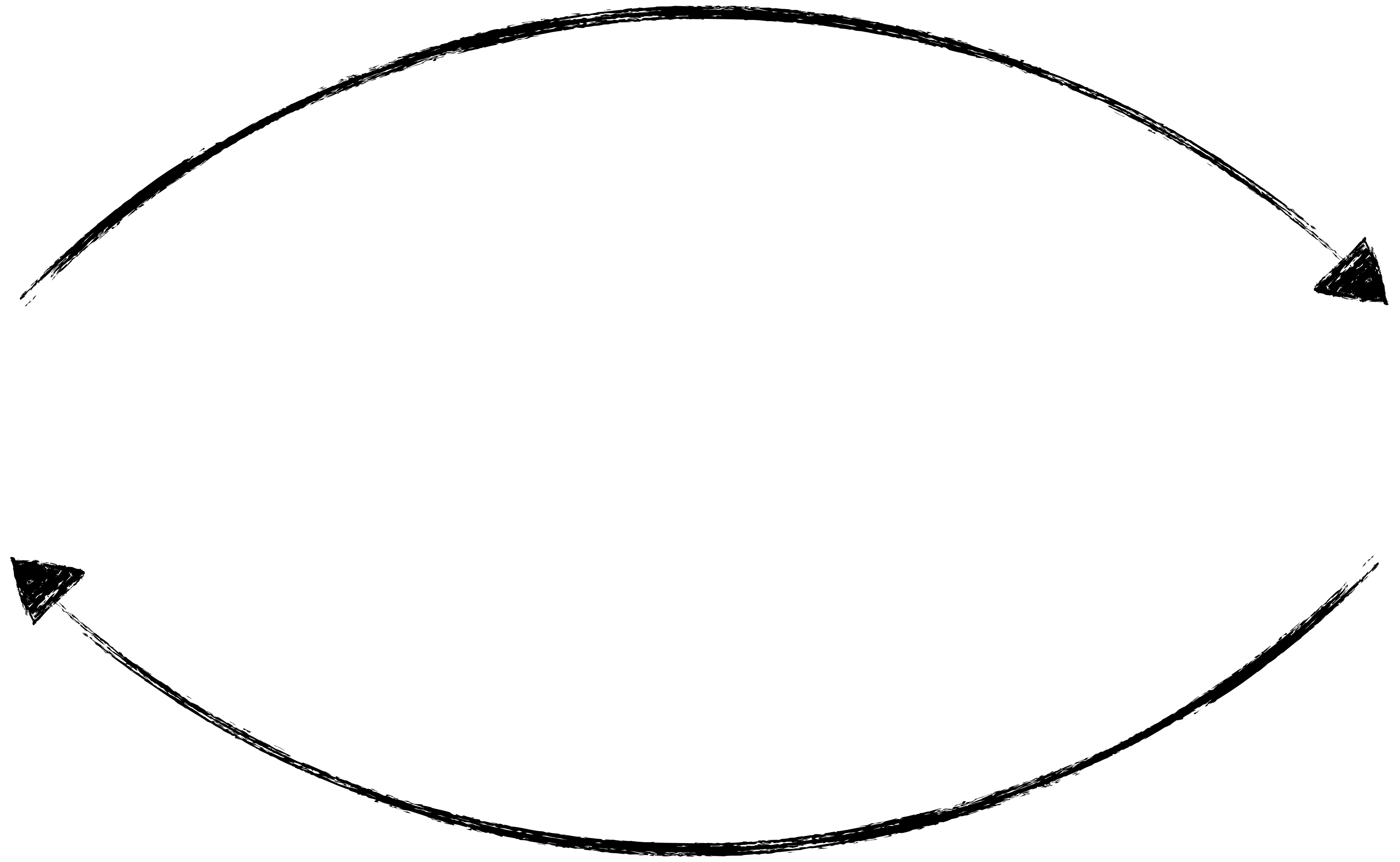
$$V(s) \leftarrow V(s) + \alpha(c + \gamma V(s') - V(s))$$

