

Microfacet models for reflection and refraction

Steve Marschner

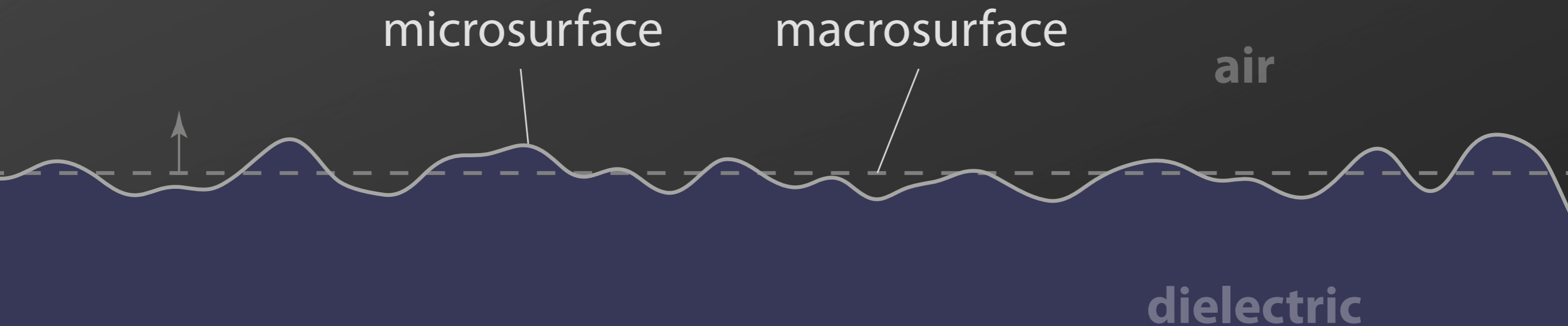
Cornell University CS 667 Fall 2007

(based on presentation for
Walter, Marschner, Li, and Torrance EGSR '07)

