

CS6670: Computer Vision

Noah Snavely

Lecture 19: Single-view modeling



Announcements

- Project 3: Eigenfaces
 - due tomorrow, November 11 at 11:59pm
- Quiz on Thursday, first 10 minutes of class

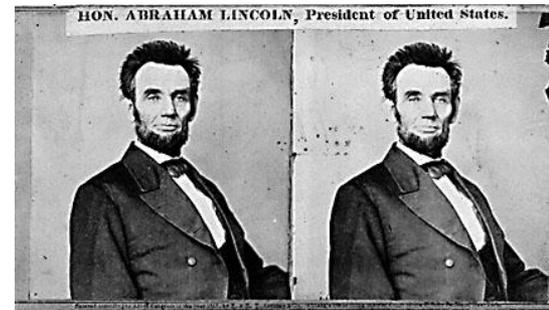
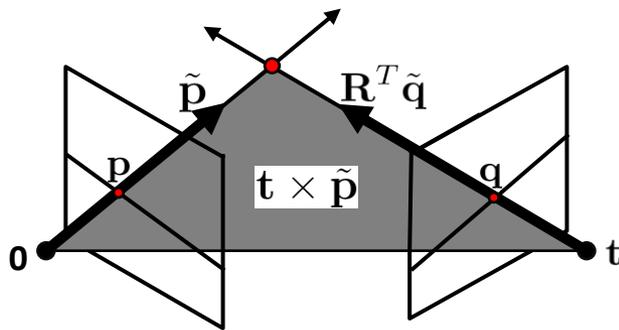
Announcements

- Final projects
 - Feedback in the next few days
 - Midterm reports due November 24
 - Final presentations tentatively scheduled for the final exam period:

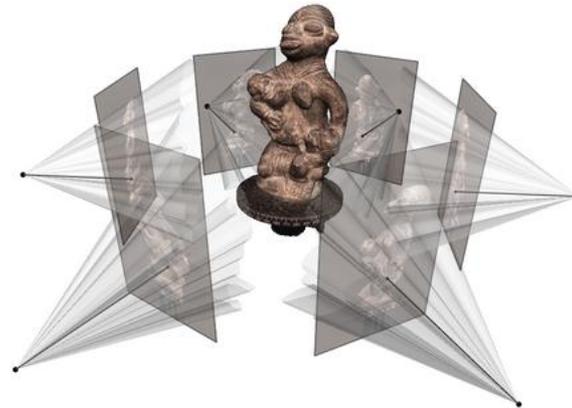
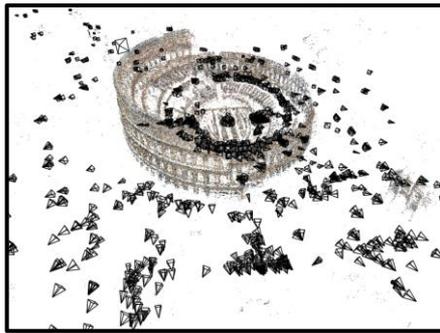
Wed, December 16, 7:00 PM - 9:30 PM

Multi-view geometry

- We've talked about two views



- And many views



- What can we tell about geometry from one view?

Projective geometry



[Ames Room](#)

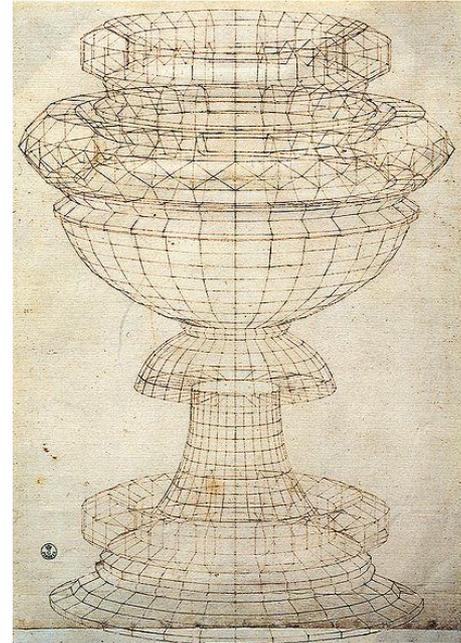
Readings

- Mundy, J.L. and Zisserman, A., Geometric Invariance in Computer Vision, Appendix: Projective Geometry for Machine Vision, MIT Press, Cambridge, MA, 1992, (read 23.1 - 23.5, 23.10)
 - available online: <http://www.cs.cmu.edu/~ph/869/papers/zisser-mundy.pdf>

Projective geometry—what's it good for?

Uses of projective geometry

- Drawing
- Measurements
- Mathematics for projection
- Undistorting images
- Camera pose estimation
- Object recognition

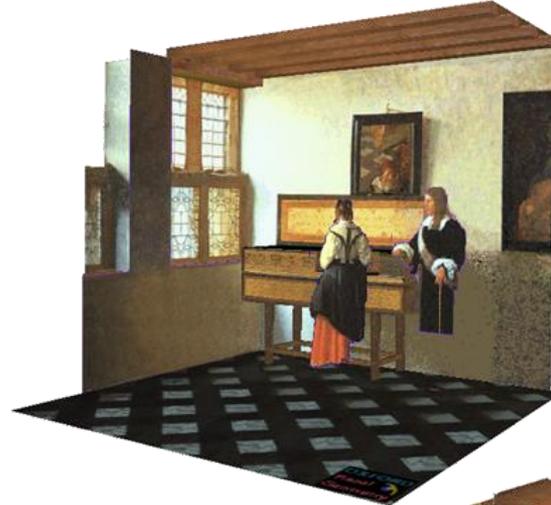


[Paolo Uccello](#)

Applications of projective geometry

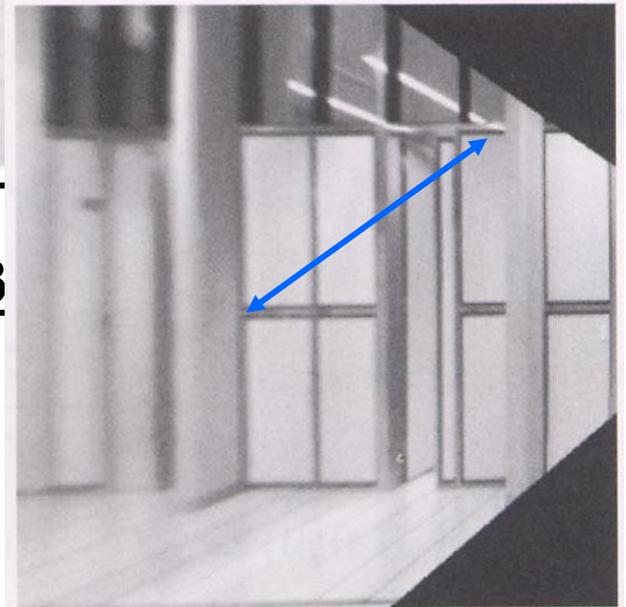
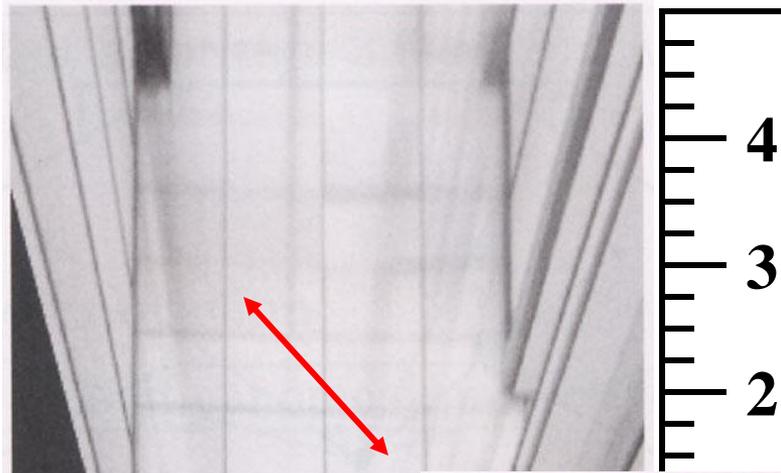
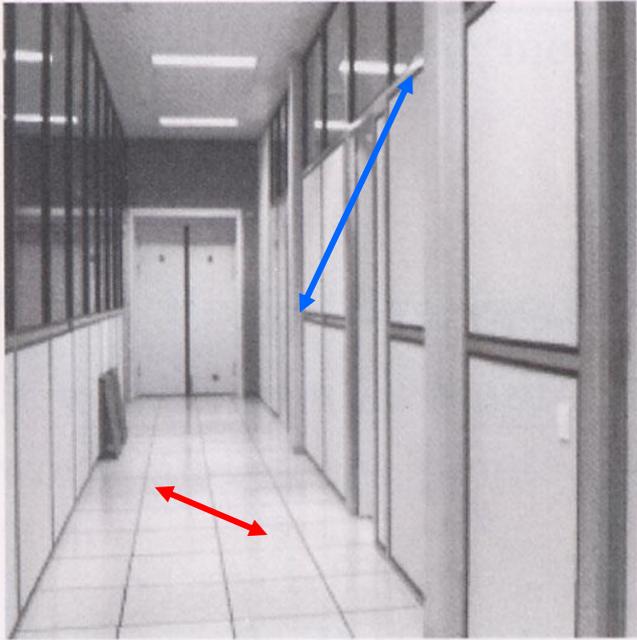


Vermeer's *Music Lesson*



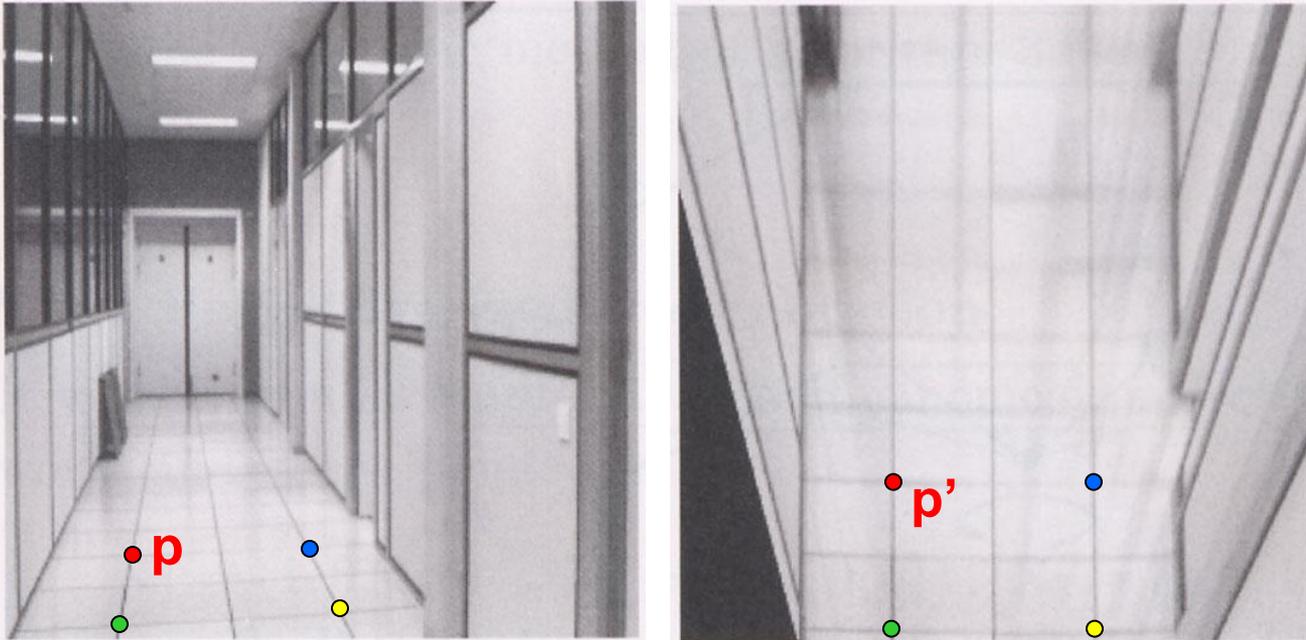
Reconstructions by Criminisi et al.

