The Economics of an Environmental Disaster: The Aral Sea
by
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Abstract
The Aral Sea was once the fourth largest lake on Earth. In the 1960s, the former Soviet Union began diverting water from the Syr Darya and the Amu Darya rivers to irrigate cotton. These two rivers are the primary source of fresh water for the Aral Sea. In less than four decades, the Aral Sea shrank to a surface area less than half its size in 1960. In addition to the loss of valuable fisheries, navigation, and potable water, the exposed sea bed contained a toxic, alkaline, dust; the result of fertilizer and pesticide run-off from agriculture. Wind produces toxic dust storms which have severely impacted public health. We develop a hydro-economic model of the Aral Sea. The economic objective is to maximize the discounted net value of water diversions less the social cost from lower Aral Sea volumes. The optimization problem has a unique, optimal, steady-state. We prove that the optimal approach is the Most Rapid Approach Path (the MRAP). There are six parameters to our model specification. We calibrate the model and calculate the optimal volume and diversion rate. With the collapse of the Soviet Union, the return to larger sea volumes will require the cooperation of five Central Asian states: Kazakhstan, Uzbekistan, Turkmenistan, Kyrgyzstan, and Tajikistan. The model reveals a critical parameter ratio on which cooperative discussions and future analysis should focus.
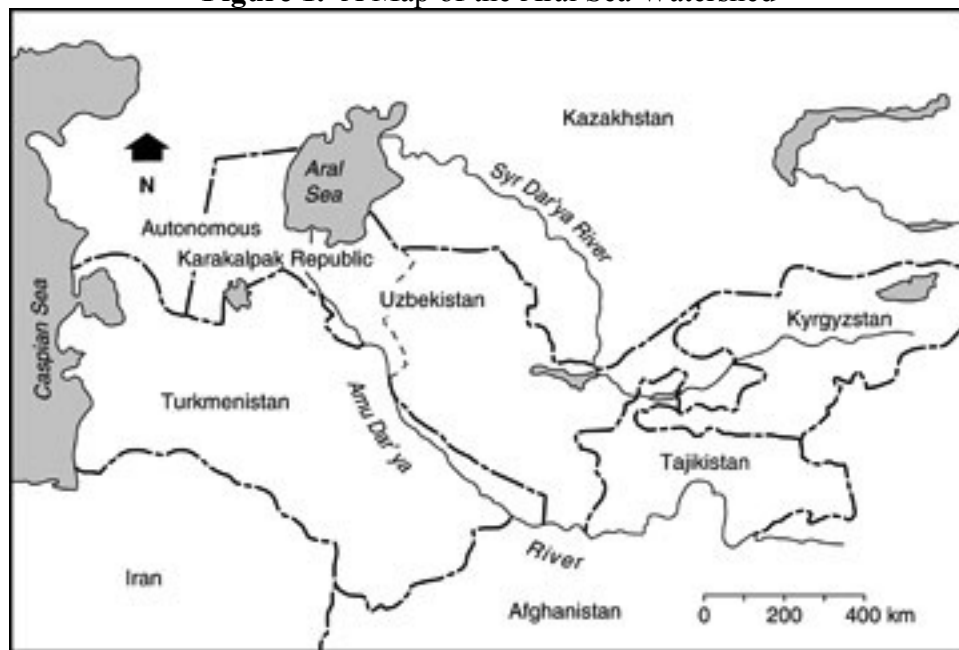
I. History and Background

The Aral Sea was once a vast terminal lake lying in the heart of Central Asia (Figure 1). The volume, size, and quality of its water depended critically on two major rivers, the Amu Darya; $2,600$ km in length and draining $692,300$ km^2 and the Syr Darya; $2,212$ km in length and draining $493,000$ km^2 . As a terminal lake, with no surface outflow, the surface area and volume of water in the Aral Sea is determined by the balance between the inflow of water and evaporation. From the mid-17th Century until the early 1960s, the total inflow from the Syr Darya and Amu Darya averaged 56 km^3 per year. Combining this flow with an average precipitation of 9 km^3 per year implied an average total inflow of 65 km^3 for the 300 years prior to 1960. This inflow balanced the loss from evaporation and the Aral Sea was more or less in steady state with volume at $1,089$ km^3 and a surface area at $67,500$ km^2 . See Micklin (2010).

