CS6670: Computer Vision

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Lecture 5: Projection

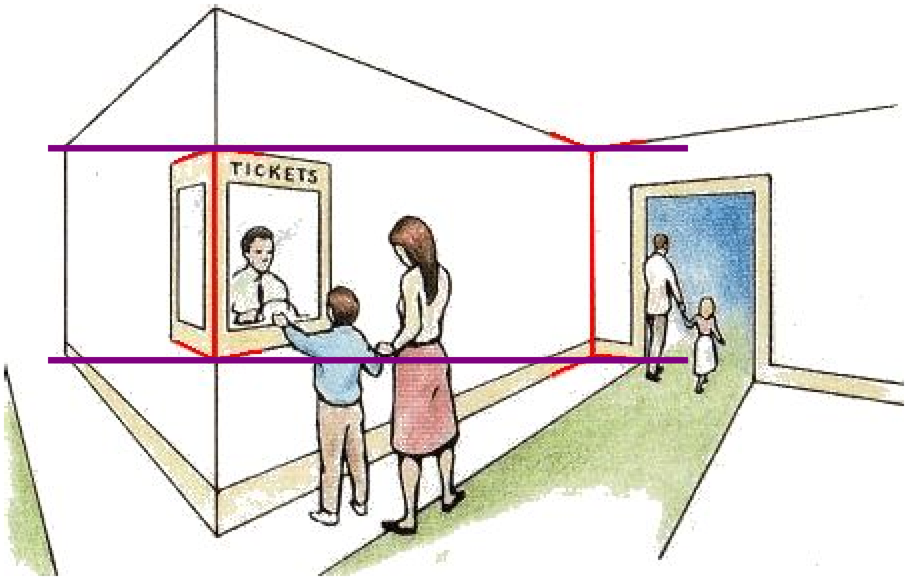
Projection

- Reading: Szeliski 2.1

Projection

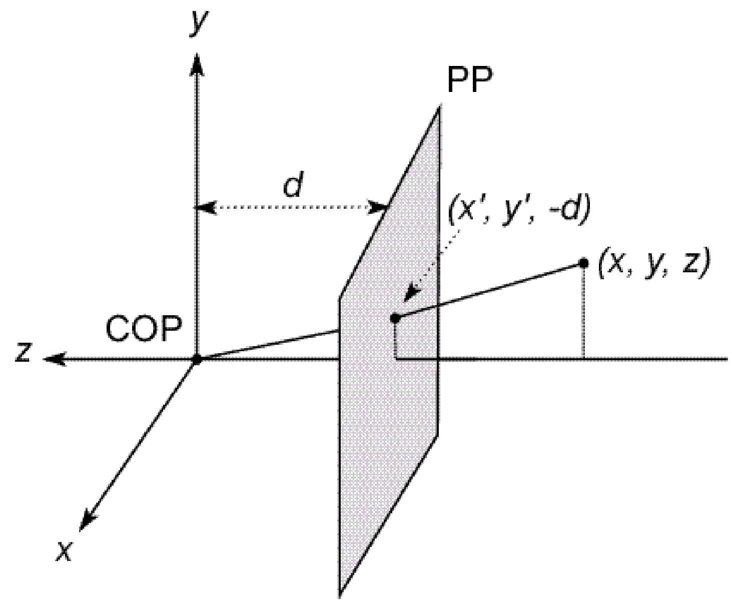
- Reading: Szeliski 2.1

Müller-Lyer Illusion



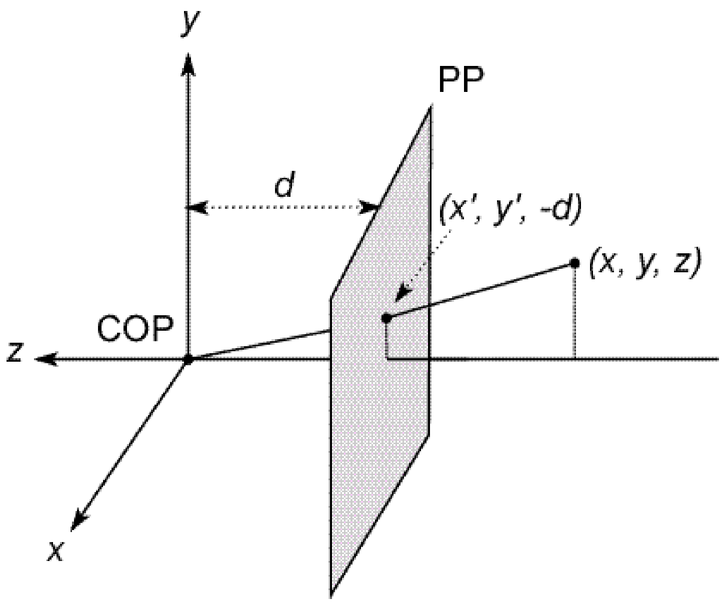
http://www.michaelbach.de/ot/sze_muelue/index.html