Microfacet scattering models

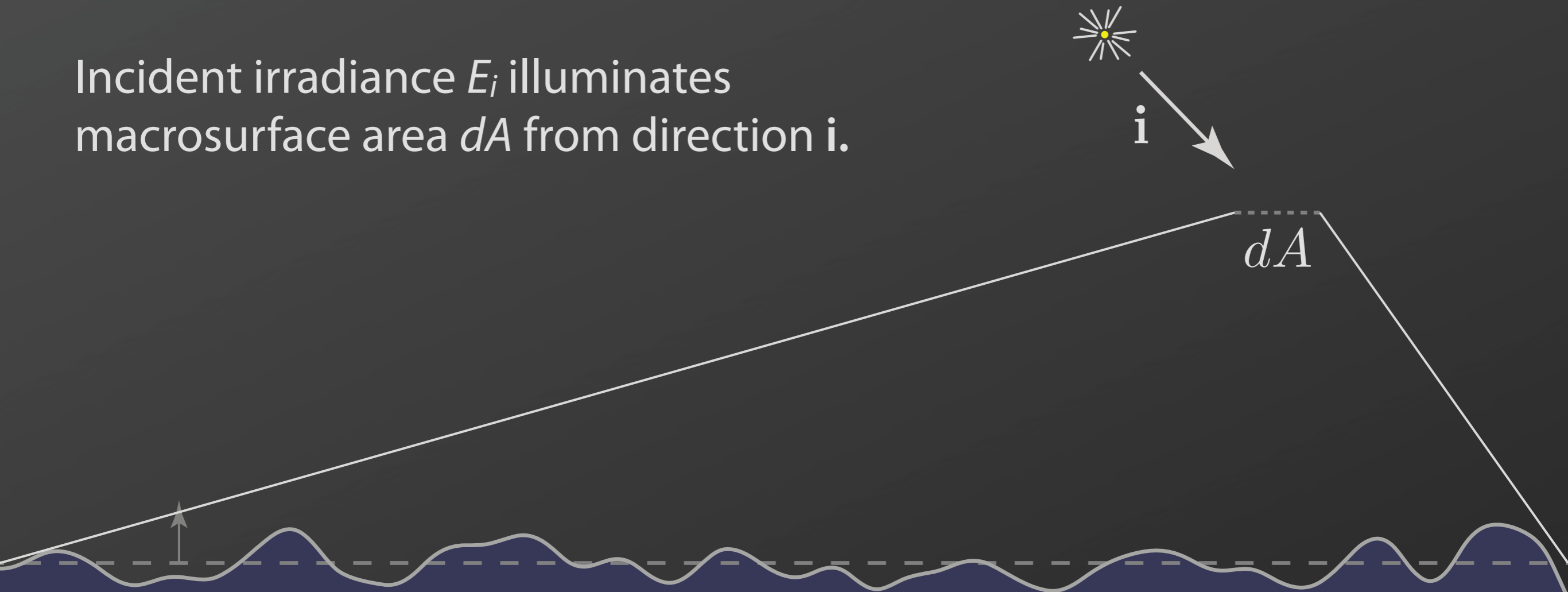
Rough dielectric surface

- smooth at wavelength scale
- rough at microscale
- flat at macroscale



Microfacet scattering models

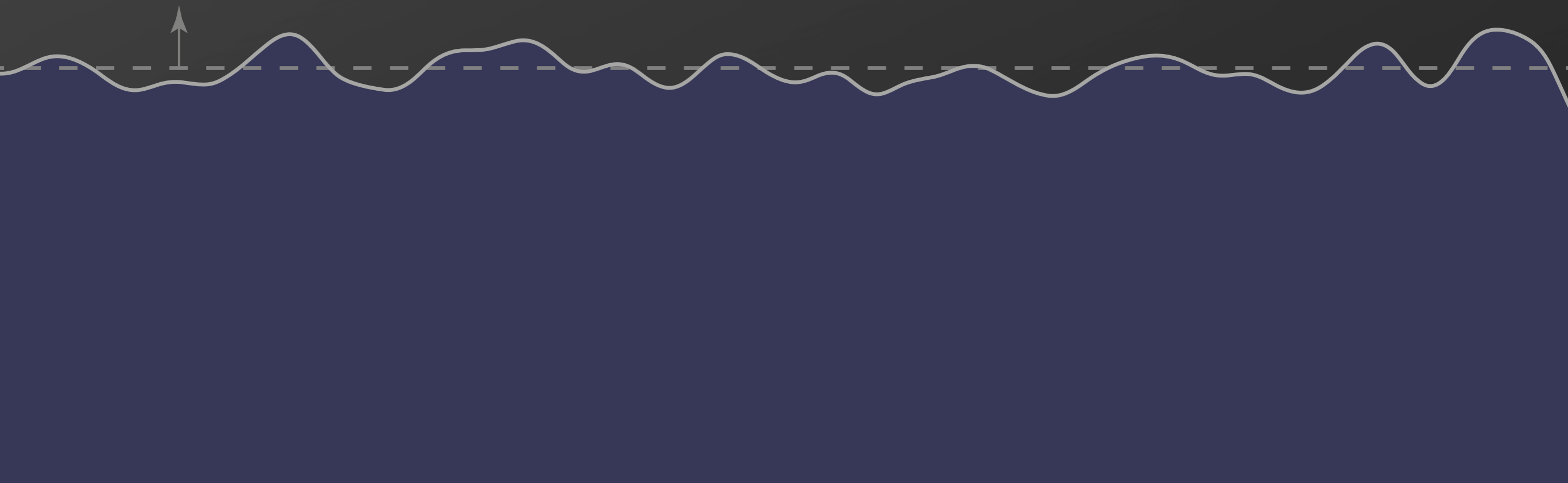
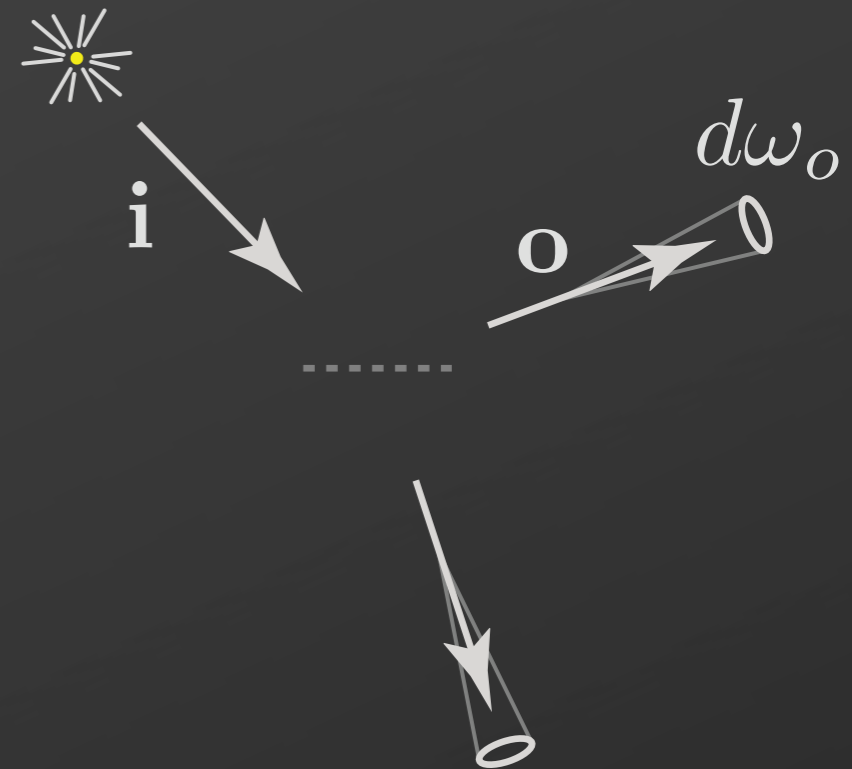
Incident irradiance E_i illuminates macrosurface area dA from direction \mathbf{i} .



Microfacet scattering models

Incident irradiance E_i illuminates macrosurface area dA from direction \mathbf{i} .

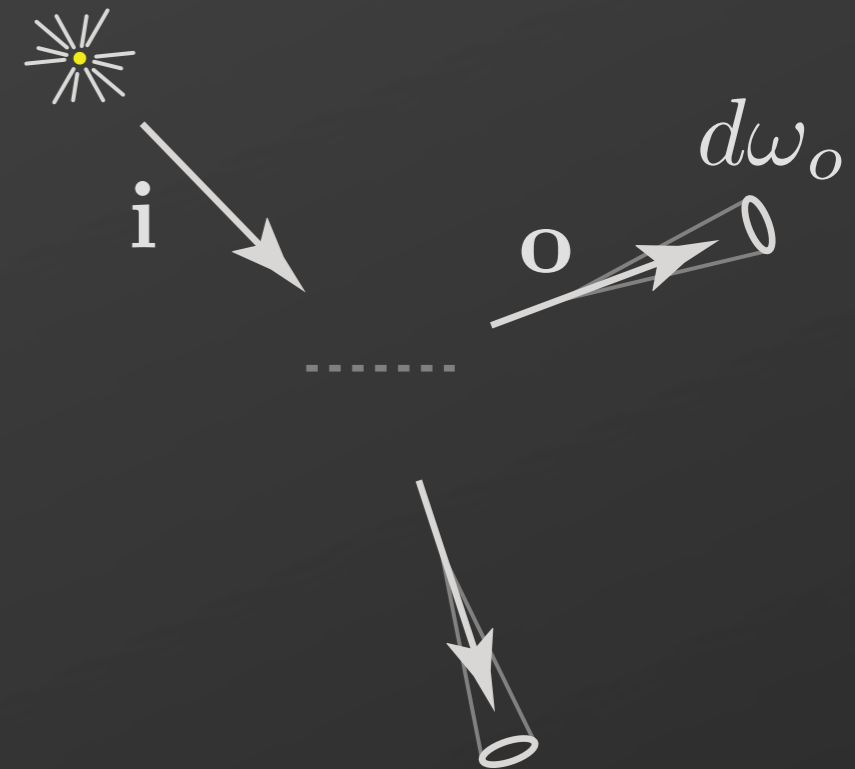
Scattered radiance L_r or L_t measured in direction \mathbf{o} in solid angle $d\omega_o$.



Microfacet scattering models

Incident irradiance E_i illuminates macrosurface area dA from direction \mathbf{i} .

Scattered radiance L_r or L_t measured in direction \mathbf{o} in solid angle $d\omega_o$.



$$f_s(\mathbf{i}, \mathbf{o}) = \frac{L_{r,t}}{E_i}$$

Bidirectional Scattering Distribution Function

“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

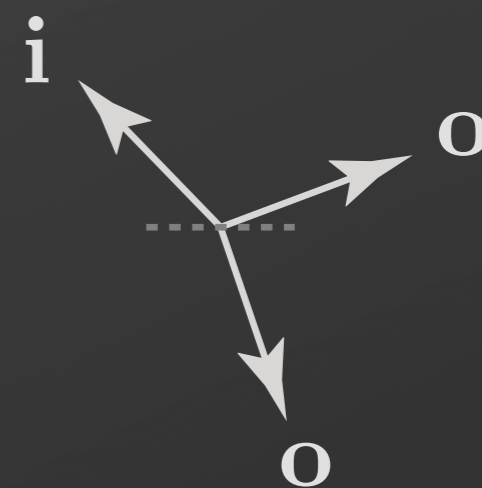
$$D(\mathbf{m})$$

shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$



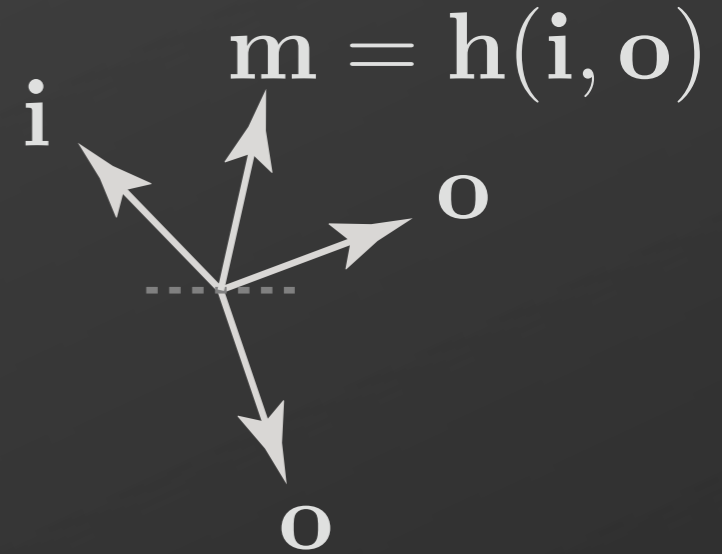
“half-vector” function
 $\mathbf{h}(\mathbf{i}, \mathbf{o})$

normal distribution
 $D(\mathbf{m})$

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attenuation
 $\rho(\mathbf{i}, \mathbf{o})$

Gives the one microsurface normal \mathbf{m} that will scatter light from \mathbf{i} to \mathbf{o} .