Works with partial segments!
(s,a,r,s')

Actor-Critic Framework

Actor

Critic



Policy improvement of π

Estimates value functions A^{π}

Actor-Critic Framework

Start with an arbitrary initial policy $\pi_\theta(a \mid s)$

while *not converged* **do**

Roll-out $\pi_\theta(a \mid s)$ to collect trajectories $D = \{s^i, a^i, r^i, s^i_+\}_{i=1}^N$

Fit value function $\hat{V}^{\pi_\theta}(s^i)$ using TD, i.e. minimize $(r^i + \gamma \hat{V}^{\pi_\theta}(s^i_+) - \hat{V}^{\pi_\theta}(s^i))^2$

Compute advantage $\hat{A}^{\pi_\theta}(s^i, a^i) = r(s^i, a^i) + \gamma \hat{V}^{\pi_\theta}(s^i_+) - \hat{V}^{\pi_\theta}(s^i)$

Compute gradient

$$\nabla_\theta J(\theta) = \frac{1}{N}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a^i_t \mid s^i_t)\, \hat{A}^{\pi_\theta}(s^i, a^i)\right]$$

Update parameters $\qquad \theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Fix #2: Take small steps

Start with an arbitrary initial policy $\pi_\theta(a \,|\, s)$

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Compute gradient

$$\nabla_\theta J(\theta) = \frac{1}{N}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a^i_t \,|\, s^i_t)\, \hat{A}^{\pi_\theta}(s^i, a^i)\right]$$

s.t. $KL(\pi(\theta + \Delta\theta) \,||\, \pi(\theta)) \leq \epsilon$

Update parameters $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

How??

Natural Gradient Descent (rediscovered as TRPO)

Start with an arbitrary initial policy $\pi_\theta(a \mid s)$

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$$\cancel{\text{s.t. } KL(\pi(\theta + \Delta\theta) \,||\, \pi(\theta)) \le \epsilon}$$

$$\approx \Delta\theta^T G(\theta) \Delta\theta \le \epsilon$$

Update parameters $\quad \theta \leftarrow \theta + \alpha G(\theta)^{-1} \nabla_\theta J(\theta)$

$G(\theta)$ is Fischer Information Matrix

$$G(\theta) = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta \nabla_\theta \log \pi_\theta^T\right]$$

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~~s.t.~~ $\cancel{KL(\pi(\theta + \Delta\theta) \,||\, \pi(\theta)) \leq \epsilon}$

$$\approx \Delta\theta^T G(\theta) \Delta\theta \leq \epsilon$$

Update parameters $\quad \theta \leftarrow \theta + \alpha G(\theta)^{-1} \nabla_\theta J(\theta)$

Don't directly compute the inverse,
use conjugate gradient to solve $G(\theta)x = \nabla_\theta J(\theta)$

$G(\theta)$ is Fischer Information Matrix

$$G(\theta) = \mathbb{E}_{\pi_\theta}\left[\nabla_\theta \log \pi_\theta \nabla_\theta \log \pi_\theta^T\right]$$

Proximal Policy Optimization (PPO)

Computing Fischer Information matrix is expensive and slow!

Idea: Instead of taking small steps, change the loss function so there is no benefit in taking large steps!

Proximal Policy Optimization (PPO)

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Idea: Instead of taking small steps, change the loss function so there is no benefit in taking large steps!

Instead of defining gradient, we will define a surrogate loss function (Lets say we are at iteration k)

$$\mathscr{L}(\theta) = \mathbb{E}_{s,a\sim\pi_{\theta_k}}\left[\frac{\pi_\theta}{\pi_{\theta_k}} A^{\pi_{\theta_k}}(s,a)\right]$$

Proximal Policy Optimization (PPO)

Computing Fischer Information matrix is expensive and slow!

Idea: Instead of taking small steps, change the loss function so there is no benefit in taking large steps!