Modeling projection



- The coordinate system
  - We will use the pin-hole model as an approximation
  - Put the optical center (**C**enter **O**f **P**rojection) at the origin
  - Put the image plane (**P**rojection **P**lane) *in front* of the COP
    - Why?
  - The camera looks down the *negative* z axis
    - we need this if we want right-handed-coordinates

Modeling projection



- Projection equations
  - Compute intersection with PP of ray from (x,y,z) to COP
  - Derived using similar triangles (on board)

$$(x, y, z) \rightarrow (-d\frac{x}{z},\ -d\frac{y}{z},\ -d)$$

- We get the projection by throwing out the last coordinate:

$$(x, y, z) \rightarrow (-d\frac{x}{z},\ -d\frac{y}{z})$$

Homogeneous coordinates

- Is this a linear transformation?
  - no—division by z is nonlinear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Perspective Projection

Projection is a matrix multiply using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \quad -d\frac{y}{z})$$

divide by third coordinate

This is known as **perspective projection**

- The matrix is the **projection matrix**
- Can also formulate as a 4x4 (today's reading does this)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \quad -d\frac{y}{z})$$

divide by fourth coordinate

Perspective Projection

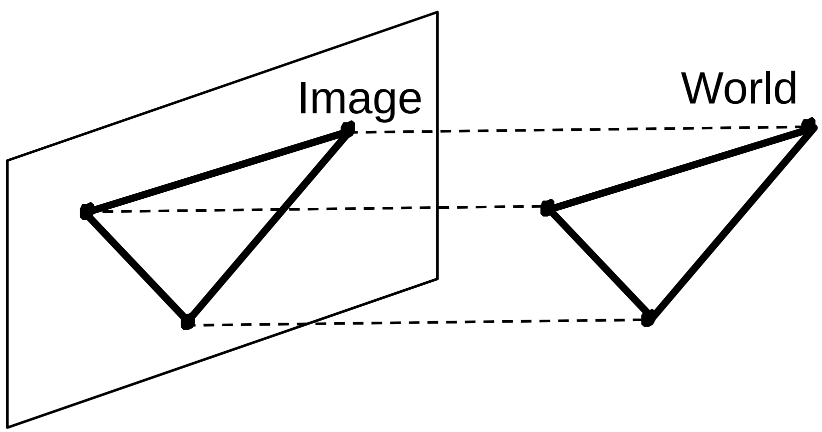
- How does scaling the projection matrix change the transformation?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \quad -d\frac{y}{z})$$

$$\begin{bmatrix} -d & 0 & 0 & 0 \\ 0 & -d & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} -dx \\ -dy \\ z \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \quad -d\frac{y}{z})$$

Orthographic projection

- Special case of perspective projection
  - Distance from the COP to the PP is infinite



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Variants of orthographic projection

- Scaled orthographic
  - Also called "weak perspective"

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/d \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1/d \end{bmatrix} \Rightarrow (dx, dy)$$

- Affine projection
  - Also called "paraperspective"

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

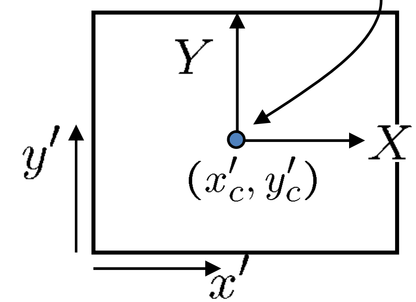
Camera parameters

A camera is described by several parameters

- Translation T of the optical center from the origin of world coords
- Rotation R of the image plane
- focal length f, principle point (x'_c, y'_c), pixel size (s_x, s_y)
- blue parameters are called "extrinsics," red are "intrinsics"

Projection equation

$$\mathbf{X} = \begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{\Pi X}$$



- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

identity matrix

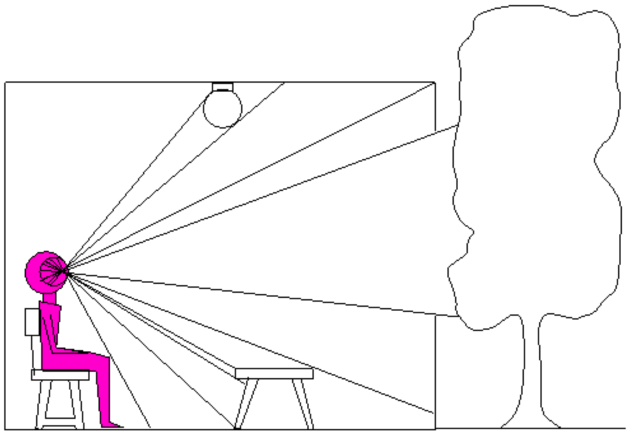
$$\mathbf{\Pi} = \begin{bmatrix} -fs_x & 0 & x'_c \\ 0 & -fs_y & y'_c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R}_{3x3} & \mathbf{0}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{3x3} & \mathbf{T}_{3x1} \\ \mathbf{0}_{1x3} & 1 \end{bmatrix}$$

intrinsics　　projection　　rotation　　translation

- The definitions of these parameters are **not** completely standardized
  - especially intrinsics—varies from one book to another