Measurements on planes



Approach: unwarp then measure
What kind of warp is this?

Image rectification



To unwarped (rectify) an image

- solve for homography \mathbf{H} given \mathbf{p} and \mathbf{p}'
- solve equations of the form: $w\mathbf{p}' = \mathbf{H}\mathbf{p}$
 - linear in unknowns: w and coefficients of \mathbf{H}
 - \mathbf{H} is defined up to an arbitrary scale factor
 - how many points are necessary to solve for \mathbf{H} ?

Solving for homographies

$$\begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$x'_i = \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

$$y'_i = \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

$$x'_i(h_{20}x_i + h_{21}y_i + h_{22}) = h_{00}x_i + h_{01}y_i + h_{02}$$

$$y'_i(h_{20}x_i + h_{21}y_i + h_{22}) = h_{10}x_i + h_{11}y_i + h_{12}$$

Solving for homographies

$$x'_i(h_{20}x_i + h_{21}y_i + h_{22}) = h_{00}x_i + h_{01}y_i + h_{02}$$

$$y'_i(h_{20}x_i + h_{21}y_i + h_{22}) = h_{10}x_i + h_{11}y_i + h_{12}$$

$$\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -x'_i x_i & -x'_i y_i & -x'_i \\ 0 & 0 & 0 & x_i & y_i & 1 & -y'_i x_i & -y'_i y_i & -y'_i \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solving for homographies

$$\begin{bmatrix}
 x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\
 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\
 & & & & & \vdots & & & \\
 x_n & y_n & 1 & 0 & 0 & 0 & -x'_n x_n & -x'_n y_n & -x'_n \\
 0 & 0 & 0 & x_n & y_n & 1 & -y'_n x_n & -y'_n y_n & -y'_n
 \end{bmatrix}
 \begin{bmatrix}
 h_{00} \\
 h_{01} \\
 h_{02} \\
 h_{10} \\
 h_{11} \\
 h_{12} \\
 h_{20} \\
 h_{21} \\
 h_{22}
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 \vdots \\
 0 \\
 0
 \end{bmatrix}$$

A

$2n \times 9$

h

9

0

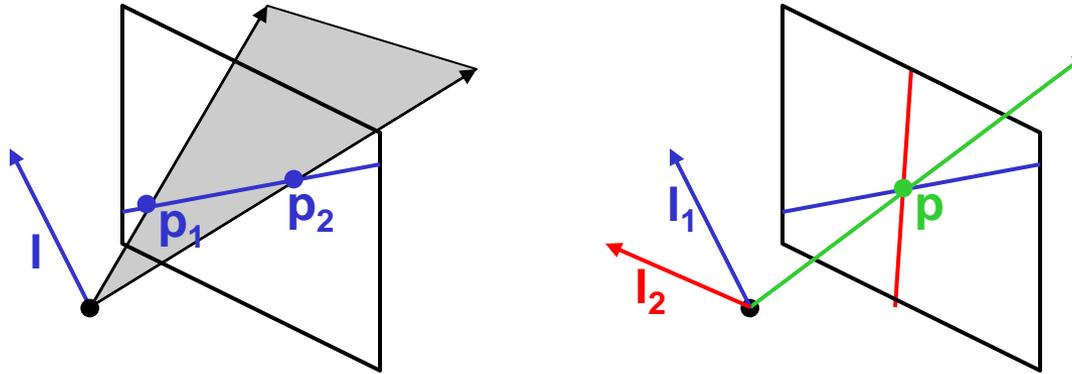
2n

Defines a least squares problem: minimize $\|\mathbf{A}\mathbf{h} - \mathbf{0}\|^2$

- Since \mathbf{h} is only defined up to scale, solve for unit vector $\hat{\mathbf{h}}$
- Solution: $\hat{\mathbf{h}} =$ eigenvector of $\mathbf{A}^T \mathbf{A}$ with smallest eigenvalue
- Works with 4 or more points

Point and line duality

- A line \mathbf{l} is a homogeneous 3-vector
- It is \perp to every point (ray) \mathbf{p} on the line: $\mathbf{l} \cdot \mathbf{p} = 0$



What is the line \mathbf{l} spanned by rays \mathbf{p}_1 and \mathbf{p}_2 ?

- \mathbf{l} is \perp to \mathbf{p}_1 and $\mathbf{p}_2 \Rightarrow \mathbf{l} = \mathbf{p}_1 \times \mathbf{p}_2$
- \mathbf{l} can be interpreted as a *plane normal*

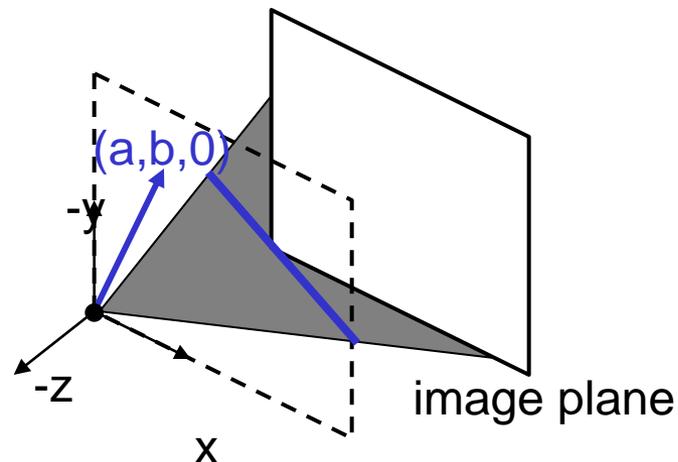
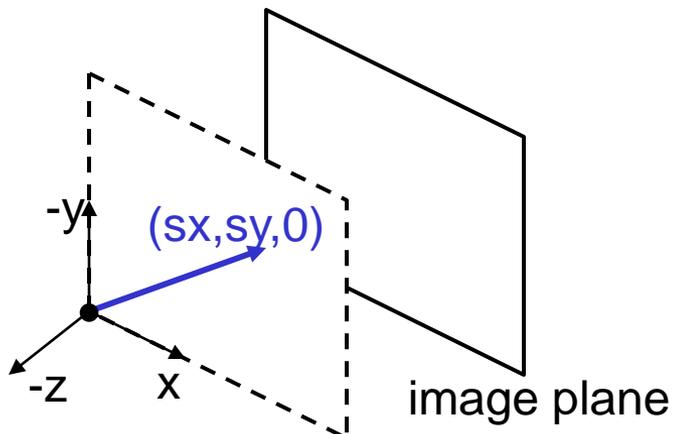
What is the intersection of two lines \mathbf{l}_1 and \mathbf{l}_2 ?

- \mathbf{p} is \perp to \mathbf{l}_1 and $\mathbf{l}_2 \Rightarrow \mathbf{p} = \mathbf{l}_1 \times \mathbf{l}_2$

Points and lines are *dual* in projective space

- given any formula, can switch the meanings of points and lines to get another formula

Ideal points and lines



Ideal point (“point at infinity”)

- $p \cong (x, y, 0)$ – parallel to image plane
- It has infinite image coordinates

Ideal line

- $l \cong (a, b, 0)$ – parallel to image plane
- Corresponds to a line in the image (finite coordinates)
 - goes through image origin (*principle point*)

3D projective geometry

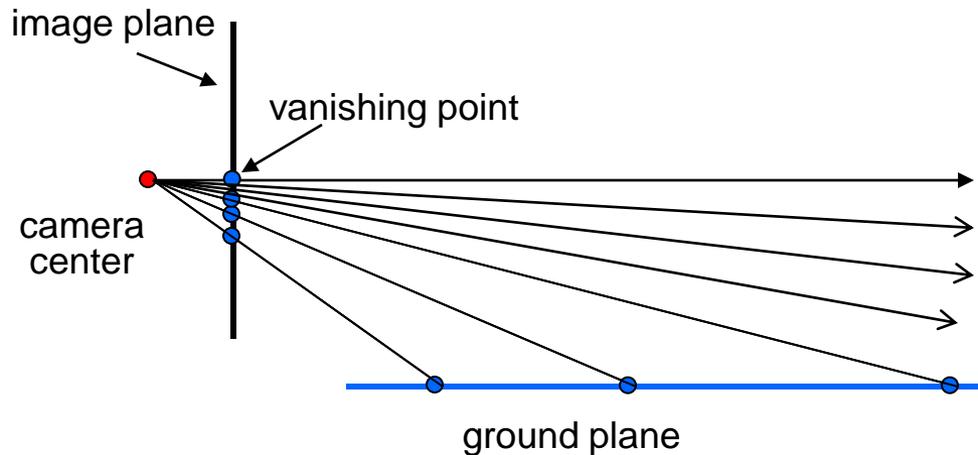
These concepts generalize naturally to 3D

- Homogeneous coordinates
 - Projective 3D points have four coords: $\mathbf{P} = (X, Y, Z, W)$
- Duality
 - A plane \mathbf{N} is also represented by a 4-vector
 - Points and planes are dual in 3D: $\mathbf{N} \cdot \mathbf{P} = 0$
 - Three points define a plane, three planes define a point
- Projective transformations
 - Represented by 4x4 matrices \mathbf{T} : $\mathbf{P}' = \mathbf{T}\mathbf{P}$, $\mathbf{N}' = \mathbf{T}^{-T} \mathbf{N}$

3D to 2D: “perspective” projection

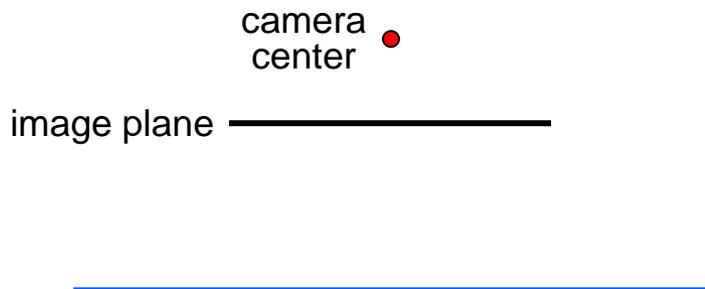
Projection: $\mathbf{p} = \begin{bmatrix} wx \\ wy \\ w \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{P}\mathbf{P}$

Vanishing points (1D)