What used to be the world's fourth largest lake in 1960 began steadily shrinking over the next five decades. Beginning in the 1960s, the former Soviet Union implemented a series of irrigation projects to support the cultivation of cotton, viewed as "white gold,"

and chosen for its ability to generate foreign exchange. The average annual river flow into the Aral Sea dropped to about 10 km^3 from 1975 to 1985 [Gleick, (1993)]. In some years there was no flow from the Syr Darya into the northern portion of the Aral Sea. As the lake began shrinking in surface area and volume, shorelines receded, the salinity of the water increased, salt-sensitive fish populations declined, and the delta areas of the Syr Darya and Amu Darya, once home to diverse flora and fauna, experienced rapid decline and destruction. The exposed seabed contained high concentrations of salts and chemicals from agricultural runoff. Wind created toxic dust storms, severely impacting human health. With the decline of the fishing industry and lack of alternative employment, people moved from towns and villages, now kilometers from the remnant sea, to other areas in the Soviet Union in hopes of finding a better life.

Figure 1. A Map of the Aral Sea Watershed

The Aral Sea divided into two water bodies in 1987, the North Aral Sea and the South Aral Sea, with Syr Darya flowing into the former and Amu Darya flowing into the latter. Between 1960 and 2003, the level of the North Aral Sea fell by about 12 meters and the South Aral Sea by 23 meters, with the total area of both seas decreasing by a staggering 74 percent and the total volume decreasing by 84 percent from its 1960 level [Micklin, (2002)]. In addition to the rapid and obvious changes in sea level and surface area, the climate of the region changed, with the summers becoming longer and hotter and the winters becoming colder.

With the fall of the Soviet Union in 1989, the problem of water management in the Aral Sea basin now rests with five sovereign, Central Asian States: Uzbekistan, Turkmenistan, Kazakhstan, Tajikistan, and Kyrgyzstan. It is perhaps ironic that the task of restoration or partial restoration of the Aral Sea might have been easier under the former Soviet Union. The newly-independent Central Asian states were left without the central authority to make water allocation decisions. Not surprisingly, their new status left these countries in

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competition for water and with an incentive for over-consumption. In the region, all countries rely heavily on water for economic progress, with downstream states, such as Uzbekistan using water for irrigation and the upstream countries of Kyrgyzstan and Tajikistan using water to generate electricity. As such, Kyrgyzstan and Tajikistan, located in the mountain zone of the basin, together have access to about 80 percent of the river flow within the basin. These states are large "net exporters" of water to the downstream countries, Uzbekistan, Kazakhstan and Turkmenistan, which consumed about 83 percent of basin flow in 1995 [Micklin, (2002)]. The competing international water claims on the two transnational rivers raises the potential for international conflict. The restoration or partial restoration of the Aral Sea will require the cooperation of these five countries. *The overarching objective of this paper is to provide a simple, transparent, dynamic model which might focus discussion and foster cooperation.*

In the next section we formulate our model. In Section III we calibrate the model. In Section IV we provide some numerical results that reveal a critical ratio: p/γ, the ratio of the value of water in irrigation, p, to a damage coefficient, γ. Section V provides a brief conclusion.

II. The Hydro-Economic Model

Our hydro-economic model will use the following notation. V_t will denote the volume of water in the Aral Sea in year t, and D_t the volume of diversions from the Syr Darya and Amu Darya rivers also in year t. The term $(I_{1960} - D_t)$ is the *net inflow* into the Aral Sea in period t. The term I_{1960} is the estimated flow from the Syr Darya and Amu Darya rivers plus precipitation in the year 1960, prior to diversions for irrigation. The estimated flow from both rivers in 1960 was $56\ km^3$. To this volume it is estimated that precipitation added another $9\ km^3$, yielding and estimate of $I_{1960} = 65\ km^3$. The volume of water in the Aral Sea in 1960 was estimated at $V_{1960} = 1,089\ km^3$. These values seem to be widely accepted by hydrologists and geologists. See Micklin (2010).