Clip the loss if the policy π_θ deviates too much from π_{θ_k}

$$\mathscr{L}(\theta) = \mathbb{E}_{s,a\sim\pi_{\theta_k}}\left[\min\left(\frac{\pi_\theta}{\pi_{\theta_k}} A^{\pi_{\theta_k}}(s,a), \text{clip}\left(\frac{\pi_\theta}{\pi_{\theta_k}}, 1-\epsilon, 1+\epsilon\right) A^{\pi_{\theta_k}}(s,a)\right)\right]$$

Fix #3: Use a reset distribution

Start with an arbitrary initial policy $\pi_\theta(a \mid s)$

while *not converged* **do**

Roll-out $\pi_\theta(a \mid s)$ to collect trajectories $D = \{s^i, a^i, r^i, s^i_+\}_{i=1}^N$

Fit value function $\hat{V}^{\pi_\theta}(s^i)$ using TD, i.e. minimize $(r^i + \gamma \hat{V}^{\pi_\theta}(s^i_+) - \hat{V}^{\pi_\theta}(s^i))^2$

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Compute gradient

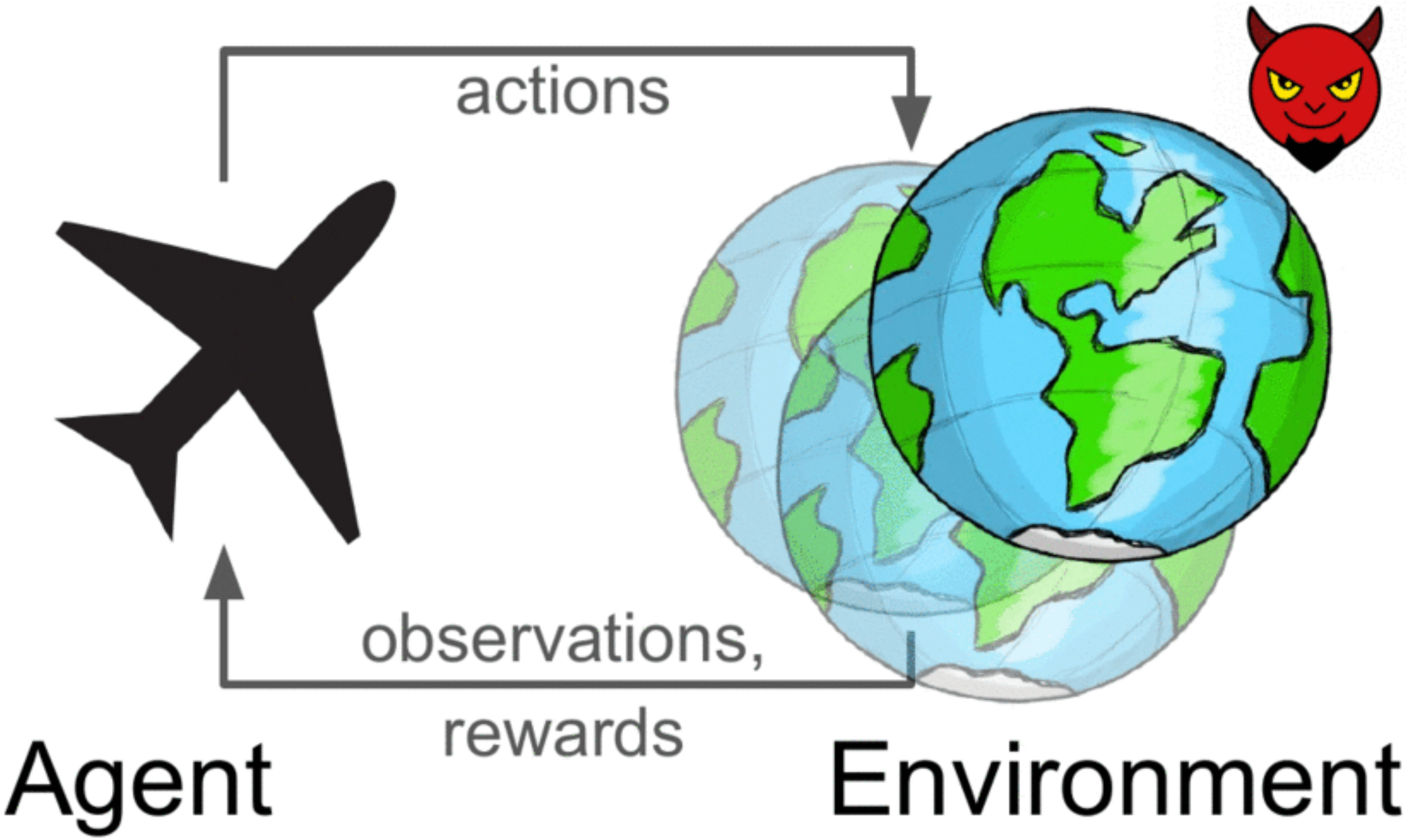
$$\nabla_\theta J(\theta) = \frac{1}{N}\left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi_\theta(a^i_t \mid s^i_t)\, \hat{A}^{\pi_\theta}(s^i, a^i)\right] \text{ s.t. } KL(\pi(\theta + \Delta\theta) \,||\, \pi(\theta)) \leq \epsilon$$