“half-vector” function
 $\mathbf{h}(\mathbf{i}, \mathbf{o})$

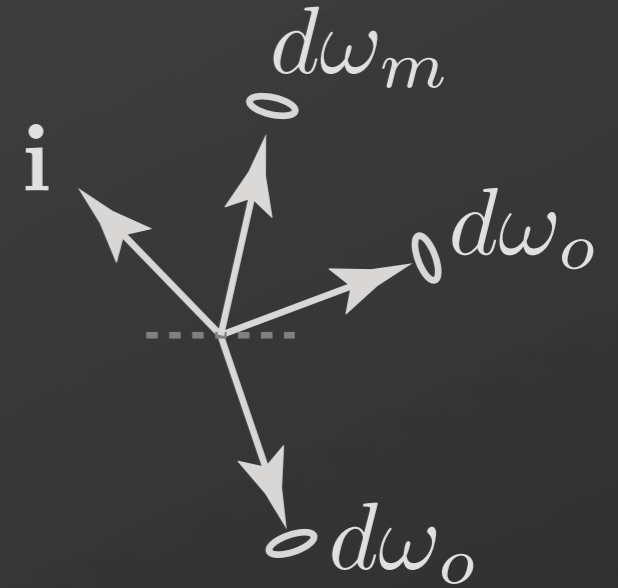
normal distribution
 $D(\mathbf{m})$

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attenuation
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Gives the one microsurface normal \mathbf{m} that will scatter light from \mathbf{i} to \mathbf{o} .

The size of the set of relevant normals $d\omega_m$ relative to the receiving solid angle $d\omega_o$ is determined by \mathbf{h} .



“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

$$D(\mathbf{m})$$

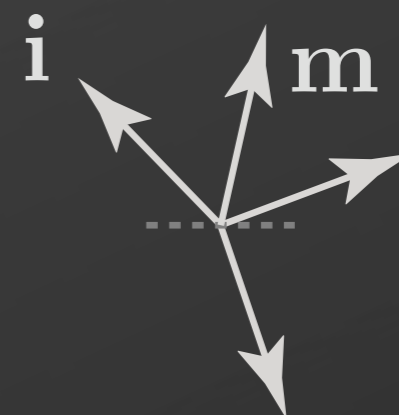
shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

Measures density of microsurface area with respect to microsurface normal.



A diagram showing a wavy surface profile. A horizontal double-headed arrow below the surface is labeled dA , representing a differential area element on the surface.
$$dA$$

“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

$$D(\mathbf{m})$$

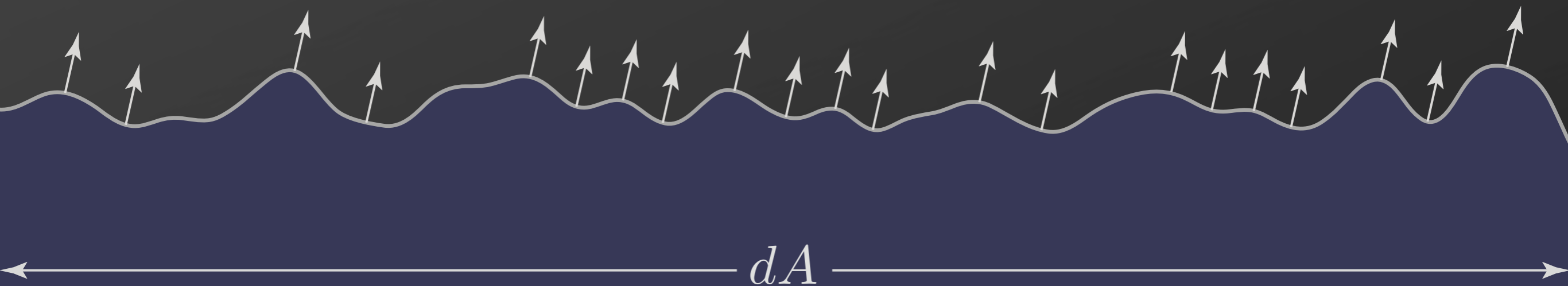
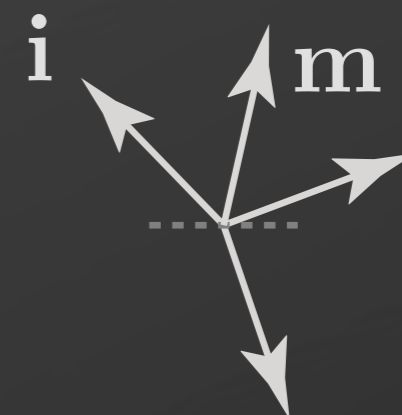
shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

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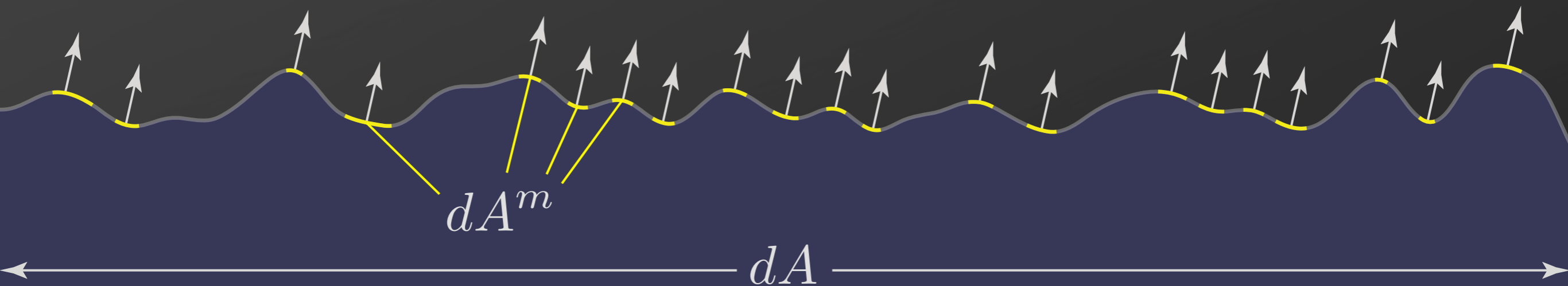
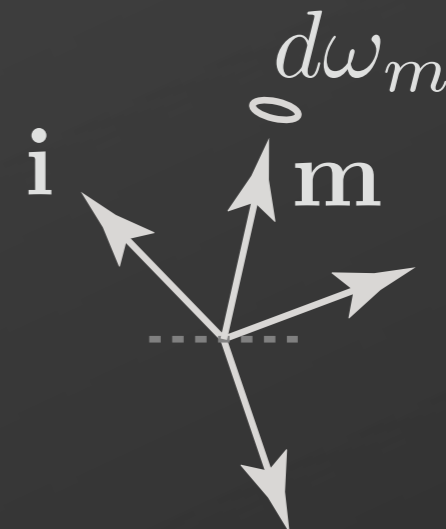
normal distribution
 $D(\mathbf{m})$

shadowing–masking
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attenuation
 $\rho(\mathbf{i}, \mathbf{o})$

Measures density of microsurface area with respect to microsurface normal.