Let $\phi(D_t)$ denote the net benefit from diversions D_t, and $\psi(V_{1960} - V_t)$ the social cost of $(V_{1960} - V_t) > 0$; the difference between the volume in 1960 and the volume in period t. The dynamics of water volume in the Aral Sea are modeled by the iterative map $V_{t+1} = (1 - \varepsilon)V_t + (I_{1960} - D_t)$, where $1 > \varepsilon > 0$ is an evaporation rate and $I_{1960} \geq D_t \geq 0$. From the mid-17th Century until the early 1960's the Aral Sea was thought to have been relatively stable with evaporation matching inflow. With $D_t = 0$, this implies $\varepsilon V_{1960} = I_{1960}$, or $\varepsilon = I_{1960}/V_{1960} = 0.05968779$.

The dynamic optimization problem of interest seeks to

$$\underset{\{D_t\}}{Maximize} \quad W = \sum_{t=0}^{\infty} \rho^t \{\phi(D_t) - \psi(V_{1960} - V_t)\}$$

$$Subject\ to \quad V_{t+1} = (1 - \varepsilon)V_t + (I_{1960} - D_t)$$

$$I_{1960} \geq D_t \geq 0, \ V_0 > 0 \ given.$$

where $\rho = 1/(1 + \delta)$ is a discount factor and $\delta > 0$ is a discount rate. The Lagrangian for this problem may be written as

$$L = \sum_{t=0}^{\infty} \rho^t \{\phi(D_t) - \psi(V_{1960} - V_t) + \rho\lambda_{t+1}[(1 - \varepsilon)V_t + (I_{1960} - D_t) - V_{t+1}]\}$$

where λ_{t+1} is the current-value shadow price on the volume of water in the Aral Sea in period $(t + 1)$. The first-order necessary conditions include

(1) $\rho\lambda_{t+1} = \phi'(D_t)$

(2) $(1 - \varepsilon)\rho\lambda_{t+1} - \lambda_t = -\psi'(V_{1960} - V_t)$

(3) $V_{t+1} = (1 - \varepsilon)V_t + (I_{1960} - D_t)$

In steady state, where $V_{t+1} = V_t = V$, $D_{t+1} = D_t = D$, and $\lambda_{t+1} = \lambda_t = \lambda$, Equations (1)-(3) require

(4) $\rho\lambda = \phi'(D)$

(5) $\rho\lambda = \psi'(V_{1960} - V)/(\delta + \varepsilon)$

(6) $\varepsilon V = I_{1960} - D$

Equation (4) requires that the discounted shadow price on water volume in the Aral Sea equal the marginal net benefit of water diverted for irrigation. Equation (5) requires that the discounted shadow price also equal the present value of marginal social cost, where the discount rate, $\delta > 0$, has been augmented by the evaporation rate, $1 > \varepsilon > 0$. Finally, for any steady state, Equation (6) requires the evaporation loss equal net inflow.

We can equate Equation (4) to Equation (5) to obtain a two-equation system that defines $[V^*, D^*]$, the optimal, steady-state Aral Sea volume and water diversion. Those two equations may be written as

(7) $\phi'(D) = \psi'(V_{1960} - V)/(\delta + \varepsilon)$

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(8) $D = I_{1960} - \varepsilon V$

Our numerical analysis will be based on the specification $\phi(D) = pD$ and
$\psi(V_{1960} - V) = (\gamma/2)(V_{1960} - V)^2$, where $p > 0$ is the marginal value (price) of a km^3 of
water used in irrigation, with the dimension $\$/km^3$, and $\gamma > 0$ is a damage coefficient
in the social cost function, with the dimension $\$/km^6$. With this specification we get an
analytic solution for V^* and can then compute D^* according to