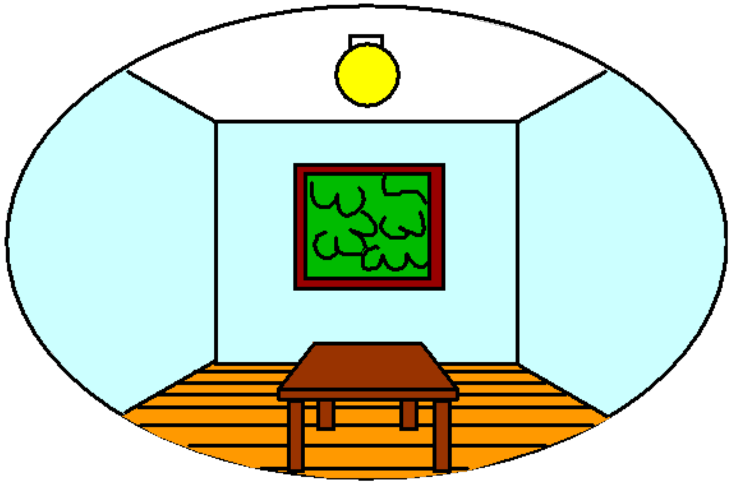
Dimensionality Reduction Machine (3D to 2D)

3D world



Point of observation

2D image



What have we lost?

- Angles
- Distances (lengths)