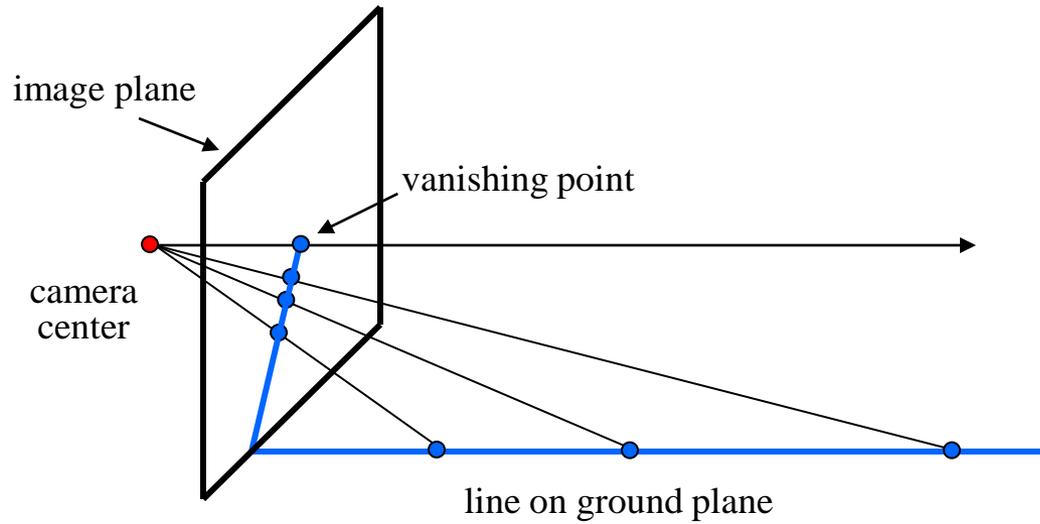


Vanishing point

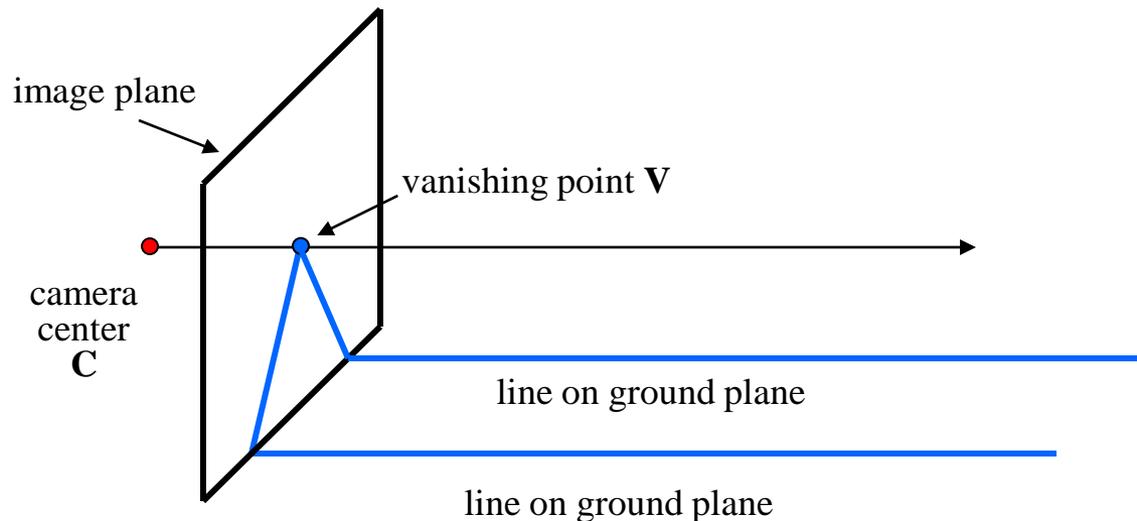
- projection of a point at infinity
- can often (but not always) project to a finite point in the image



Vanishing points (2D)



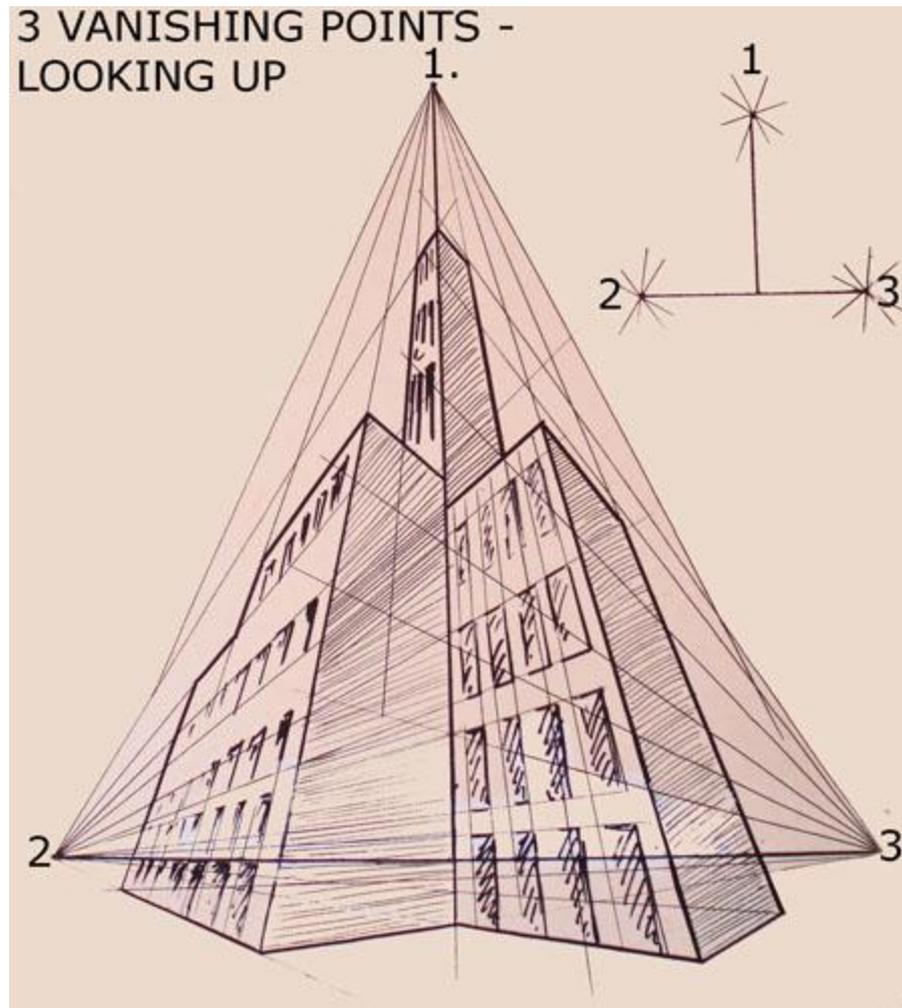
Vanishing points



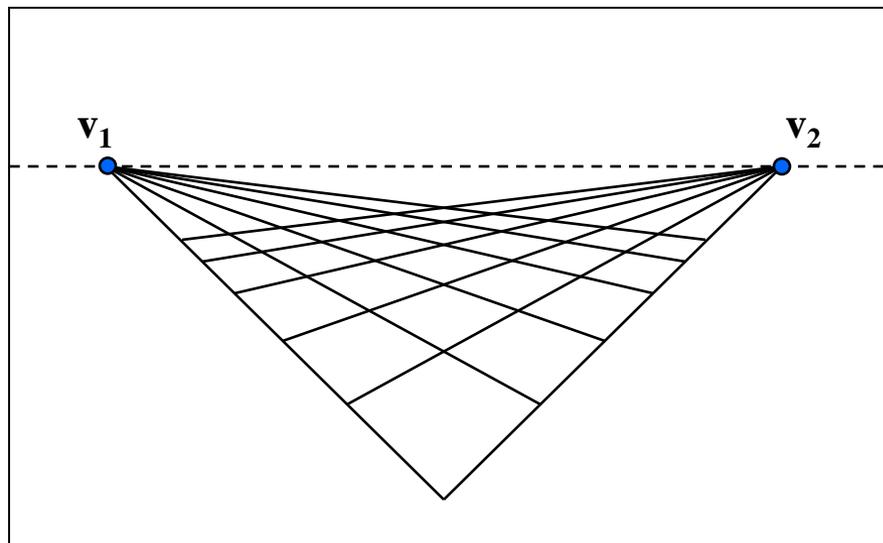
Properties

- Any two parallel lines (in 3D) have the same vanishing point v
- The ray from C through v is parallel to the lines
- An image may have more than one vanishing point
 - in fact, every image point is a potential vanishing point

Three point perspective



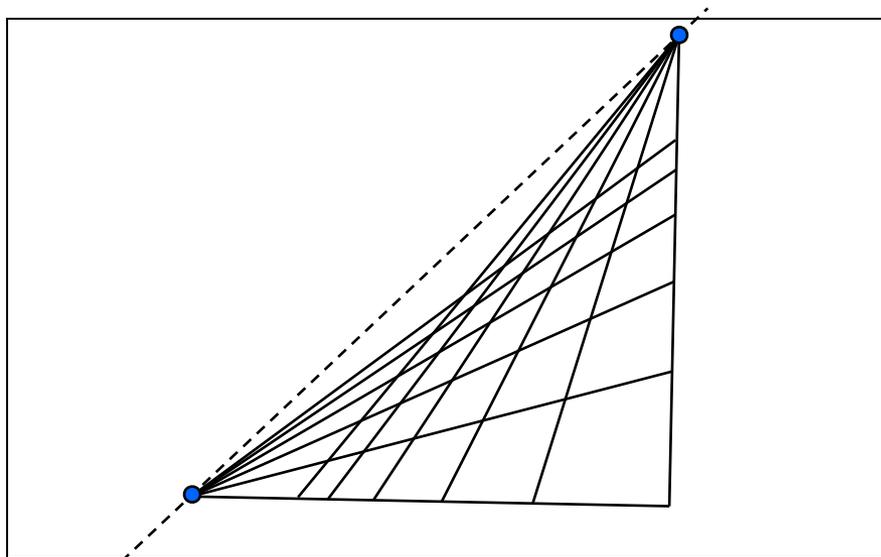
Vanishing lines



Multiple Vanishing Points

- Any set of parallel lines on the plane define a vanishing point
- The union of all of these vanishing points is the *horizon line*
 - also called *vanishing line*
- Note that different planes (can) define different vanishing lines