(9) $V^* = V_{1960} - p(\delta + \varepsilon)/\gamma$

(10) $D^* = I_{1960} - \varepsilon V^*$

Equations (9) and (10) have six parameters: $V_{1960} = 1{,}089\ km^3$, $I_{1960} = 65\ km^3$,
$\varepsilon = I_{1960}/V_{1960} = 0.05968779$, $p > 0$, $\delta > 0$, and $\gamma > 0$. For $V^* \geq 0$ it must be the
case that $V_{1960} \geq p(\delta + \varepsilon)/\gamma$. We now prove that the *most rapid approach path* (the
MRAP) is optimal when moving the hydro-economic system from V_0 to V^*.

Proposition: For the hydro-economic specification where $\phi(D_t) = pD_t$ and
$\psi(V_{1960} - V_t) = (\gamma/2)(V_{1960} - V_t)^2$, the MRAP from V_0 to V^* is optimal.

Proof: For the above hydro-economic specification, we will show that the optimization
problem may be equivalently stated as

$$\underset{\{V_t\}_{t=1}^{\infty}}{\textit{Maximize}}\quad M(V_0) + \sum_{t=1}^{\infty} \rho^t N(V_t)$$

and that $N(V_t)$ is quasi-concave, thus satisfying the sufficient condition for the MRAP
to be optimal, as proved in Spence and Starrett (1975).

First, solve the iterative map $V_{t+1} = (1 - \varepsilon)V_t + (I_{1960} - D_t)$ for

(11) $D_t = I_{1960} + (1 - \varepsilon)V_t - V_{t+1}$

Substitute the above expression for D_t into the objective functional yielding

(12) $W(V_t, V_{t+1}) = p[I_{1960} + (1 - \varepsilon)V_t - V_{t+1}] - (\gamma/2)(V_{1960} - V_t)^2$

Note that $W(V_t, V_{t+1})$ is additively separable in V_t and V_{t+1}. Then, by re-indexing, it is possible to write

(13) $\quad W = M(V_0) + \sum_{t=1}^{\infty} \rho^t N(V_t)$

where

(14) $\quad M(V_0) = p[I_{1960} + (1-\varepsilon)V_0] - (\gamma/2)(V_{1960} - V_0)^2$

and

(15) $\quad N(V_t) = p[I_{1960} - (\delta + \varepsilon)V_t] - (\gamma/2)(V_{1960} - V_t)^2$

We observe that $N(V_t)$ is strictly concave, and thus quasi-concave in V_t, thereby satisfying the Spence and Starrett (1975) sufficient condition for the MRAP to be optimal when going from V_0 to V^* □

If one evaluates $N(V_t)$ at steady state and sets $N'(V) = 0$ one obtains
$V^* = V_{1960} - p(\delta + \varepsilon)/\gamma$, the analytic solution for V^* given in Equation (9).

III. Calibration
Table 1 below reproduces some of the entries from Table 1 in Macklin (2010). In 1987 the Aral Sea separated into two seas; the "Small Aral Sea" in the north and the "Large Aral Sea" in the south., with the Syr Darya flowing into the northern portion and the Amu Darya flowing into the southern portion. After separation, a channel formed allowing spring overflow from the higher-elevation, Small Aral Sea, to reach the lower-elevation, Large Aral Sea. In 2005 a 13 km earthen dike with concrete control gates was completed allowing the surface area and volume of the Small Aral Sea to be increased and to better regulate the flow from the Small Aral Sea to the south. The surface areas and water volumes reported in Table 1 combine the surface areas and volumes for both the Small Aral and Large Aral Seas.

Table 1. Surface Area and Water Volume in the Aral Sea, All Portions Combined.

Year	Surface Area (km^2)	Percentage of 1960 Surface Area	Water Volume (km^3)	Percentage of 1960 Water Volume
1960	67,499	100	1,089	100
1971	60,200	89	925	85
1976	55,700	83	763	70
1989	39,734	59	364	33
2009	8,409	12.5	84	7.7

Source: Micklin (2010), Table 1.