The ratio of relevant microsurface area dA^m to macrosurface area dA is $D(\mathbf{m})d\omega_m$.



$$dA^m = D(\mathbf{m}) d\omega_m dA$$

“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

$$D(\mathbf{m})$$

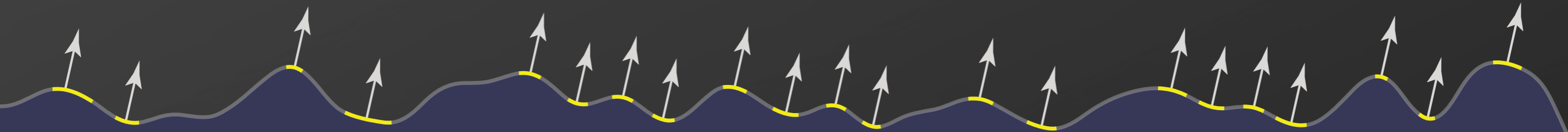
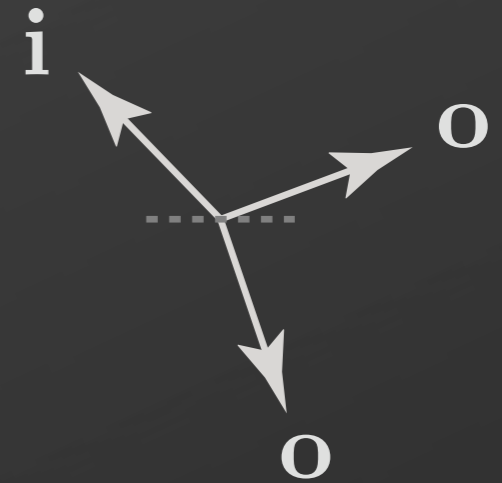
shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

Measures the fraction of points with microsurface normal \mathbf{m} that are visible in directions \mathbf{i} and \mathbf{o} .



$$dA^m = D(\mathbf{m}) d\omega_m dA$$

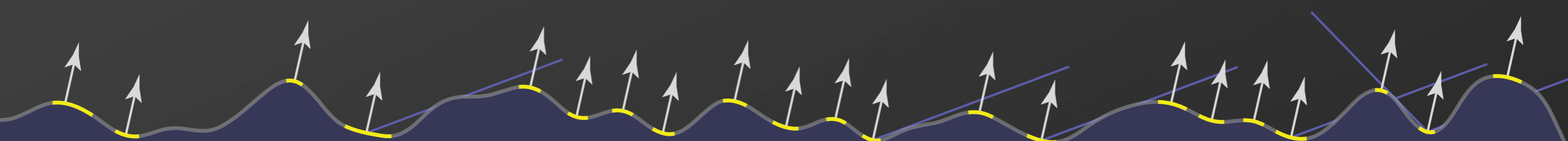
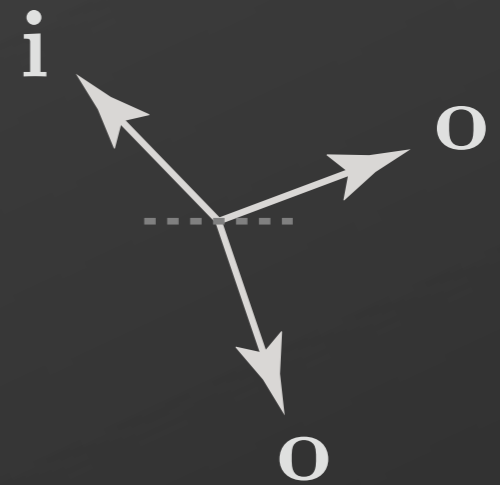
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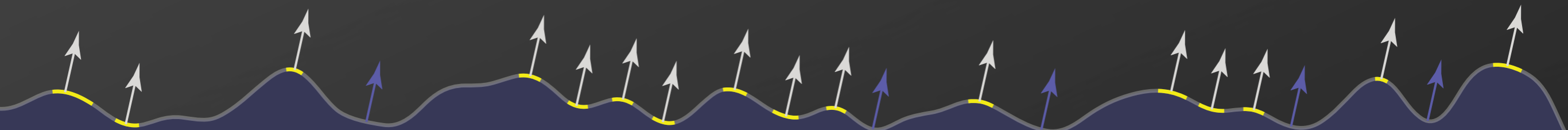
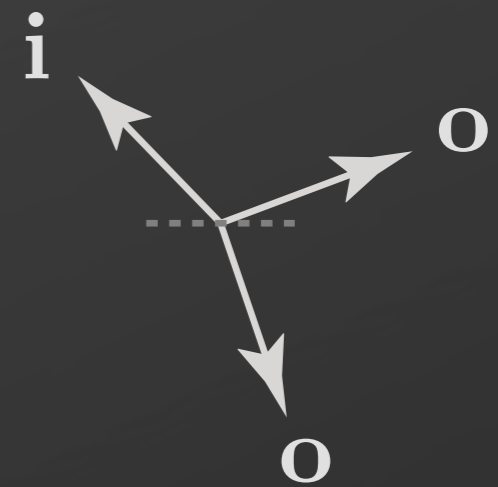
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$$dA^m = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m dA$$

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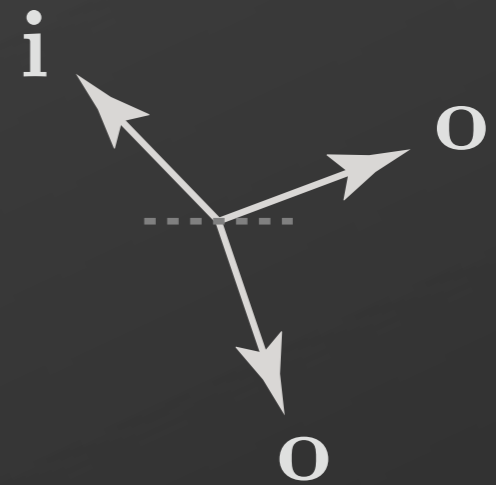
$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

Measures the fraction of points with microsurface normal \mathbf{m} that are visible in directions \mathbf{i} and \mathbf{o} .

We now know the size of the **scattering area**, which determines how much light reflects.



$$dA^m = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m dA$$

“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

$$D(\mathbf{m})$$

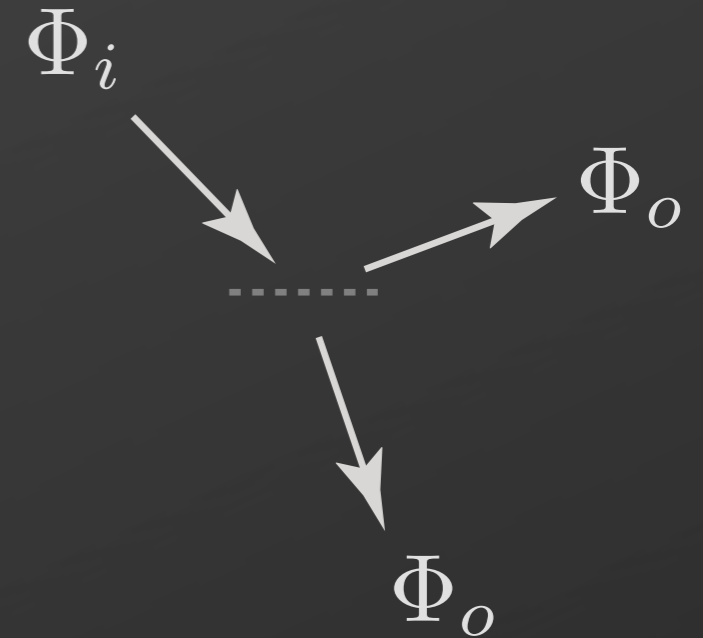
shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