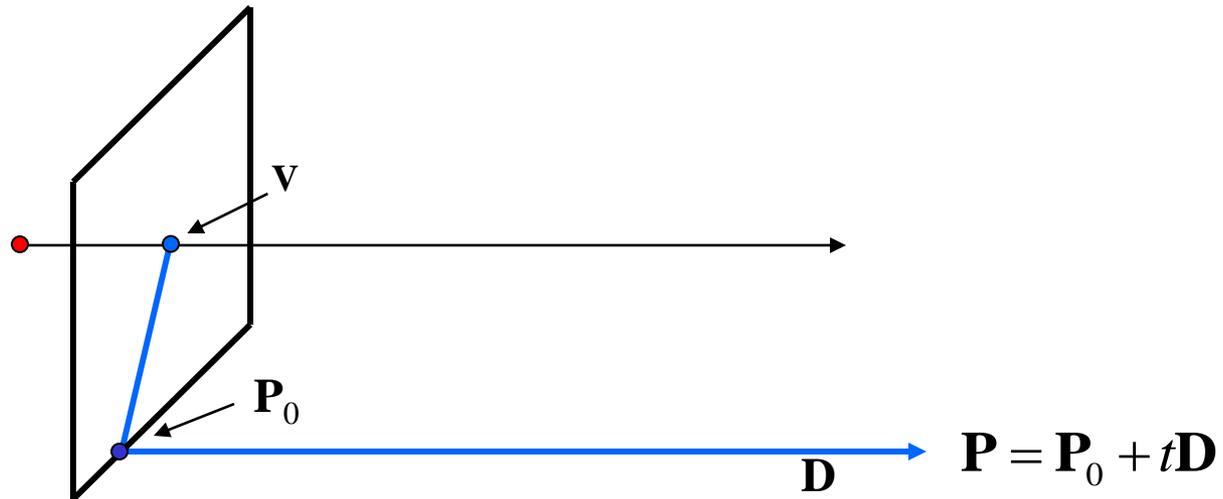
Vanishing lines



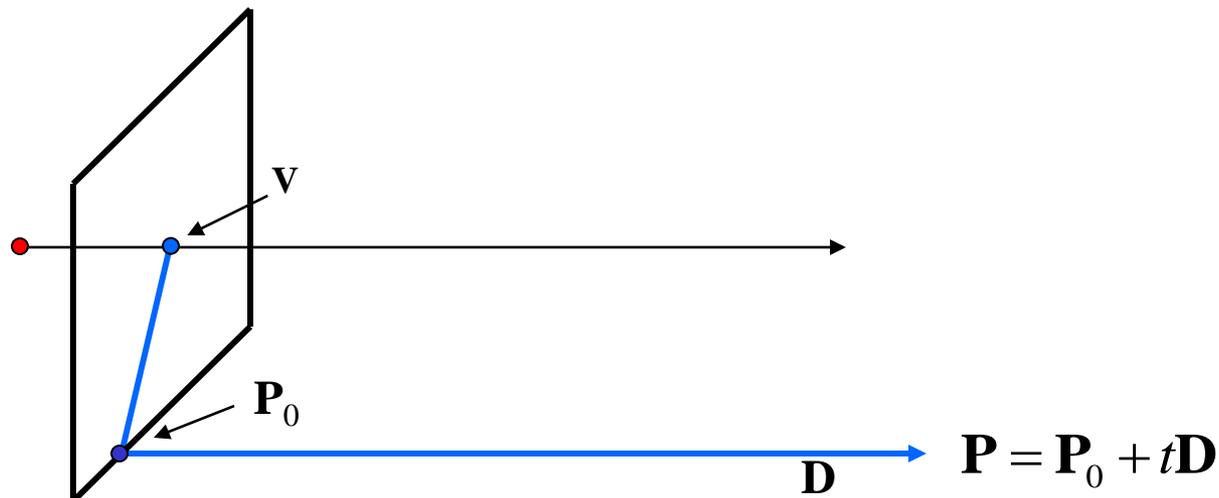
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Computing vanishing points



Computing vanishing points

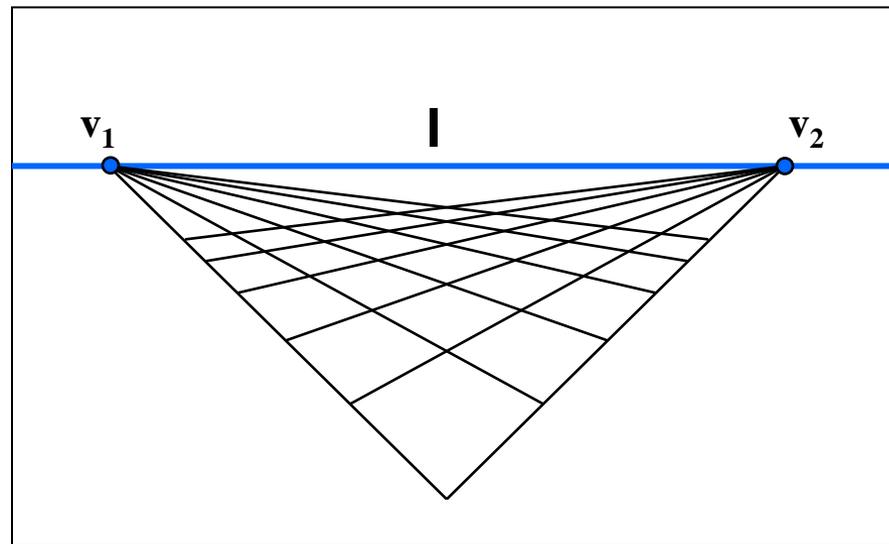
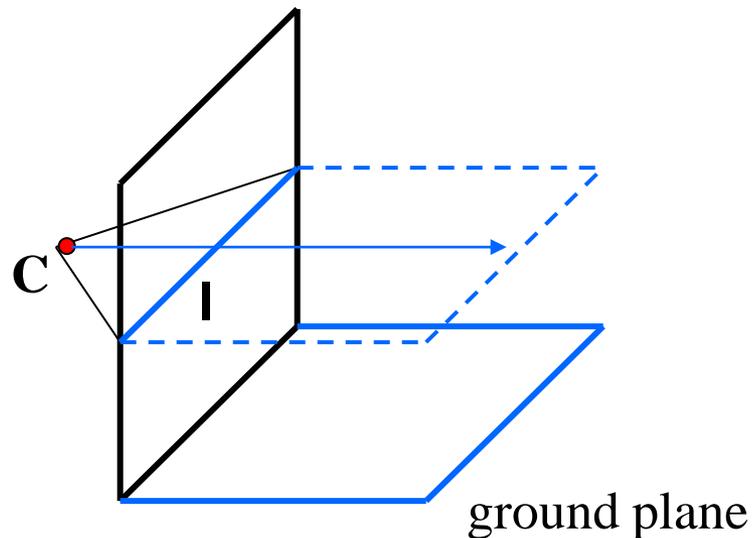


$$\mathbf{P}_t = \begin{bmatrix} P_X + tD_X \\ P_Y + tD_Y \\ P_Z + tD_Z \\ 1 \end{bmatrix} \cong \begin{bmatrix} P_X / t + D_X \\ P_Y / t + D_Y \\ P_Z / t + D_Z \\ 1/t \end{bmatrix}$$

Properties $\mathbf{v} = \mathbf{I}\mathbf{P}_\infty$

- \mathbf{P}_∞ is a point at *infinity*, \mathbf{v} is its projection
- Depends only on line *direction*
- Parallel lines $\mathbf{P}_0 + t\mathbf{D}$, $\mathbf{P}_1 + t\mathbf{D}$ intersect at \mathbf{P}_∞

Computing vanishing lines

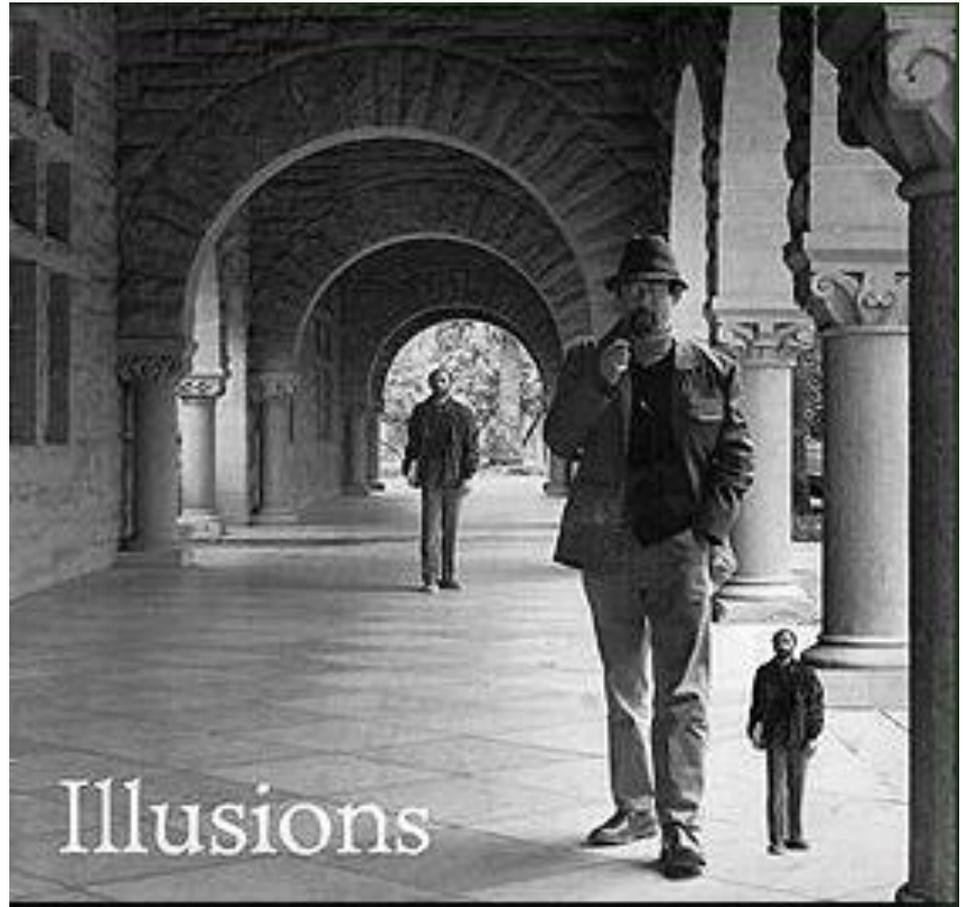
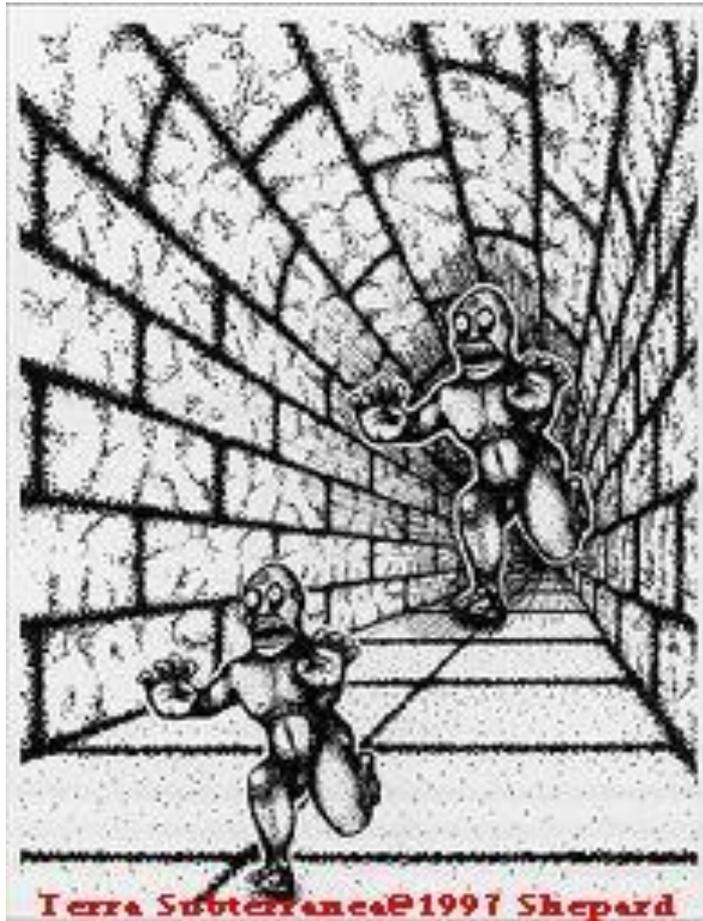