We have previously noted that from the mid-17th Century through the early 1960s the Aral Sea was more or less in steady state with $V_{1960} = 1,089\ km^3$, $I_{1960} = 65\ km^3$, and $\varepsilon = I_{1960}/V_{1960} = 0.05968779$. These widely-accepted values provide us with three of the six parameters for our hydro-economic specification. We now discuss the calibration of γ, δ, and p.

Gamma: We assume that in 2009 the social cost (foregone fisheries, navigation, drinking water, and especially the public health costs from higher than normal rates of infant mortality and esophageal and lung cancer) was $20 Billion. Using the specification for our social cost function, this implies $\$20x10^9 = (\gamma/2)(1,089\ km^3 - 84\ km^3)^2$. Solving for gamma yields $\gamma = 39,603\ (\$/km^6)$.

Delta: For the discount rate, we adopt the mean of the distribution estimated from the survey of economists conducted by Martin Weitzman (2001). The mean, risk-free rate was $\delta = 0.02$. In a more recent paper, where future discount rates are themselves uncertain, Weitzman (2010) shows that in a stochastic, Ramsey-growth model, future discount rates may be lower than $\delta = 0.02$.

Price: The remaining parameter in our hydro-economic specification is $p > 0$, the marginal value product or marginal cost to cotton growers for one km^3 of water. We could not find an estimate of this value in a Central Asian currency. Instead, we will determine the value of one km^3 of water based on the value of an acre-foot of water in Southern California. Careful conversion will reveal that there are $809,732.80$ acre feet in one km^3 of water. In 2010, the Metropolitan Water District of Southern California had a Tier 2 rate (price) for an acre-foot of water of $280. If this is comparable to the value of an acre-foot of water in irrigation in Central Asia, then the value (price) of one km^3 of water would be $226,725,184$.

IV. Numerical Results

In Table 2. we show the values for V^*, D^*, $W^* = pD^* - (\gamma/2)(V_{1960} - V^*)^2$, and T_{MIN}, where T_{MIN} is the fastest time to go from $V_{2009} = 84\ km^3$ to V^*. It is determined by setting $D_t^* = 0$ and counting the years until $V_t \geq V^*$. The MRAP policy takes the form

$$(16) \quad D_t^* = \begin{cases} 0 \text{ if } V_t < V^* \\ D^* = I_{1960} - \varepsilon V^* \text{ if } V_t = V^* \\ I_{1960} \text{ if } V_t > V^* \end{cases}$$

Table 2. Numerical Results for the Base-Case: $V_{1960} = 1{,}089 \ km^3$, $I_{1960} = 65 \ km^3$, $\varepsilon = I_{1960}/V_{1960} = 0.05968779$, $\gamma = 39{,}603$, $\delta = 0.02$, $p = \$226{,}725{,}184$, and for a Change in a Single Economic Parameter.

Base-Case or Single Parameter Change from Base-Case	$V^*(km^3)$	$D^*(km^3)$	$W^*(\$x10^9)$	T_{MIN} (Years)
Base-Case	633	27.23	2.053	13
$\delta = 0.01$	690	23.81	2.247	15
$\delta = 0.03$	576	30.65	1.728	11
$\gamma = 19{,}800$	177	54.46	4.105	2
$\gamma = 79{,}200$	861	13.62	1.026	25
$p = \$126{,}725{,}184$	834	15.23	0.641	23
$p = \$326{,}725{,}184$	432	39.24	4.262	7

From simple inspection of $V^* = V_{1960} - p(\delta + \varepsilon)/\gamma$ we see that an increase in p, δ, or ε will reduce the value of V^* while an increase in γ will increase V^*. The point elasticities of V^* with respect to p, δ, or γ are given by the expressions $\eta_p = -p(\delta + \varepsilon)/(\gamma V^*)$, $\eta_\delta = -p\delta/(\gamma V^*)$, and $\eta_\gamma = p(\delta + \varepsilon)/(\gamma V^*)$, respectively. When evaluated at the base-case, optimal volume, $V^* = 633 \ km^3$, these elasticities have the values $\eta_p = -0.721$, $\eta_\delta = -0.181$, $\eta_\gamma = 0.721$; indicating that the percentage change in V^* is smaller than the percentage change in p, δ, or γ.