Gives the fraction of the power incident on the **scattering area** dA^m that is scattered.



$$d\Phi_o^m = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) dA^m dE_i$$

$$dA^m = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m dA$$

“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

$$D(\mathbf{m})$$

shadowing–masking

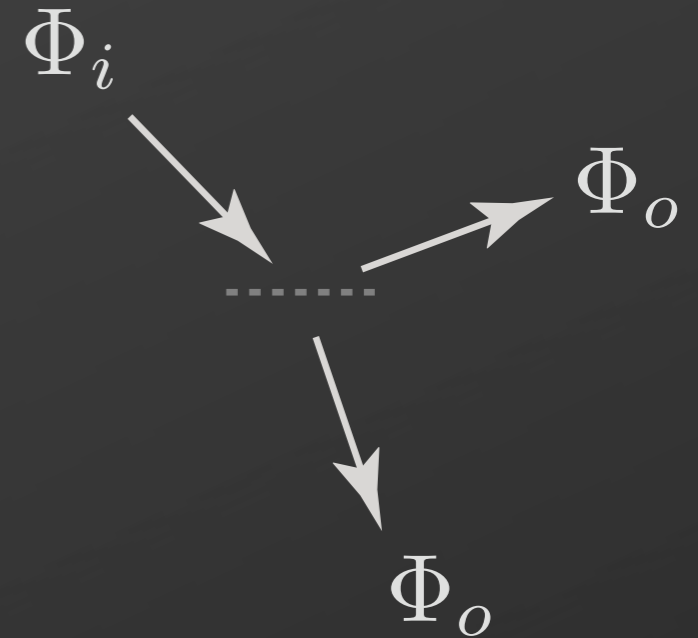
$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

Gives the fraction of the power incident on the **scattering area** dA^m that is scattered.

This scattered power is related to the incident irradiance by the attenuation and the **scattering area**, projected in the incident direction.



$$d\Phi_o^m = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) dA^m dE_i$$

$$dA^m = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m dA$$

“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

$$D(\mathbf{m})$$

shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

The BSDF is the ratio of scattered radiance to incident irradiance:

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{dL_o}{dE_i} = \frac{d\Phi_o^m / (dA |\mathbf{o} \cdot \mathbf{n}| d\omega_o)}{dE_i}$$

$$d\Phi_o^m = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) dA^m dE_i$$

$$dA^m = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m dA$$

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attenuation

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The BSDF is the ratio of scattered radiance to incident irradiance:

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$

$$d\Phi_o^m = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) dA^m dE_i$$

$$dA^m = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m dA$$

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$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

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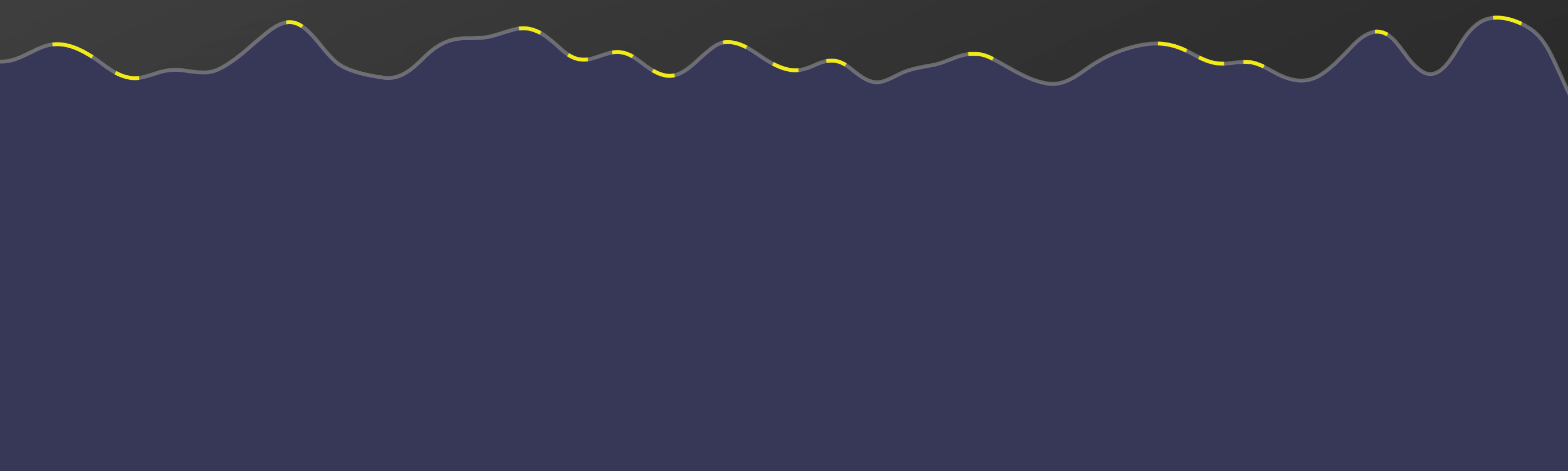
shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$



“half-vector” function

$\mathbf{h}(\mathbf{i}, \mathbf{o})$

normal distribution

$D(\mathbf{m})$

shadowing–masking

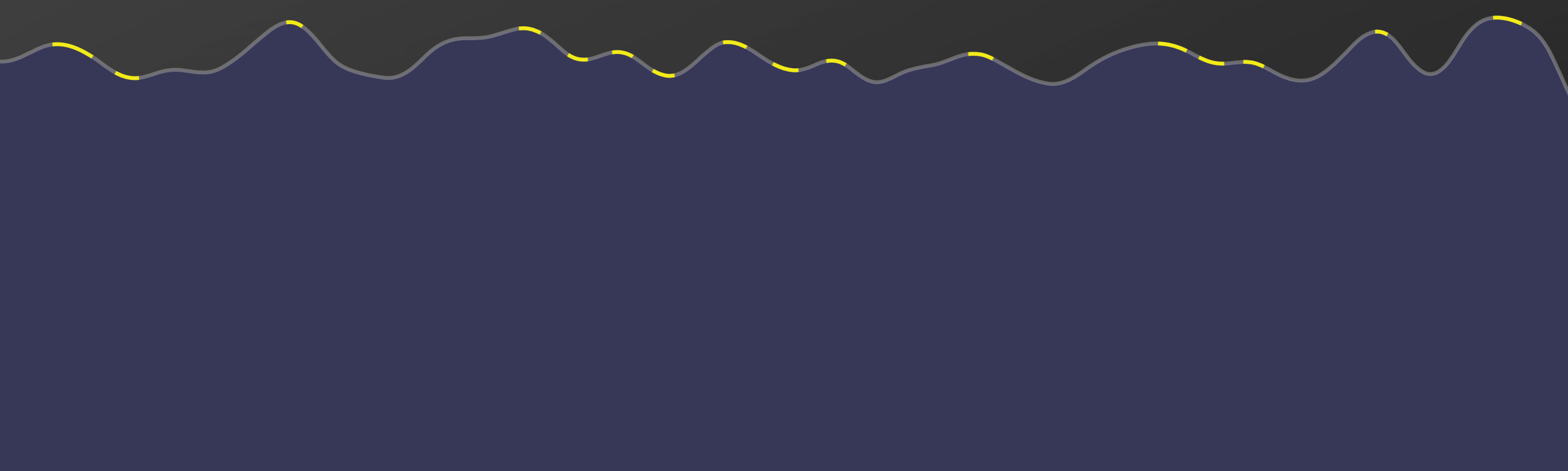
$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$