Update parameters $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Instead of rolling out from the start state, rollout from states expert visits

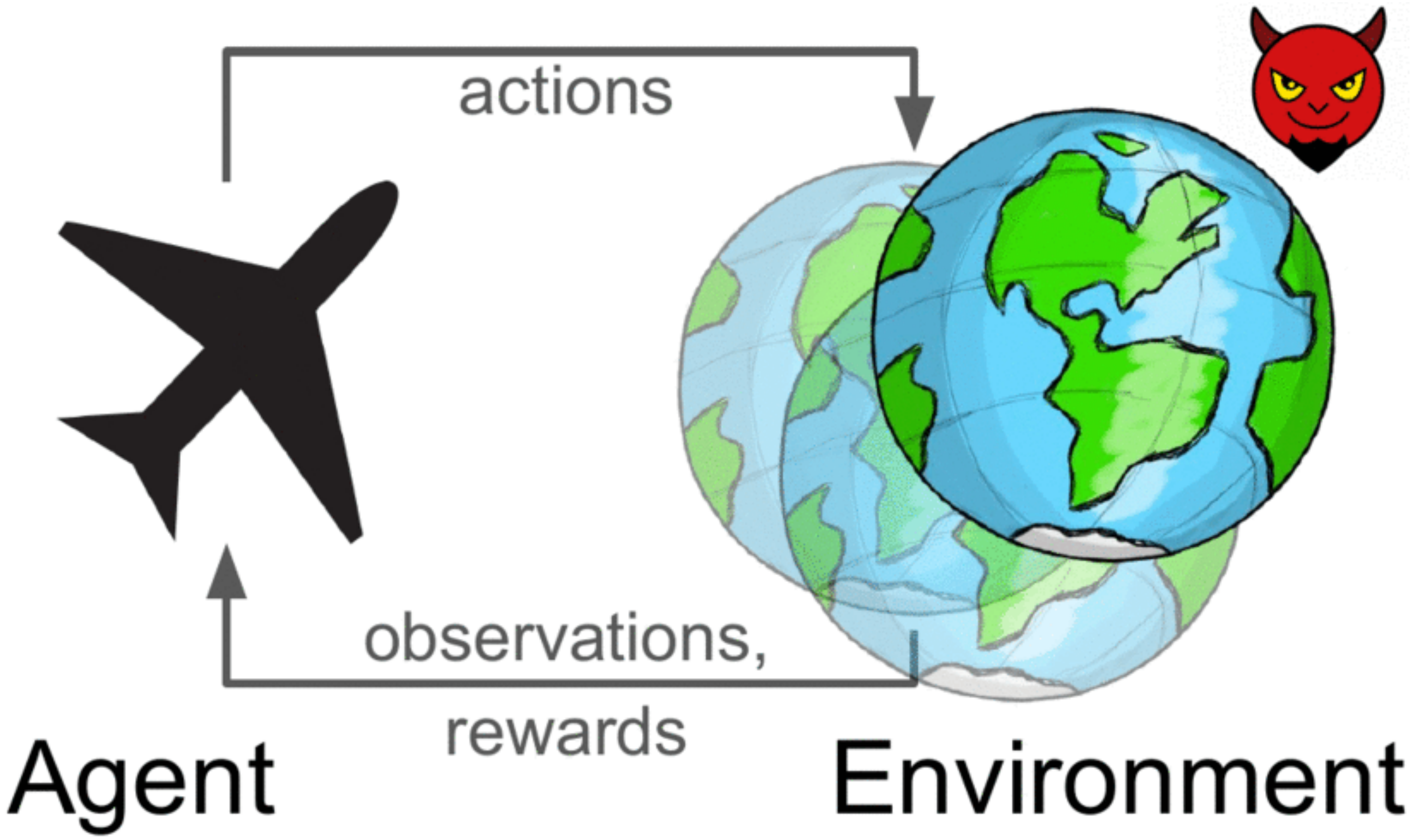
How do we make Actor-Critic more robust to randomness of the environment?