Properties

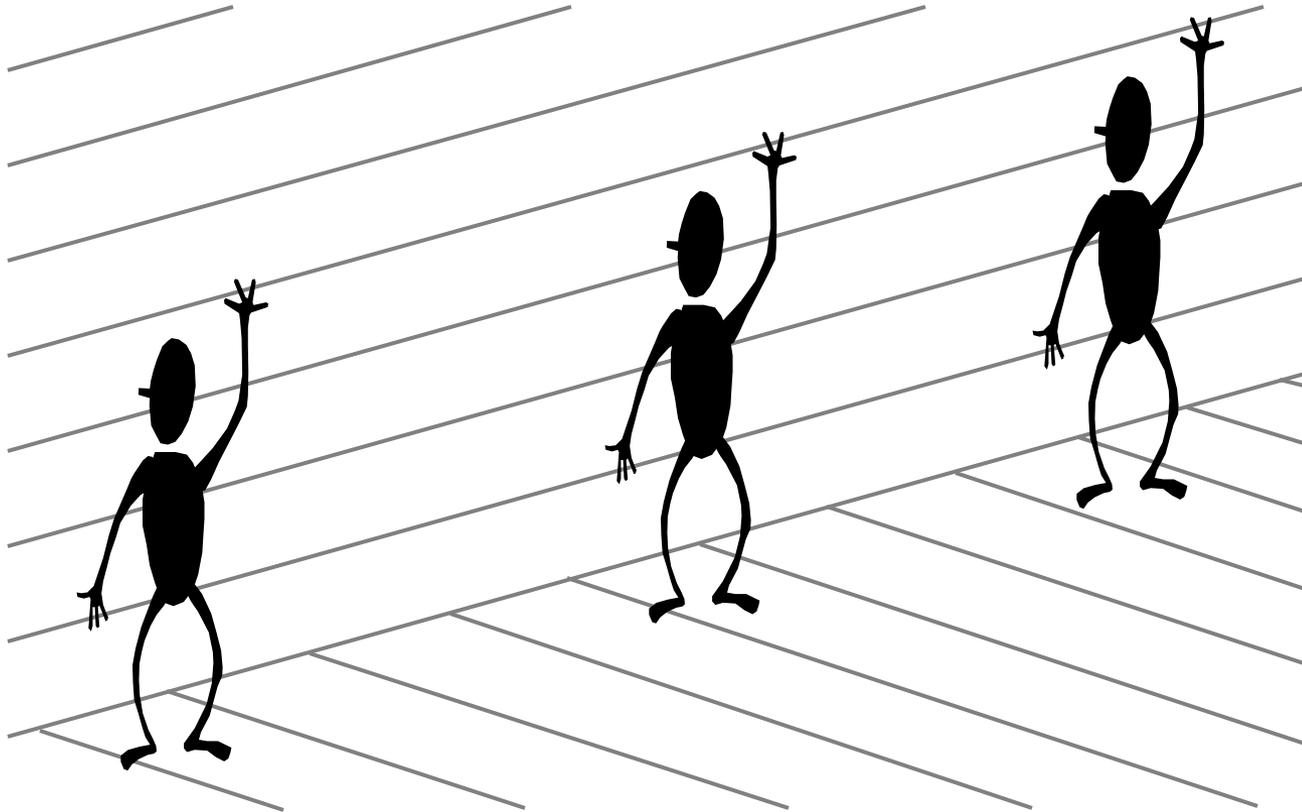
- I is intersection of horizontal plane through C with image plane
- Compute I from two sets of parallel lines on ground plane
- All points at same height as C project to I
 - points higher than C project above I
- Provides way of comparing height of objects in the scene