attenuation

$\rho(\mathbf{i}, \mathbf{o})$

Fresnel reflection

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$



“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

$$D(\mathbf{m})$$

shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

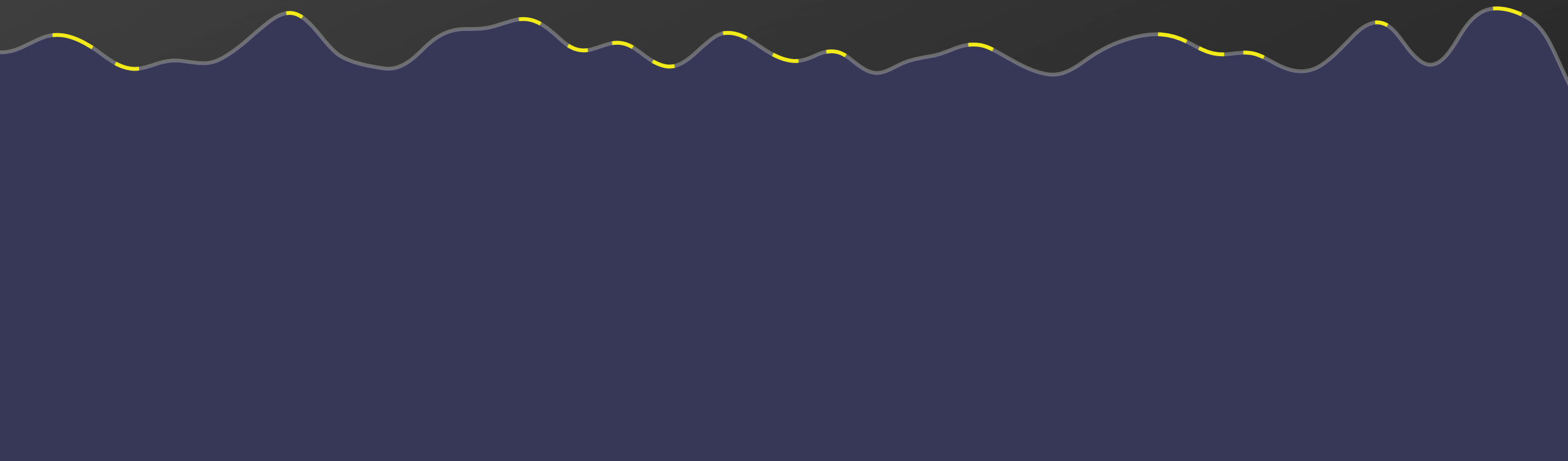
attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

Fresnel reflection

surface roughness

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$



“half-vector” function

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normal distribution

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shadowing–masking

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attenuation

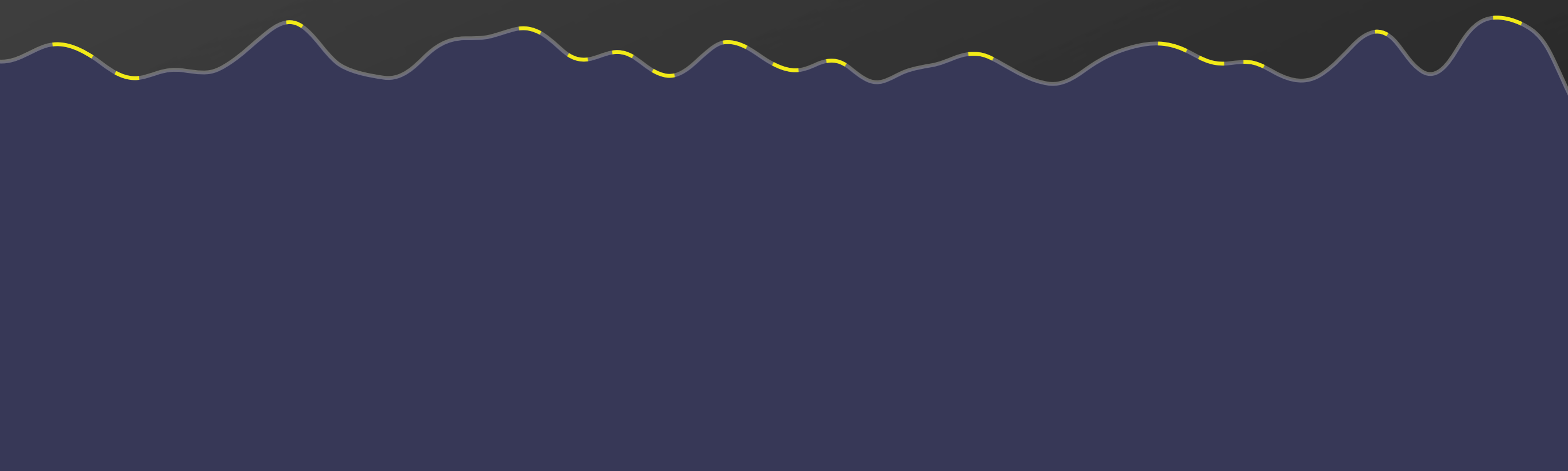
$$\rho(\mathbf{i}, \mathbf{o})$$

Fresnel reflection

surface roughness

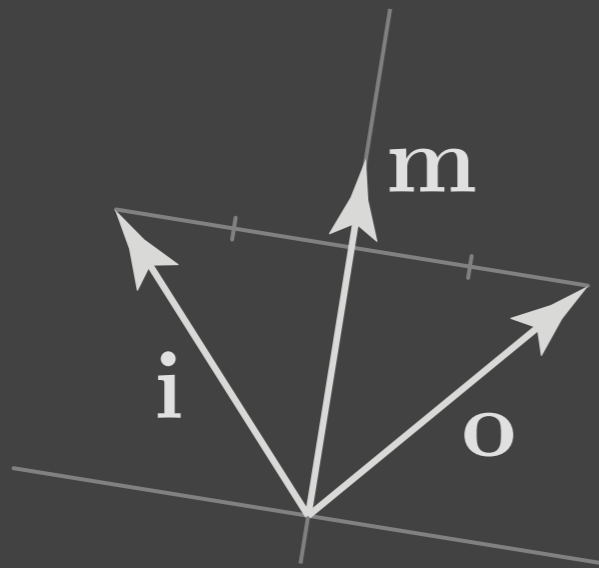
$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$

determined by geometry



Construction of half-vector

reflection



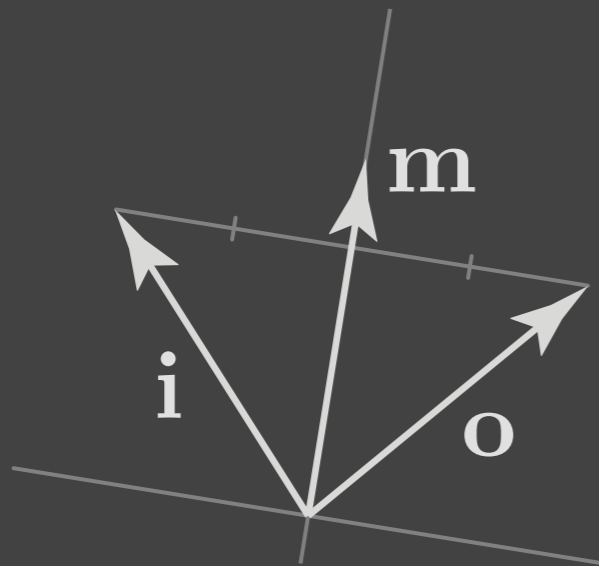
$\mathbf{i} + \mathbf{o}$ parallel to \mathbf{m}