We never see the actual environment in RL



Credit: Ben Eyesenbach

We want our policy to be robust against all possible environments that can explain the data



$$\max_{\pi} \min_{\tilde{p} \in \tilde{\mathcal{P}}, \tilde{r} \in \tilde{\mathcal{R}}} \mathbb{E}_{\tilde{p}(\mathbf{s_{t+1}} | \mathbf{s_t}, \mathbf{a_t}), \pi(\mathbf{a_t} | \mathbf{s_t})} \left[\sum_{t=1}^{T} \tilde{r}(\mathbf{s_t}, \mathbf{a_t}) \right].$$

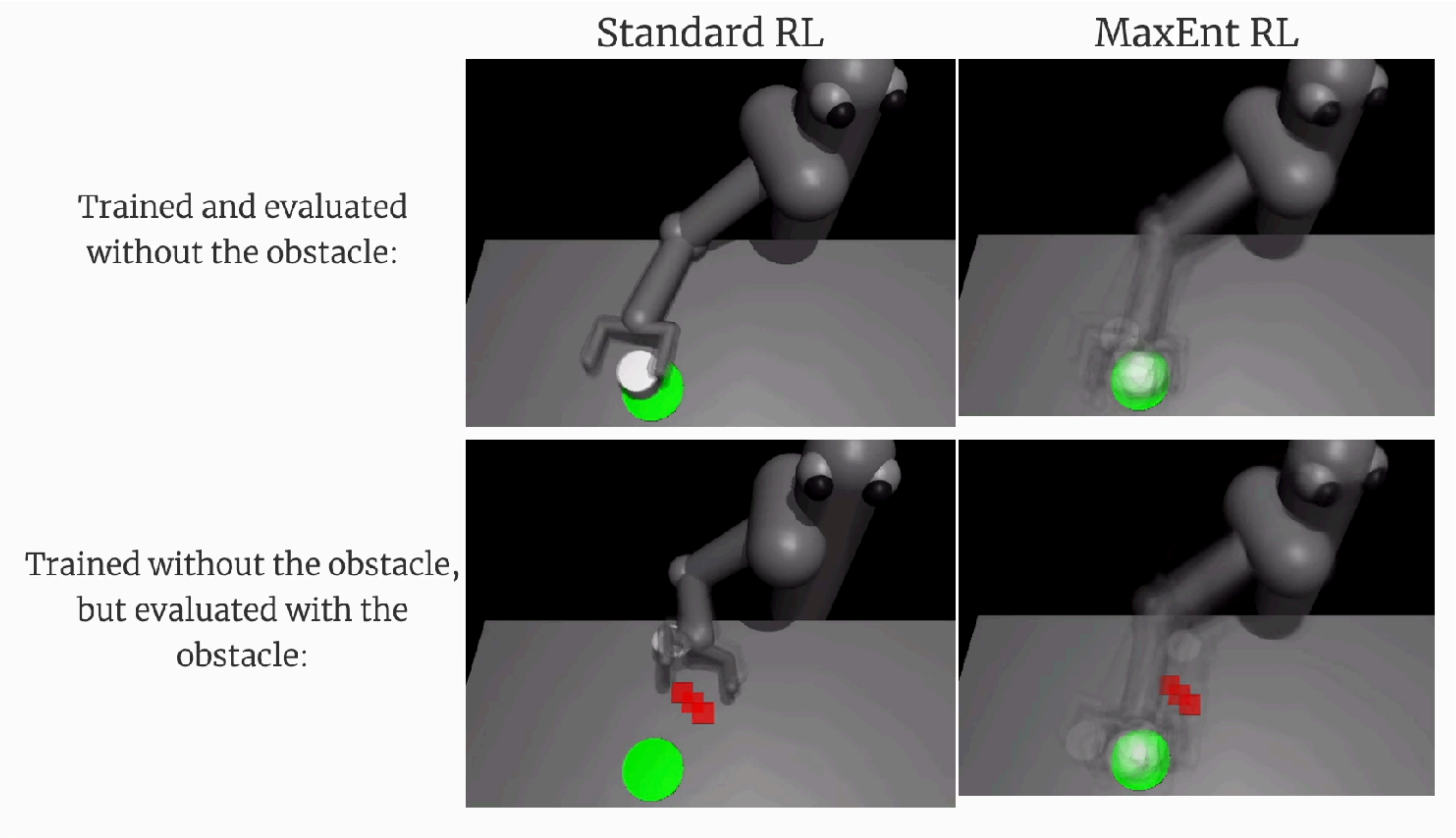
Credit: Ben Eyesenbach

Solution: Use Maximum Entropy RL!

$$J_{\text{MaxEnt}}(\pi; p, r) \triangleq \mathbb{E}_{\mathbf{a_t} \sim \pi(\mathbf{a_t}|\mathbf{s_t}), \mathbf{s_{t+1}} \sim p(\mathbf{s_{t+1}}|\mathbf{s_t}, \mathbf{a_t})} \left[\sum_{t=1}^{T} r(\mathbf{s_t}, \mathbf{a_t}) + \boxed{\alpha \mathcal{H}_\pi [\mathbf{a_t} \mid \mathbf{s_t}]} \right]$$