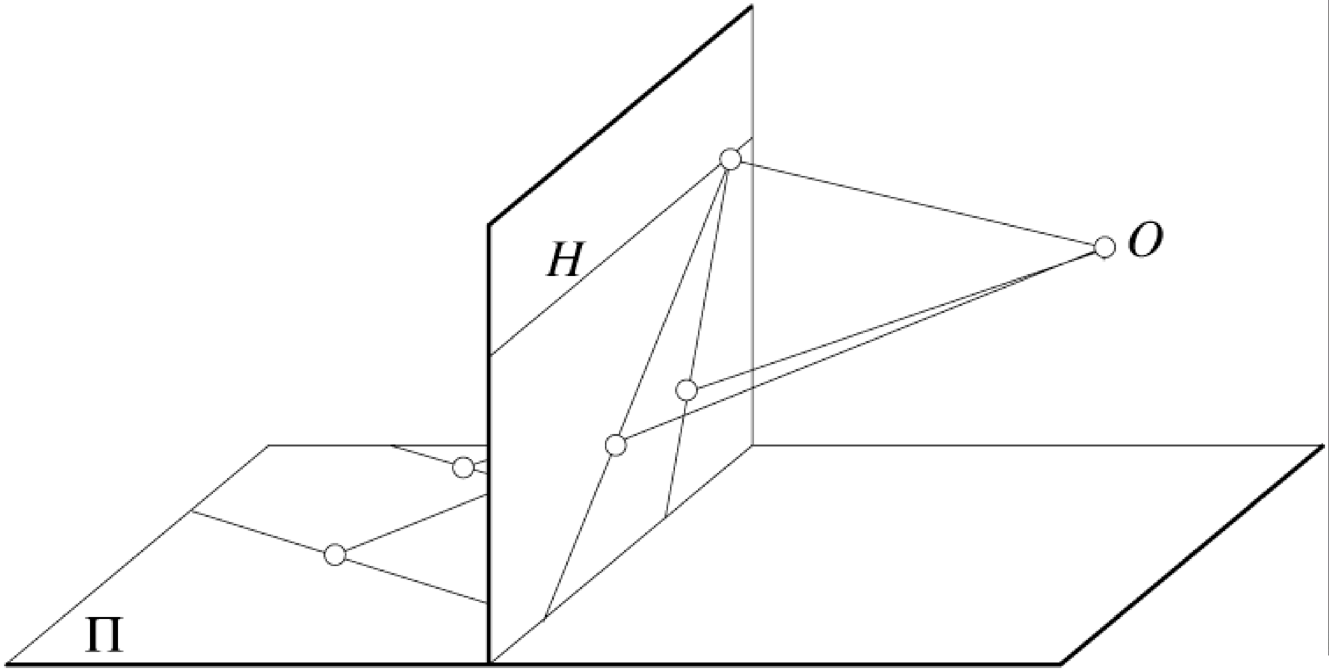
Slide by A. Efros

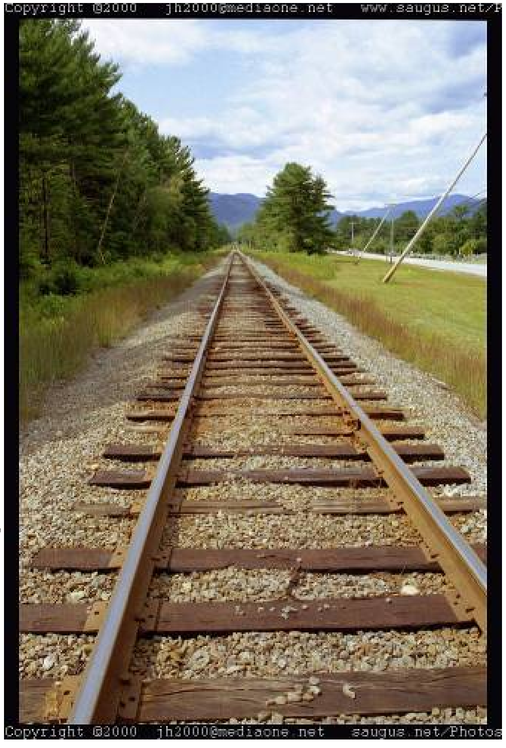
Projection properties

- Many-to-one: any points along same ray map to same point in image
- Points → points
- Lines → lines (collinearity is preserved)
  - But line through focal point projects to a point
- Planes → planes (or half-planes)
  - But plane through focal point projects to line

Projection properties

- Parallel lines converge at a vanishing point
  - Each direction in space has its own vanishing point
  - But parallels parallel to the image plane remain parallel





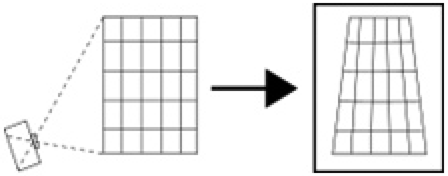
Perspective distortion

- Problem for architectural photography: converging verticals

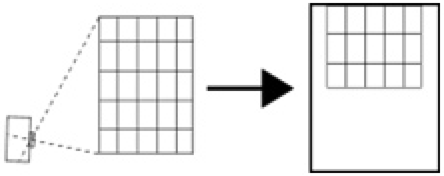
Source: F. Durand

Perspective distortion

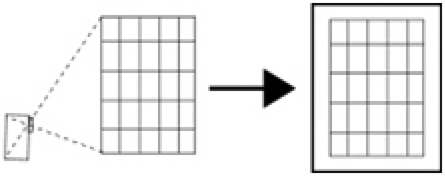
- Problem for architectural photography: converging verticals



Tilting the camera upwards results in converging verticals

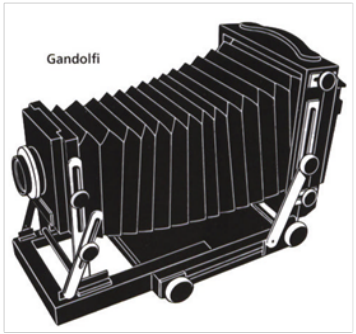


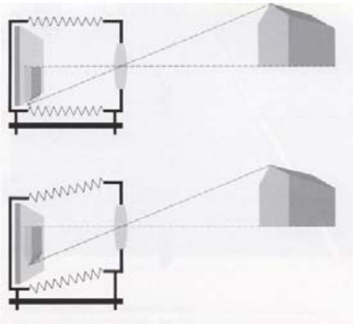
Keeping the camera level, with an ordinary lens, captures only the bottom portion of the building



Shifting the lens upwards results in a picture of the entire subject

- Solution: view camera (lens shifted w.r.t. film)





http://en.wikipedia.org/wiki/Perspective_correction_lens

Source: F. Durand

Perspective distortion

- Problem for architectural photography: converging verticals
- Result:

Source: F. Durand

Perspective distortion

- What does a sphere project to?