refraction

Construction of half-vector

reflection

$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



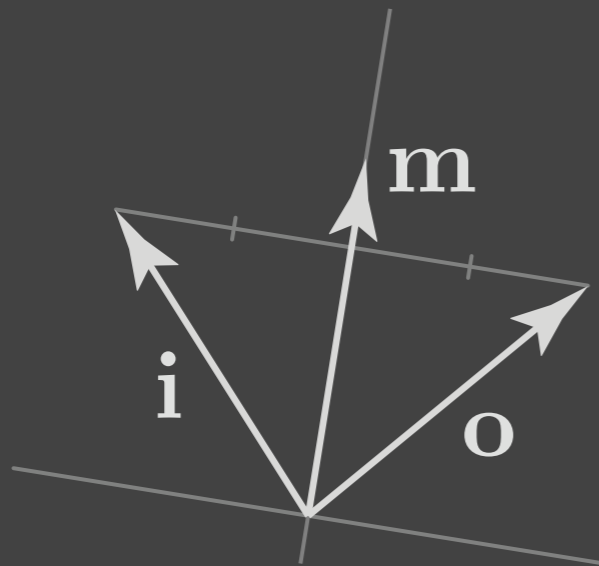
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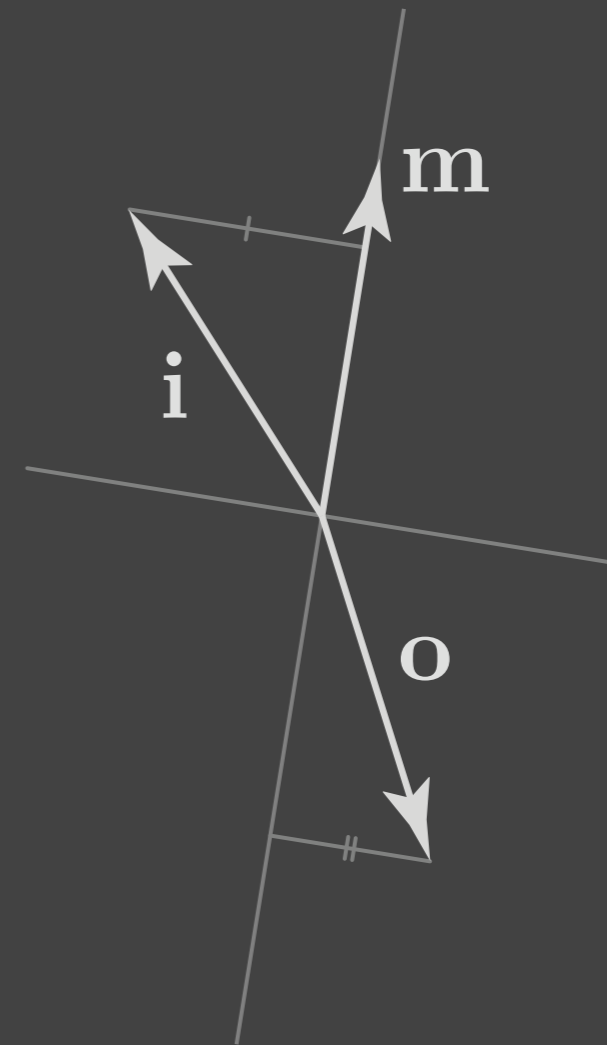
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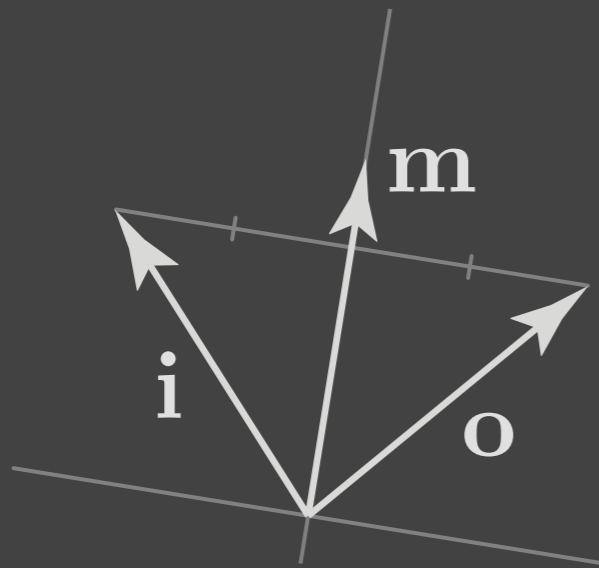
refraction



Construction of half-vector

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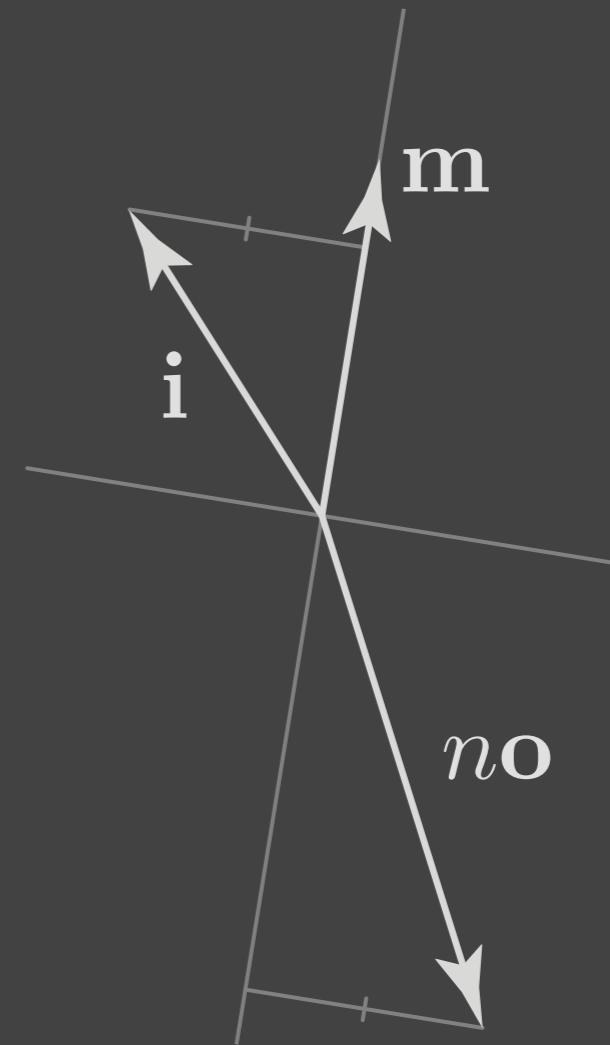
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$\mathbf{i} + \mathbf{o}$ parallel to \mathbf{m}