Intuition: There are many policies that can achieve the same cumulative rewards. MaxEntRL keeps alive all of those policies. Learns many different ways to solve the same task.

Solution: Use Maximum Entropy RL!



Haarnoja 2018

"Soft" Actor Critic

Actor

$$\pi_{\text{new}} = \arg\min_{\pi' \in \Pi} \mathrm{D}_{\mathrm{KL}} \left(\pi'(\,\cdot\,|\mathbf{s}_t) \,\middle\|\, \frac{\exp\left(Q^{\pi_{\text{old}}}(\mathbf{s}_t, \,\cdot\,)\right)}{Z^{\pi_{\text{old}}}(\mathbf{s}_t)} \right)$$

"Soft" Policy Improvement

Critic

$$\mathcal{T}^{\pi} Q(\mathbf{s}_t, \mathbf{a}_t) \triangleq r(\mathbf{s}_t, \mathbf{a}_t) + \gamma \, \mathbb{E}_{\mathbf{s}_{t+1} \sim p} \left[V(\mathbf{s}_{t+1}) \right],$$

where

$$V(\mathbf{s}_t) = \mathbb{E}_{\mathbf{a}_t \sim \pi} \left[Q(\mathbf{s}_t, \mathbf{a}_t) - \log \pi(\mathbf{a}_t|\mathbf{s}_t) \right]$$

"Soft" Value Evaluation

Haarnoja 2018

“Soft” Actor Critic