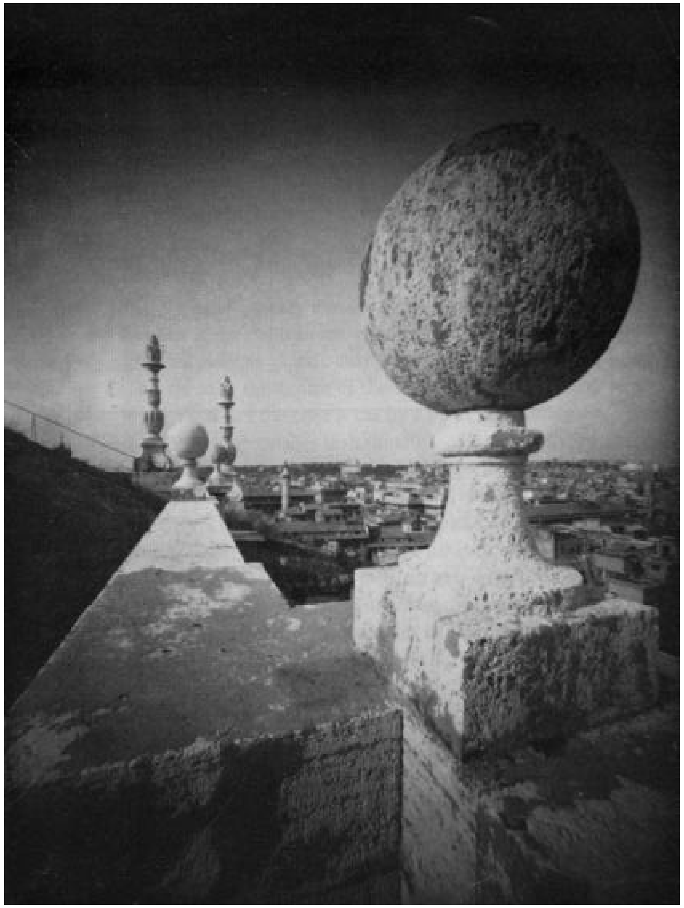
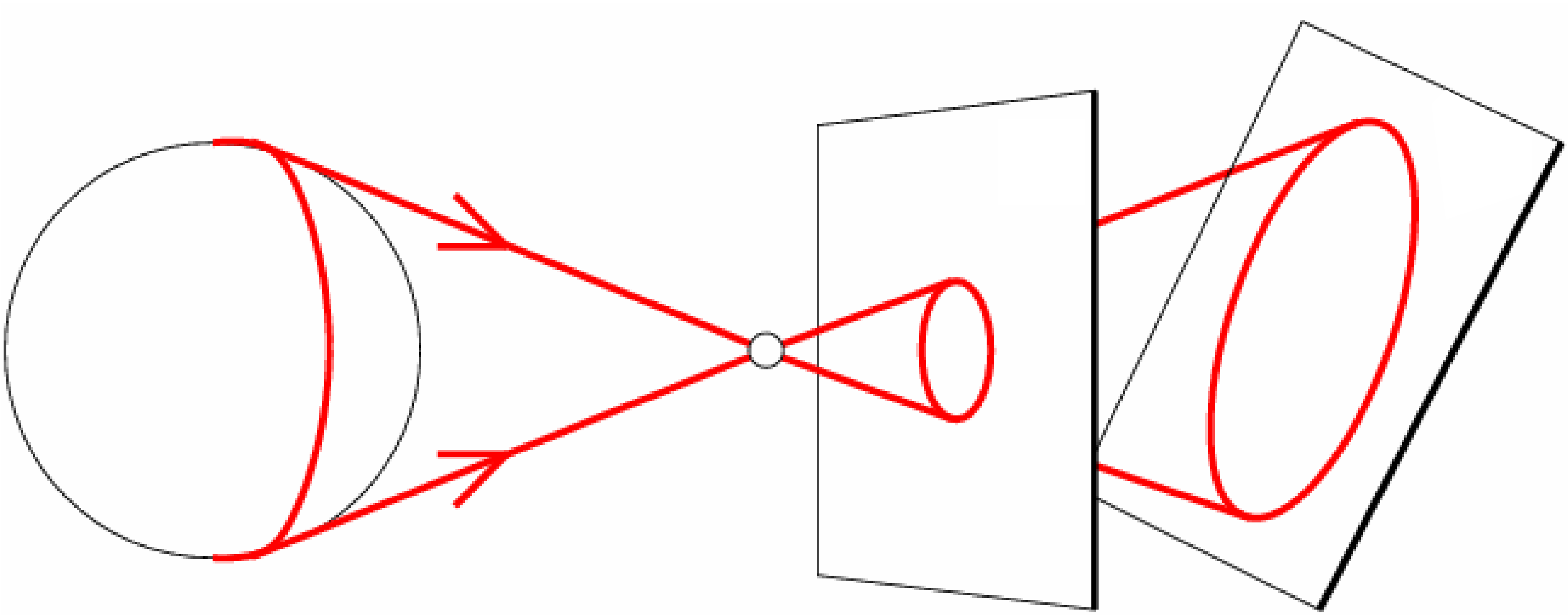