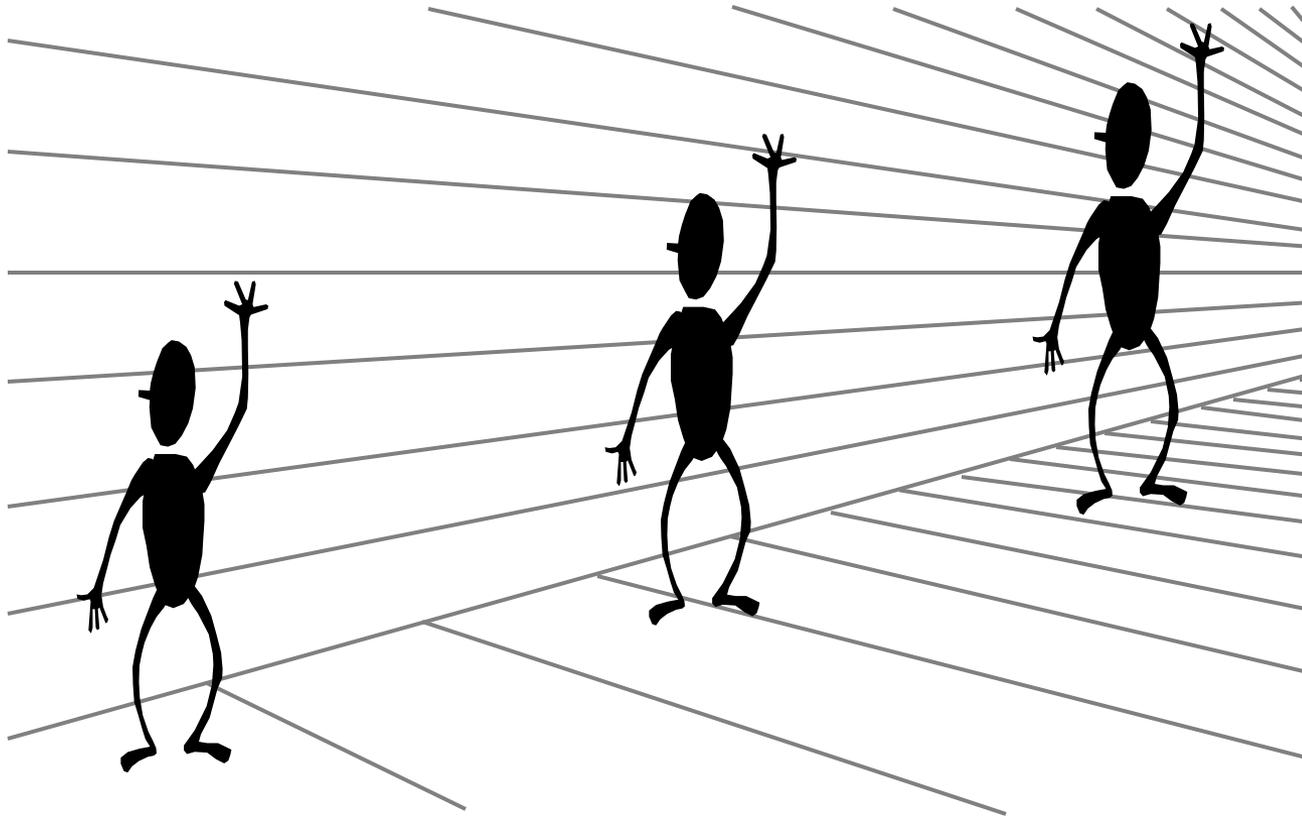
Fun with vanishing points



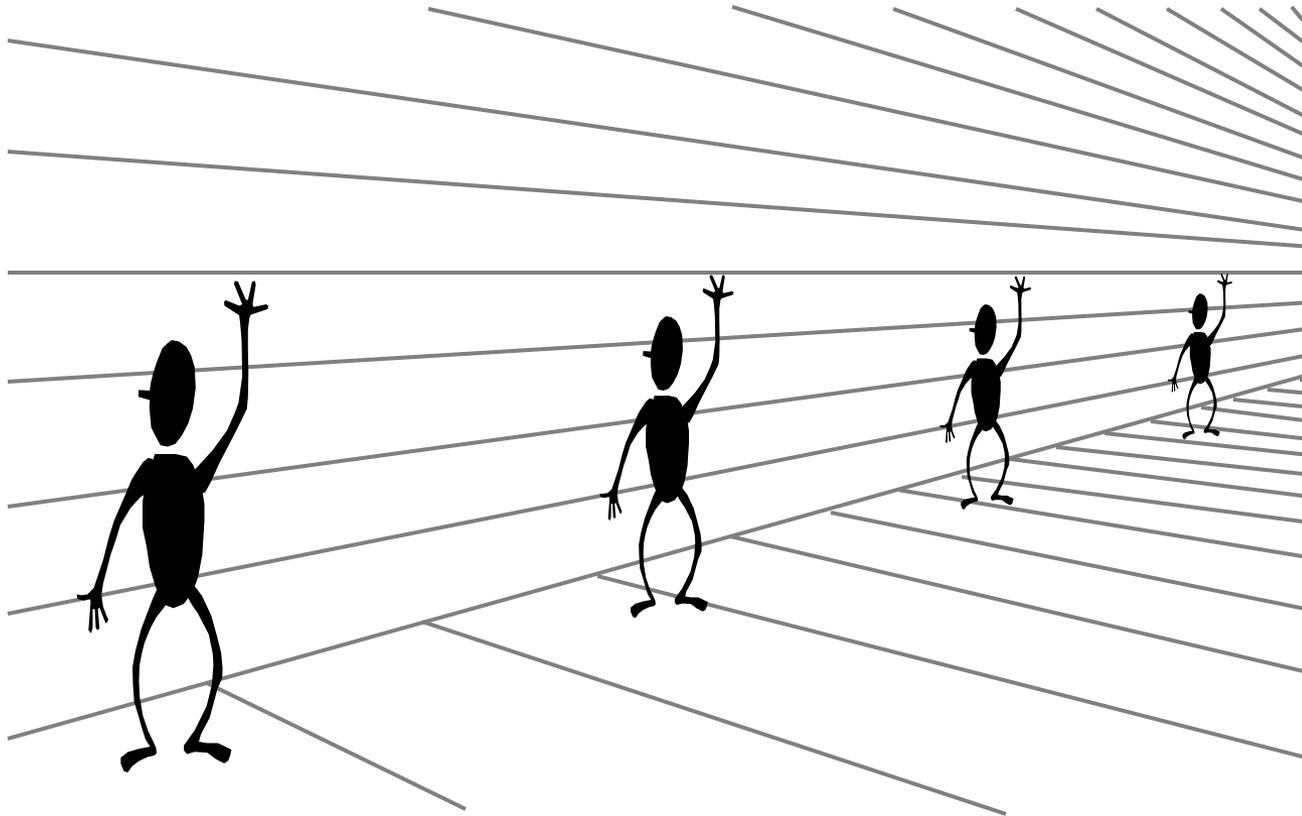
Perspective cues



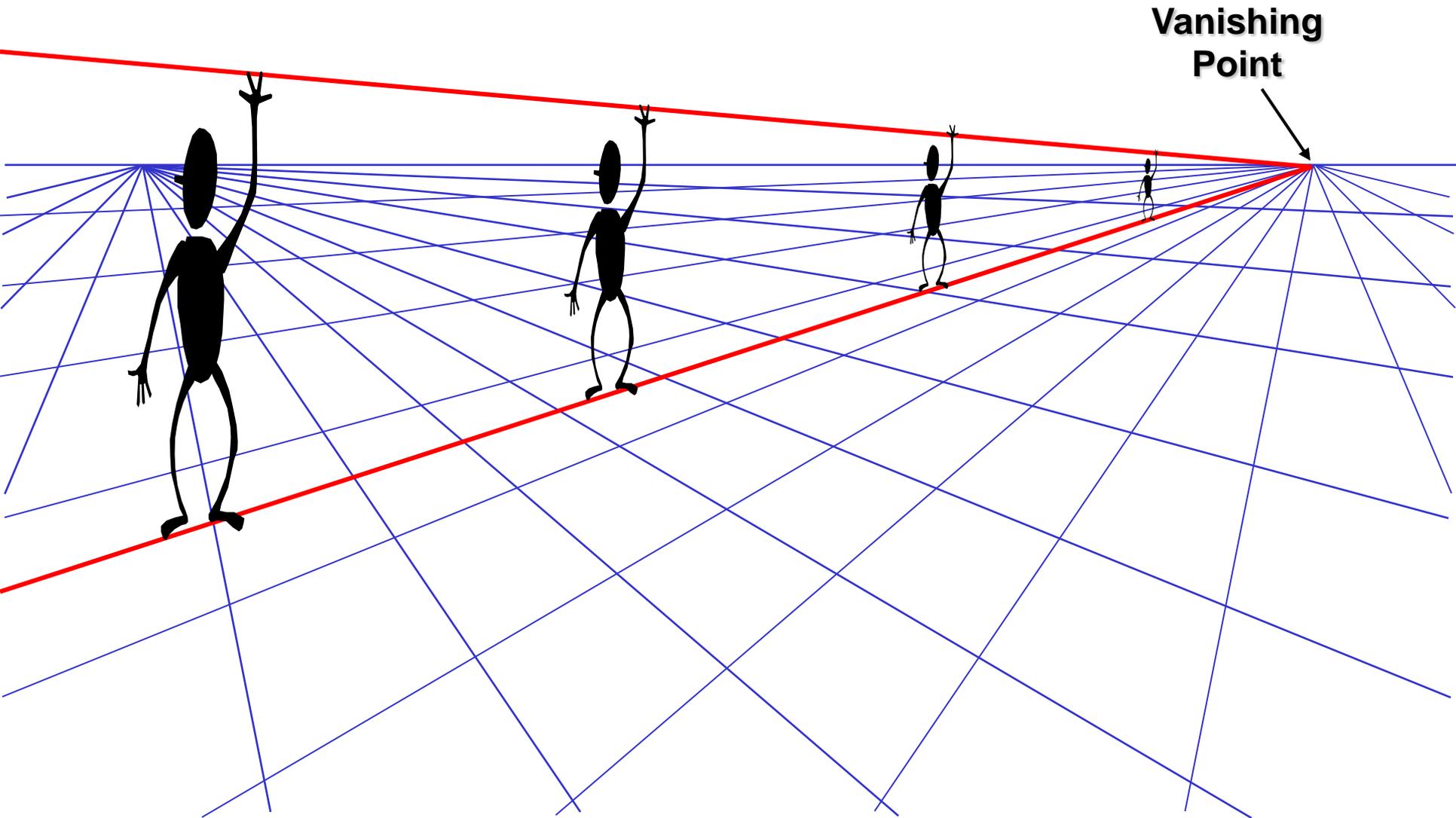
Perspective cues



Perspective cues

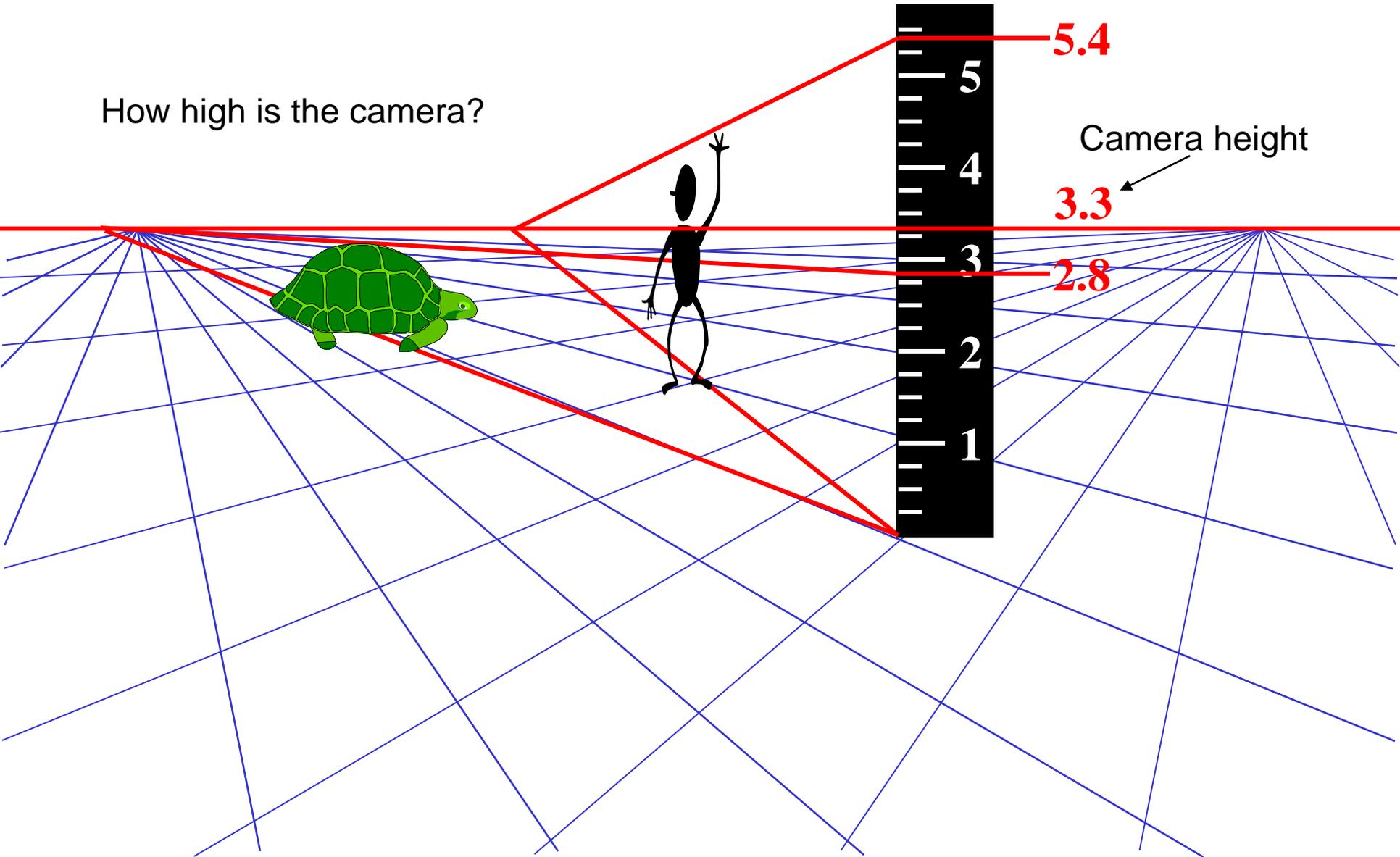


Comparing heights

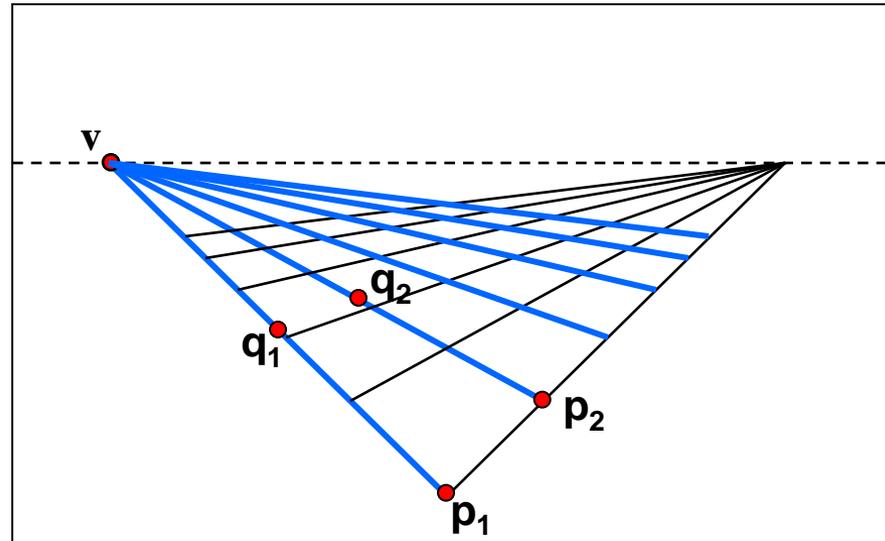


Measuring height

How high is the camera?



Computing vanishing points (from lines)



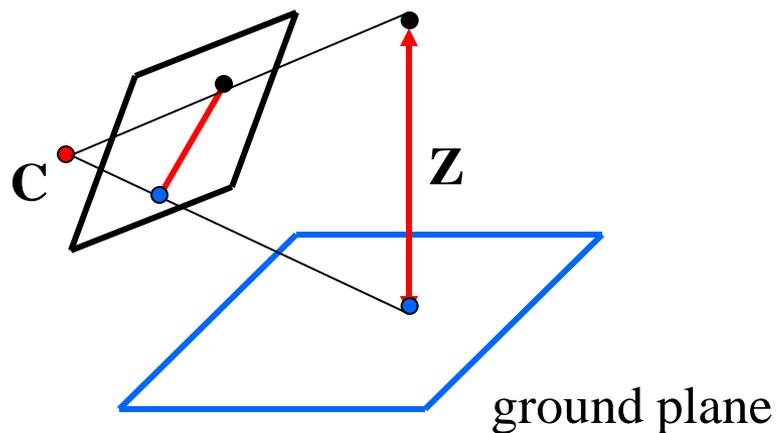
Intersect p_1q_1 with p_2q_2

$$v = (p_1 \times q_1) \times (p_2 \times q_2)$$

Least squares version

- Better to use more than two lines and compute the “closest” point of intersection
- See notes by [Bob Collins](#) for one good way of doing this:
 - <http://www-2.cs.cmu.edu/~ph/869/www/notes/vanishing.txt>

Measuring height without a ruler



Compute Z from image measurements

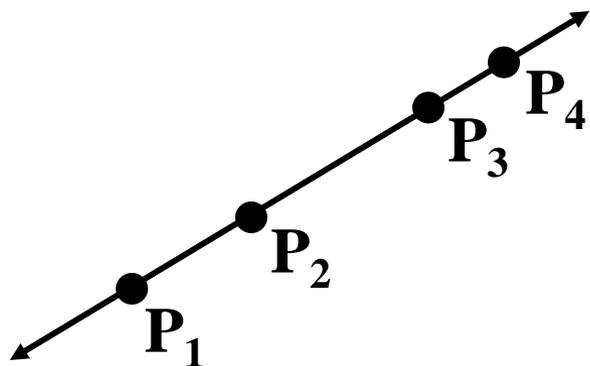
- Need more than vanishing points to do this