Large changes in the base-case parameters can result in significant changes in the optimal volume, V^*. Of the three economic parameters, the greatest uncertainty surrounds the value of γ. In Table 2 we approximately halve and then double the base-case value of γ. The lower value of γ causes the optimal volume to fall to $V^* = 177 \ km^3$, while the higher value of γ causes the optimal volume to increase to $V^* = 861 \ km^3$.

As V^* increases, the time to reach it from $V_{2009} = 84 \ km^3$ increases. If $V^* = V_{1960}$, the approach would be asymptotic. Optimality of the MRAP requires that $D_t^* = 0$ when approaching V^* from below. Halting all irrigation diversions until V_t reaches V^* is unlikely to play well in Uzbekistan and Turkmenistan, the largest cotton producers in Central Asia. If the five Central Asian countries can agree that $V_{1960} = 1{,}089 \ km^3$, $I_{1960} = 65 \ km^3$, $\varepsilon = I_{1960}/V_{1960} = 0.05968779$, and $\delta = 0.02$, then additional research and discussion should focus on the ratio p/γ, the value of one km^3 of irrigation water relative to the damage coefficient in our social cost function. This becomes the critical ratio that determines V^* and D^*.

V. Conclusions

Under the former Soviet Union, large volumes of water were diverted from the Syr Darya and Amu Darya rivers to irrigate cotton. The Aral Sea is a terminal lake, with no surface outflow, and whose surface area and volume are determined by net inflow and evaporation. The diversions to irrigate cotton caused the water level of Aral Sea to decline by 26 meters and its surface area and volume to decline by 88% and 92%, respectively. By 2009 the Aral Sea had furthered divided into four smaller bodies of water.

With the fall of the Soviet Union in 1989, the restoration of the Aral Sea, or portions of that sea, will likely require the cooperation of the five Central Asian countries, Uzbekistan, Turkmenistan, Tajikistan, Kazakhstan and Kyrgyzstan. We have developed a simple hydro-economic model of the Aral Sea. There are a total of six parameters, three hydrologic and three economic, which determine optimal Aral Sea volume and diversion of water for cotton irrigation. The value of the hydrologic parameters can be estimated from the relatively stable sea volume and river inflow from the mid-17th Century to 1960. If the five Central Asian countries can agree on a discount rate, then the optimal volume, V^*, and diversion for irrigation, D^*, will be determined by the ratio of the marginal value of a km^3 of water in irrigation, p, to our social cost parameter, γ. An agreement on the ratio p/γ would determine the optimal volume and annual diversion for irrigation. The distribution of irrigation water between the five Central Asian countries will undoubtedly be a contentious issue. The model in this paper suggests that negotiators focus first on finding a consensus value for p/γ and then strive to equitably distribute the allowable diversion, D^*.

References

Gleick, P. H. 1993. *Water in Crisis: A Guide to the World's Fresh Water Resources*, Oxford University Press, New York.

Micklin, P. 2002. "Water in the Aral Sea Basin of Central Asia: Cause of Conflict or Cooperation?" *Eurasian Geography and Economics*, 43(7):505-528.

Micklin, P. 2010. "The Past, Present, and Future Aral Sea," *Lakes & Reservoirs: Research & Management*, 15(3):193-213.

Spence, M., and D. Starrett. 1975. "Most Rapid Approach Paths in Accumulation Problems," *International Economic Review*, 16(2):388-403.

Weitzman, M. L. 2001. "Gamma Discounting," *American Economic Review*, 91(1):260-271.

Weitzman, M. L. 2010. "Risk-Adjusted Gamma Discounting," *Journal of Environmental Economics and Management*, 60(1):1-13.