Image source: F. Durand

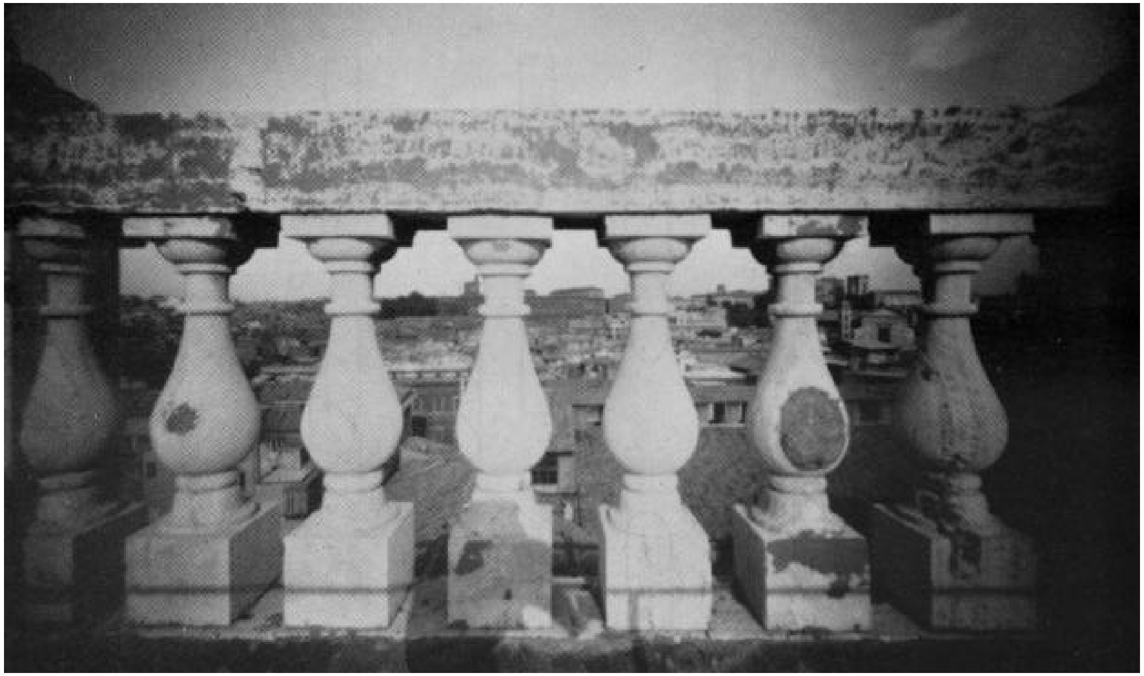
Perspective distortion

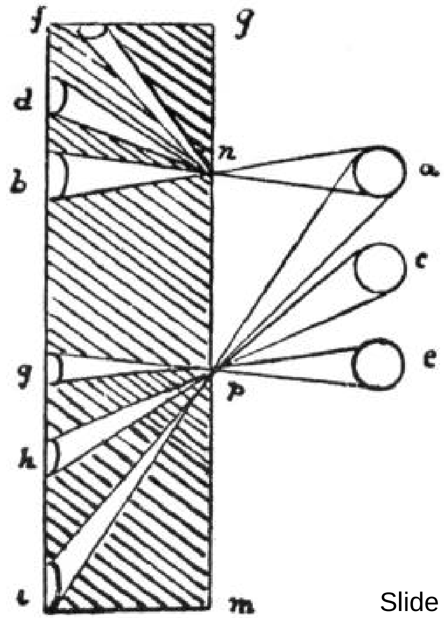
- What does a sphere project to?



Perspective distortion

- The exterior columns appear bigger
- The distortion is not due to lens flaws
- Problem pointed out by Da Vinci