The cross ratio

A Projective Invariant

- Something that does not change under projective transformations (including perspective projection)

The *cross-ratio* of 4 collinear points



$$\frac{\| \mathbf{P}_3 - \mathbf{P}_1 \| \| \mathbf{P}_4 - \mathbf{P}_2 \|}{\| \mathbf{P}_3 - \mathbf{P}_2 \| \| \mathbf{P}_4 - \mathbf{P}_1 \|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

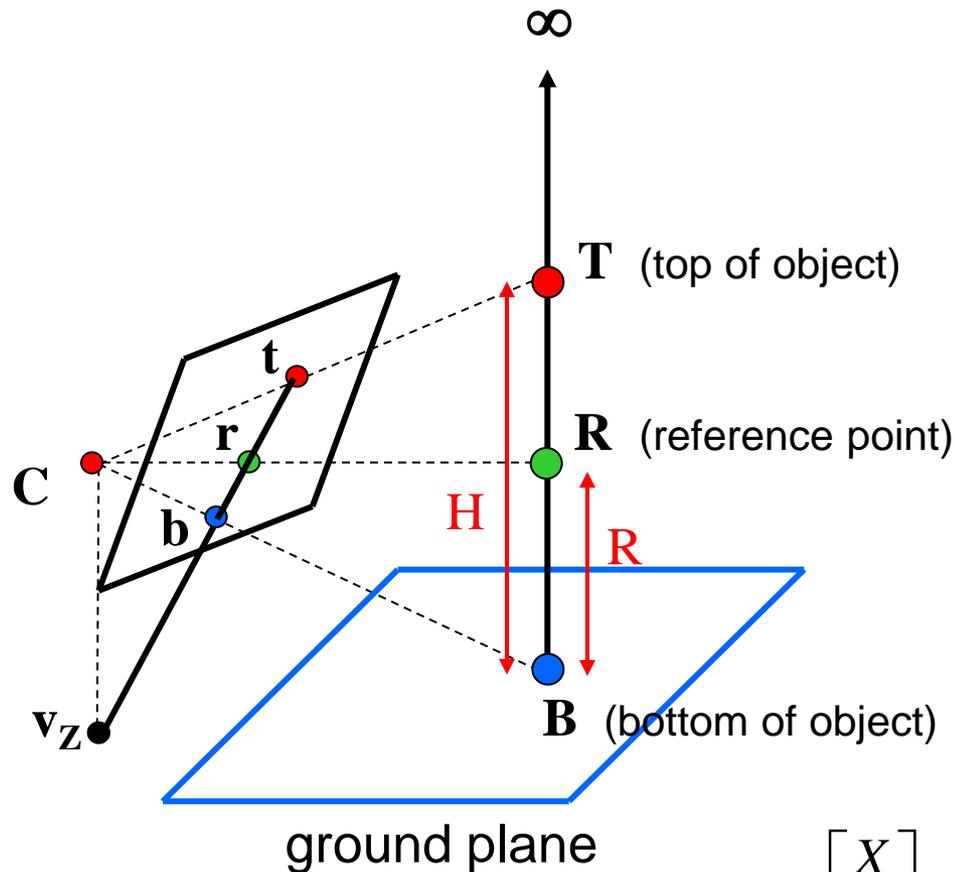
Can permute the point ordering

$$\frac{\| \mathbf{P}_1 - \mathbf{P}_3 \| \| \mathbf{P}_4 - \mathbf{P}_2 \|}{\| \mathbf{P}_1 - \mathbf{P}_2 \| \| \mathbf{P}_4 - \mathbf{P}_3 \|}$$

- $4! = 24$ different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Measuring height



scene points represented as $\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$

$$\frac{\|\mathbf{T} - \mathbf{B}\| \|\infty - \mathbf{R}\|}{\|\mathbf{R} - \mathbf{B}\| \|\infty - \mathbf{T}\|} = \frac{H}{R}$$

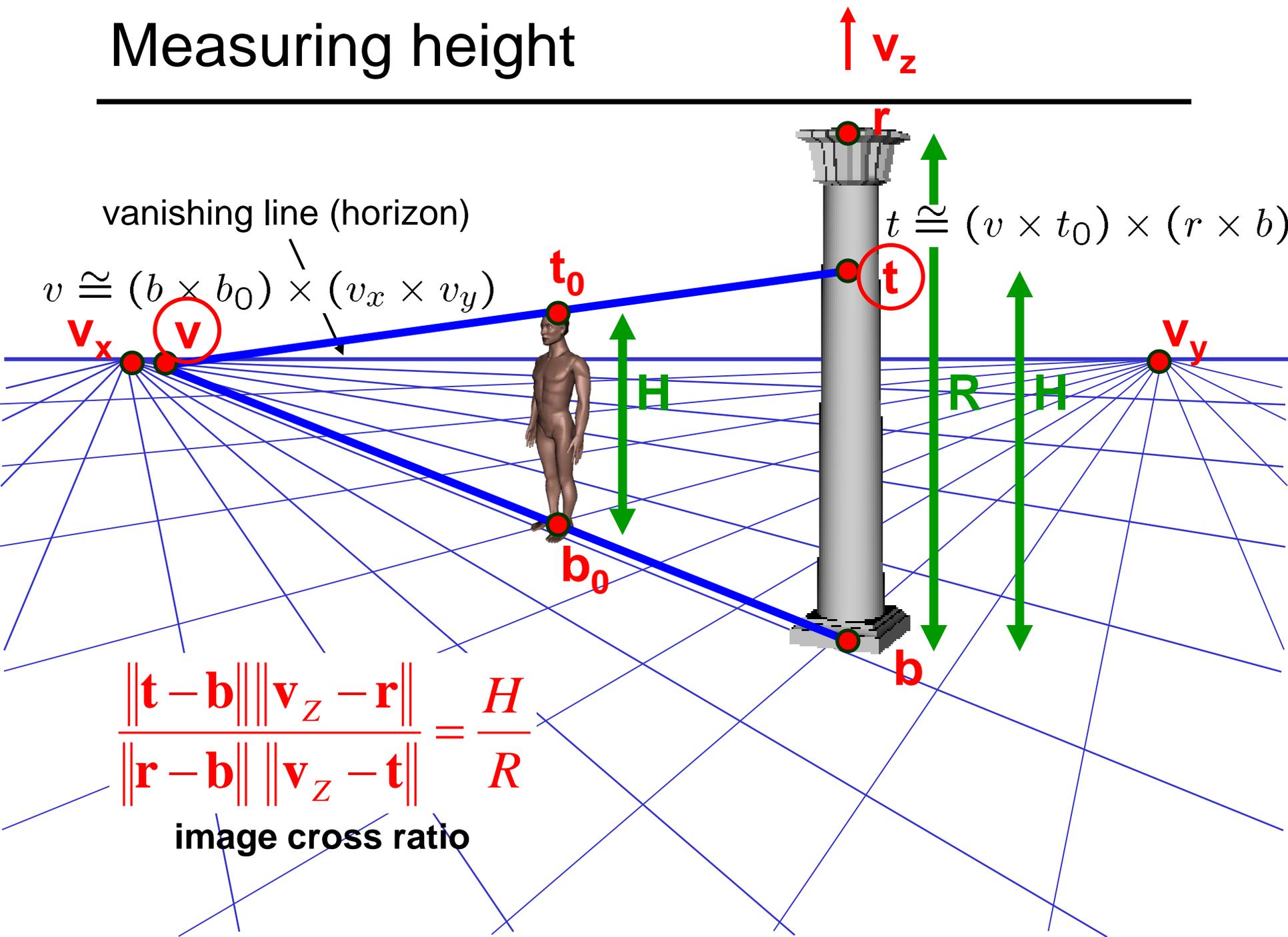
scene cross ratio

$$\frac{\|\mathbf{t} - \mathbf{b}\| \|\mathbf{v}_Z - \mathbf{r}\|}{\|\mathbf{r} - \mathbf{b}\| \|\mathbf{v}_Z - \mathbf{t}\|} = \frac{H}{R}$$

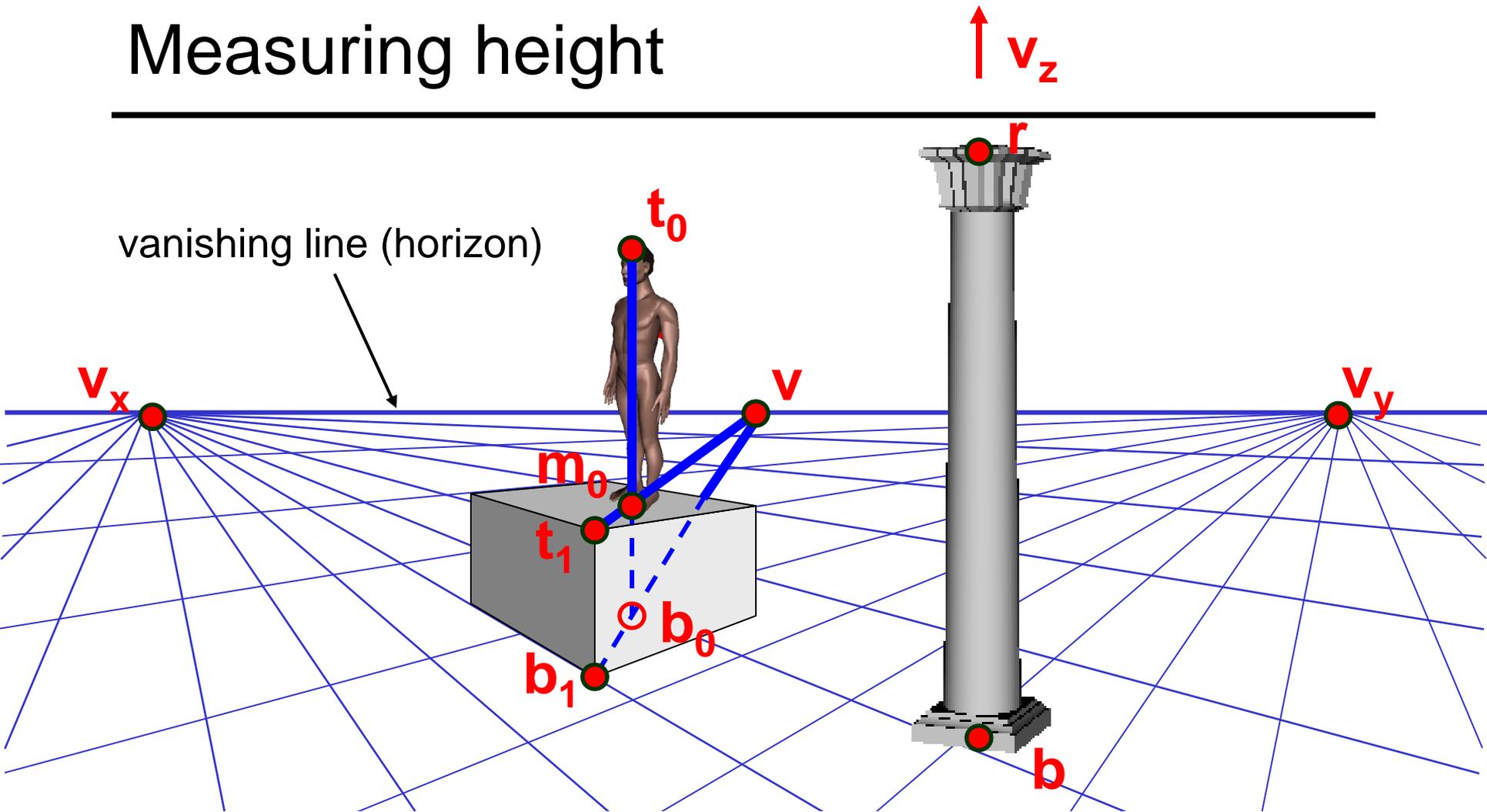
image cross ratio

image points as $\mathbf{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

Measuring height



Measuring height



What if the point on the ground plane \mathbf{b}_0 is not known?

- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find \mathbf{b}_0 as shown above

3D Modeling from a photograph



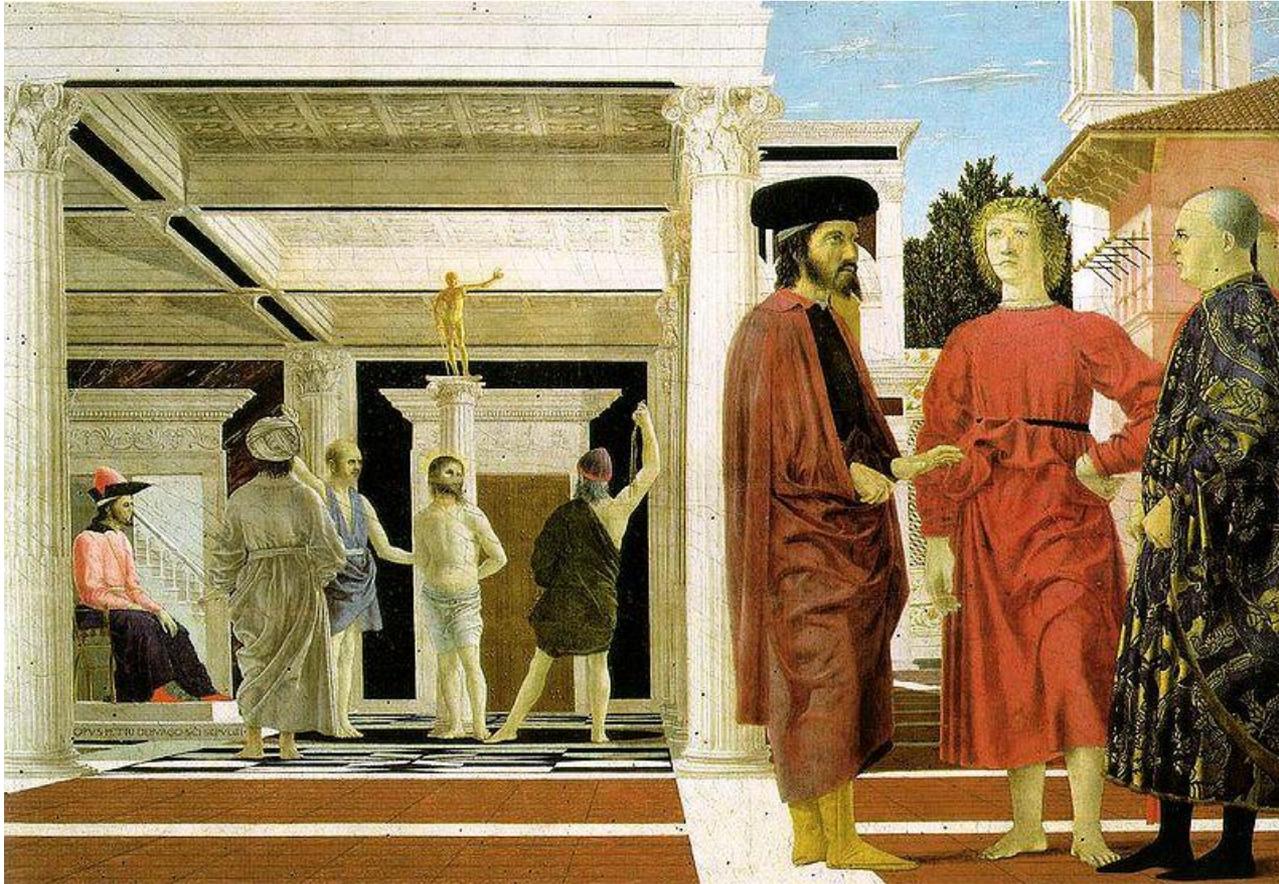
St. Jerome in his Study, H. Steenwick

3D Modeling from a photograph



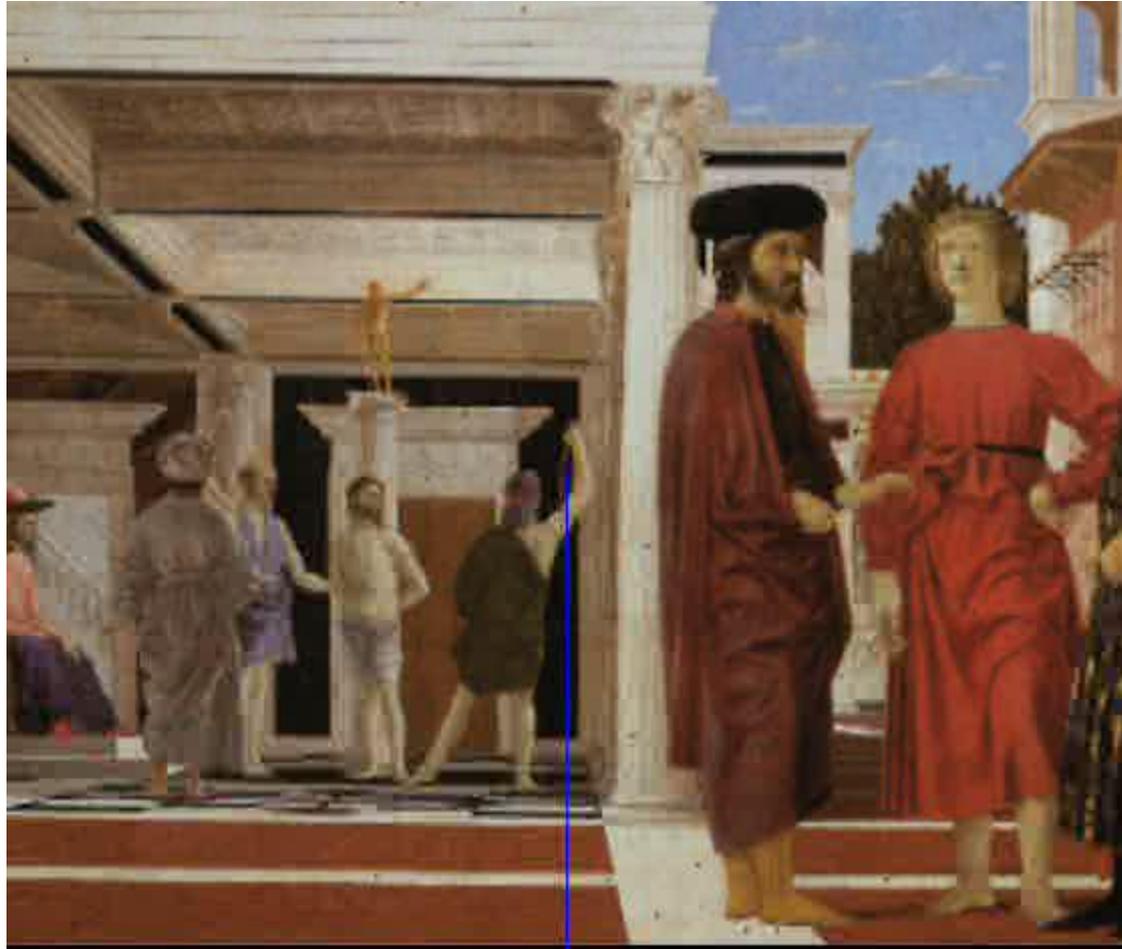
video by Antonio Criminisi

3D Modeling from a photograph



Flagellation, Piero della Francesca

3D Modeling from a photograph



video by Antonio Criminisi

3D Modeling from a photograph

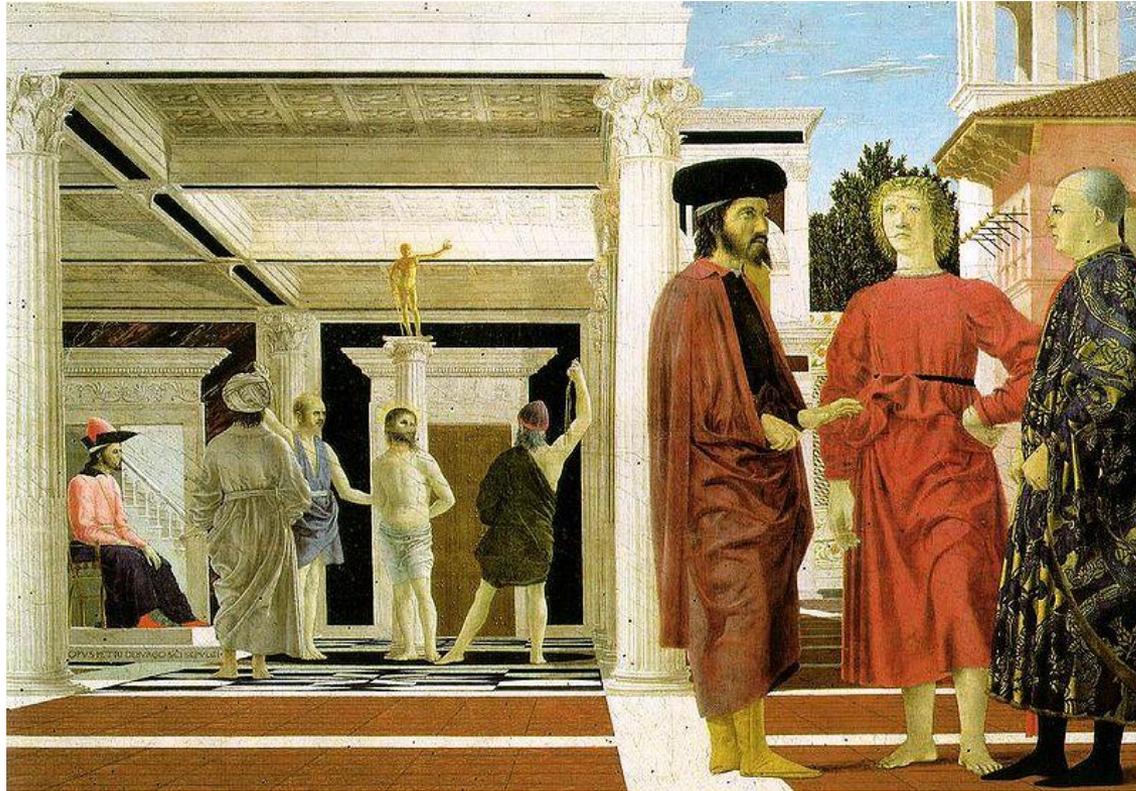


figure by Antonio Criminisi