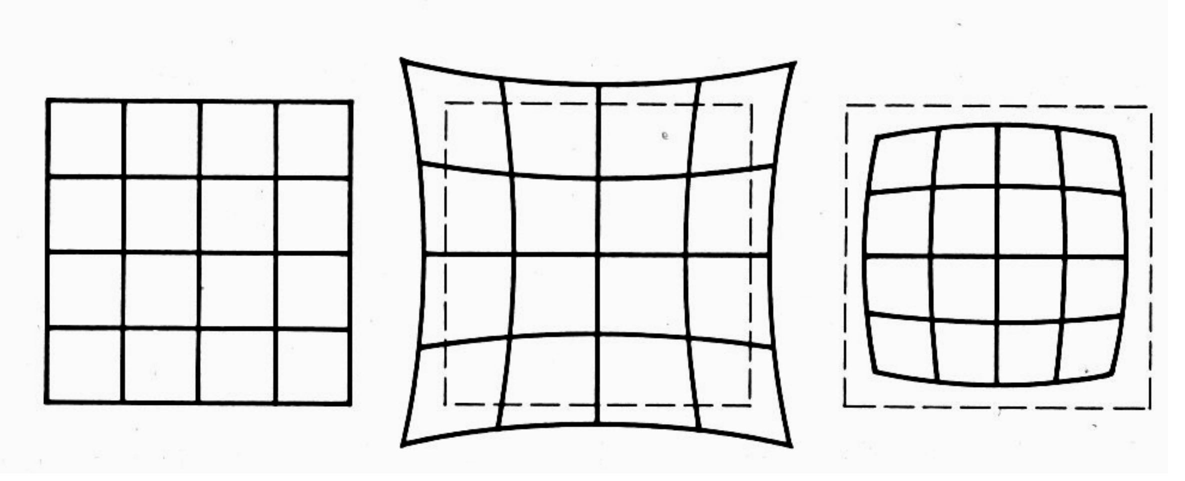




Slide by F. Durand

Perspective distortion: People

Distortion

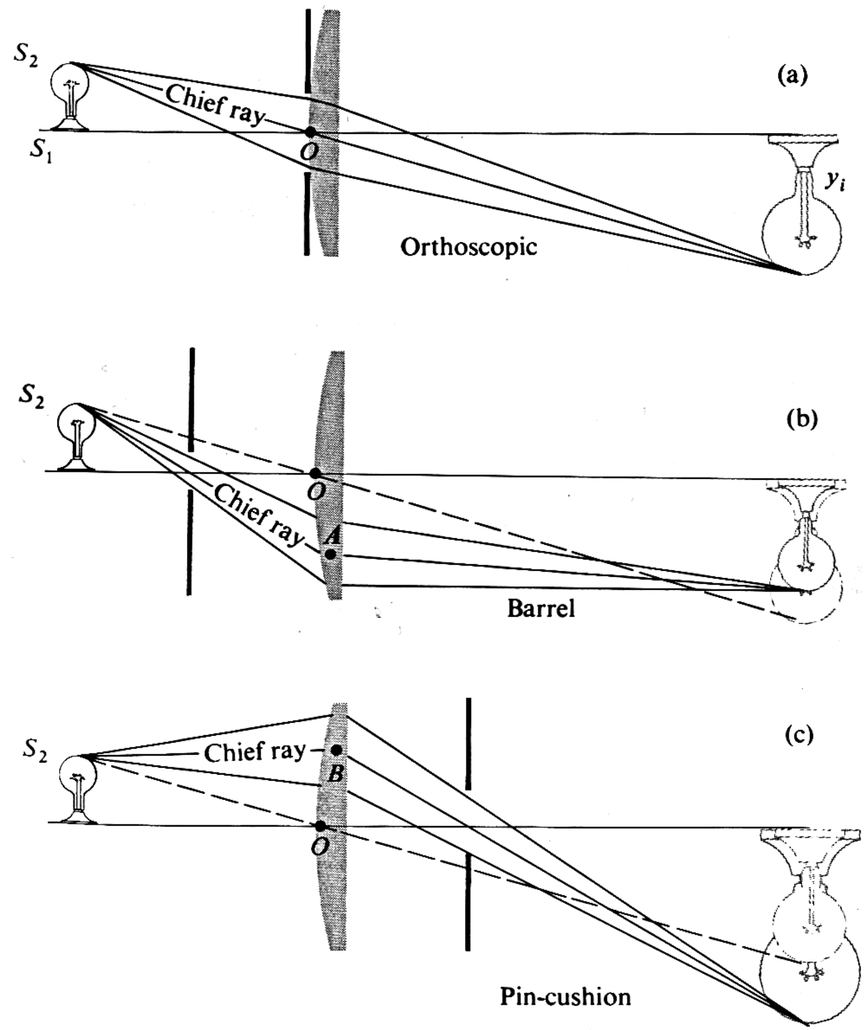


- Radial distortion of the image
  - Caused by imperfect lenses
  - Deviations are most noticeable for rays that pass through the edge of the lens

Correcting radial distortion

from [Helmut Dersch]

Distortion


S_2
S_1
Chief ray
O
y_i
(a)
Orthoscopic
S_2
Chief ray
O
A
(b)
Barrel
S_2
Chief ray
B
O
(c)
Pin-cushion

Modeling distortion

Project $(\widehat{x}, \widehat{y}, \widehat{z})$ to "normalized" image coordinates

$$x'_n = \widehat{x}/\widehat{z}$$
$$y'_n = \widehat{y}/\widehat{z}$$

Apply radial distortion

$$r^2 = {x'_n}^2 + {y'_n}^2$$
$$x'_d = x'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)$$
$$y'_d = y'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)$$

Apply focal length translate image center

$$x' = f x'_d + x_c$$
$$y' = f y'_d + y_c$$

- To model lens distortion
  - Use above projection operation instead of standard projection matrix multiplication

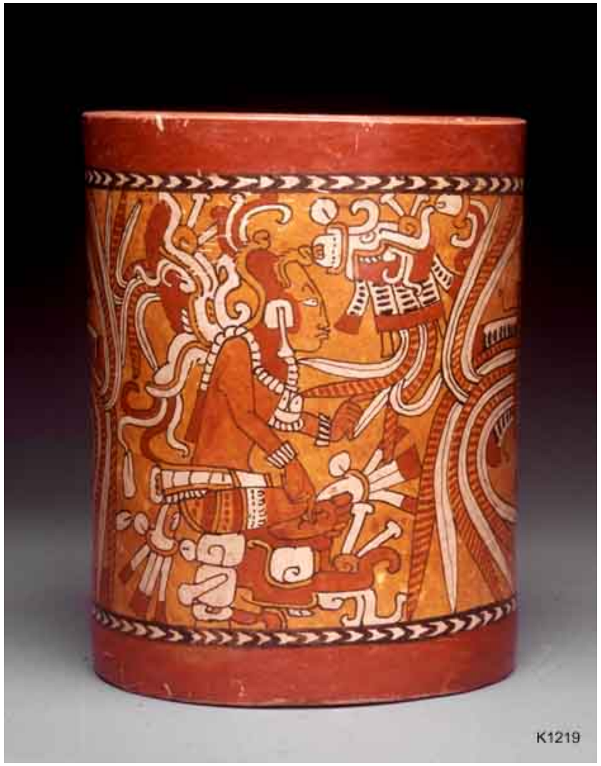
Other types of projection

- Lots of intriguing variants…
- (I’ll just mention a few fun ones)

360 degree field of view…

- Basic approach
  - Take a photo of a parabolic mirror with an orthographic lens (Nayar)
  - Or buy one a lens from a variety of omnicam manufacturers…
    - See http://www.cis.upenn.edu/~kostas/omni.html

Rotating sensor (or object)

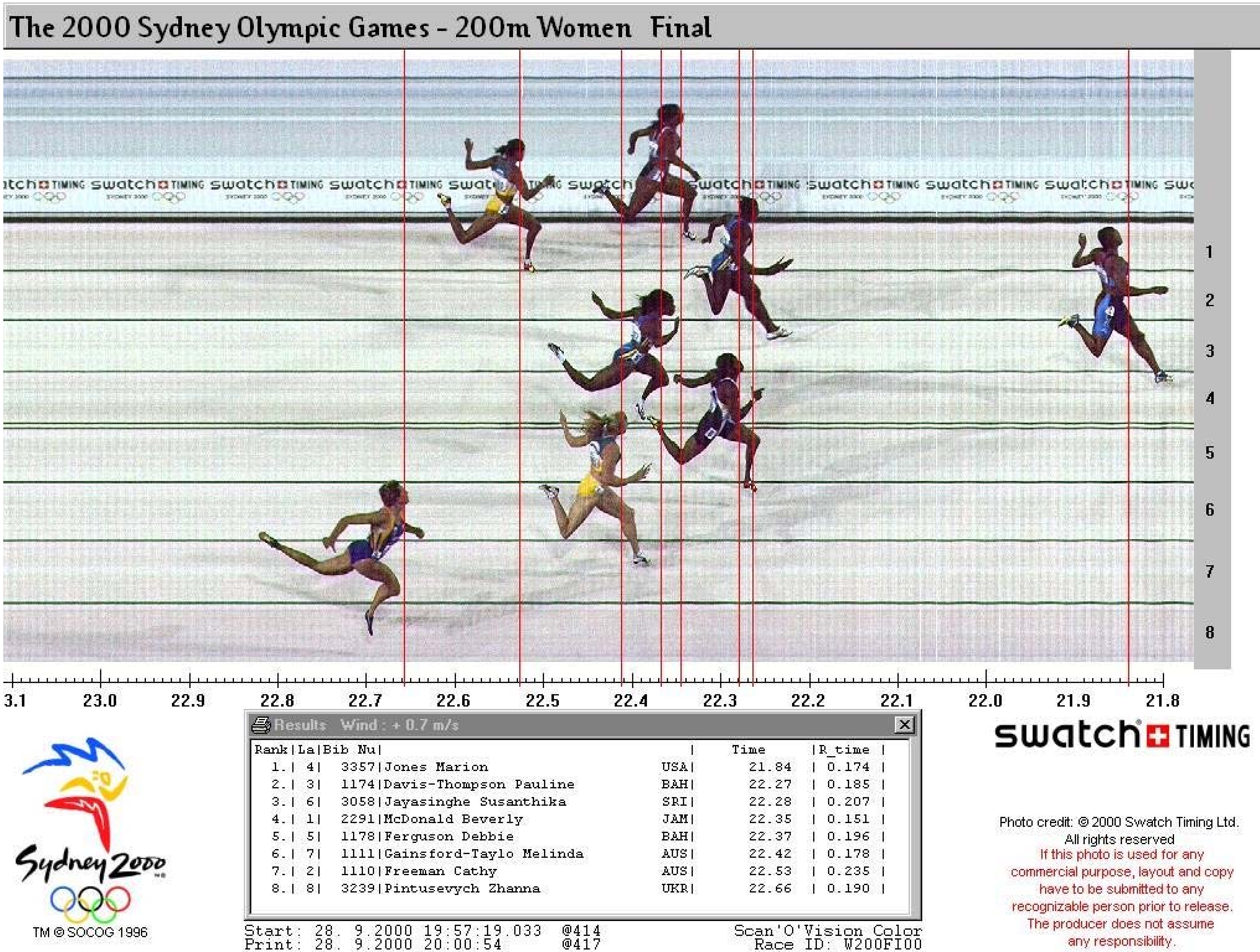


Rollout Photographs © Justin Kerr

http://research.famsi.org/kerrmaya.html

Also known as “cyclographs”, “peripheral images”

Photofinish



Questions?