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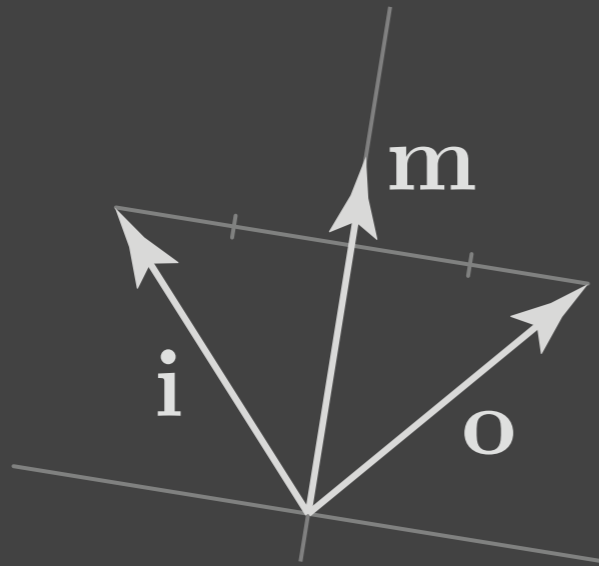
$\mathbf{i} + n\mathbf{o}$ parallel to \mathbf{m}



Construction of half-vector

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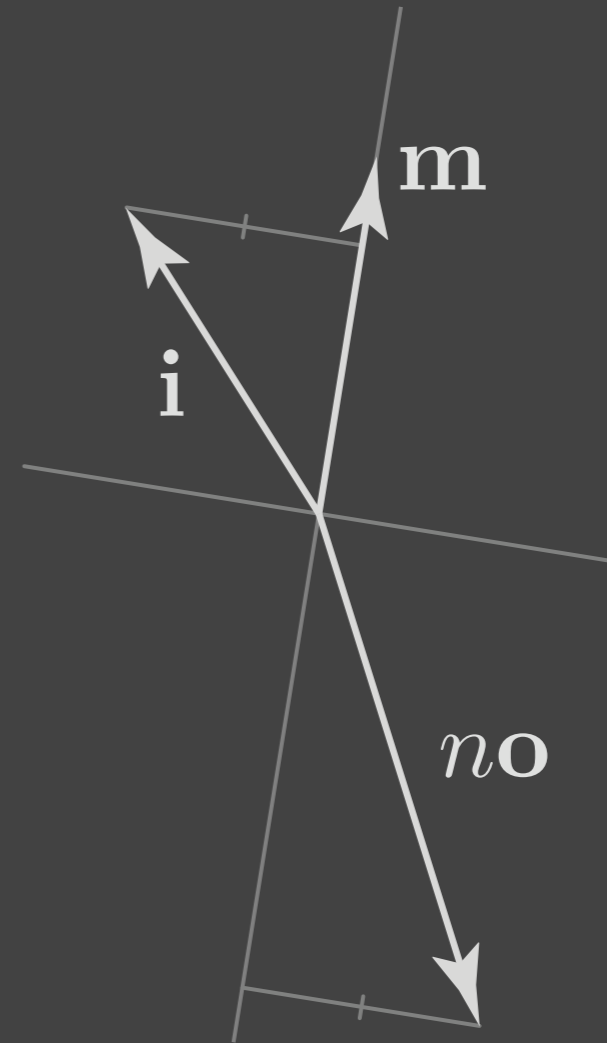
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$\mathbf{i} + \mathbf{o}$ parallel to \mathbf{m}

refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



$\mathbf{i} + n\mathbf{o}$ parallel to \mathbf{m}

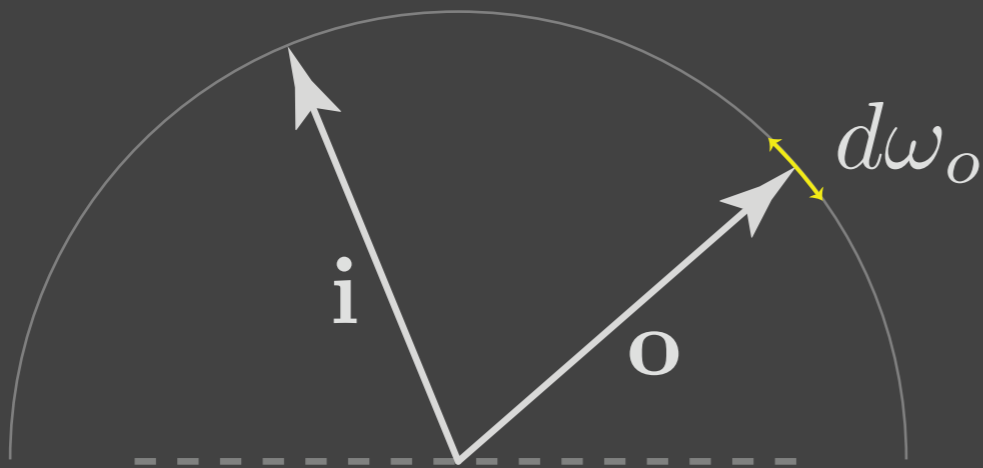
Construction of half-vector solid angle

reflection

$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$

refraction

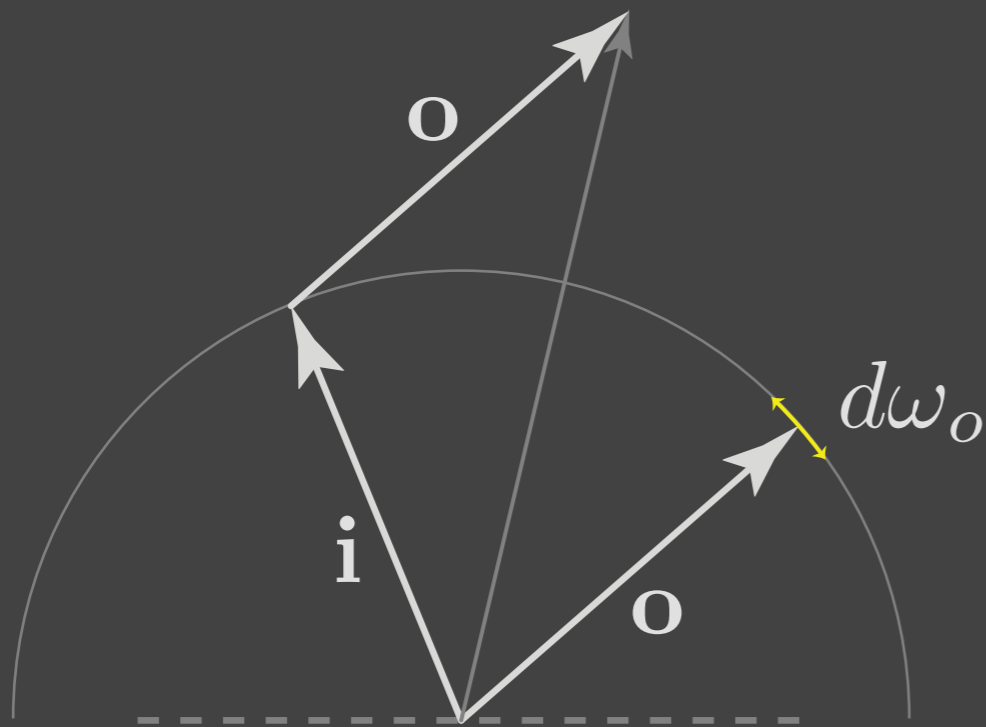
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Construction of half-vector solid angle

reflection

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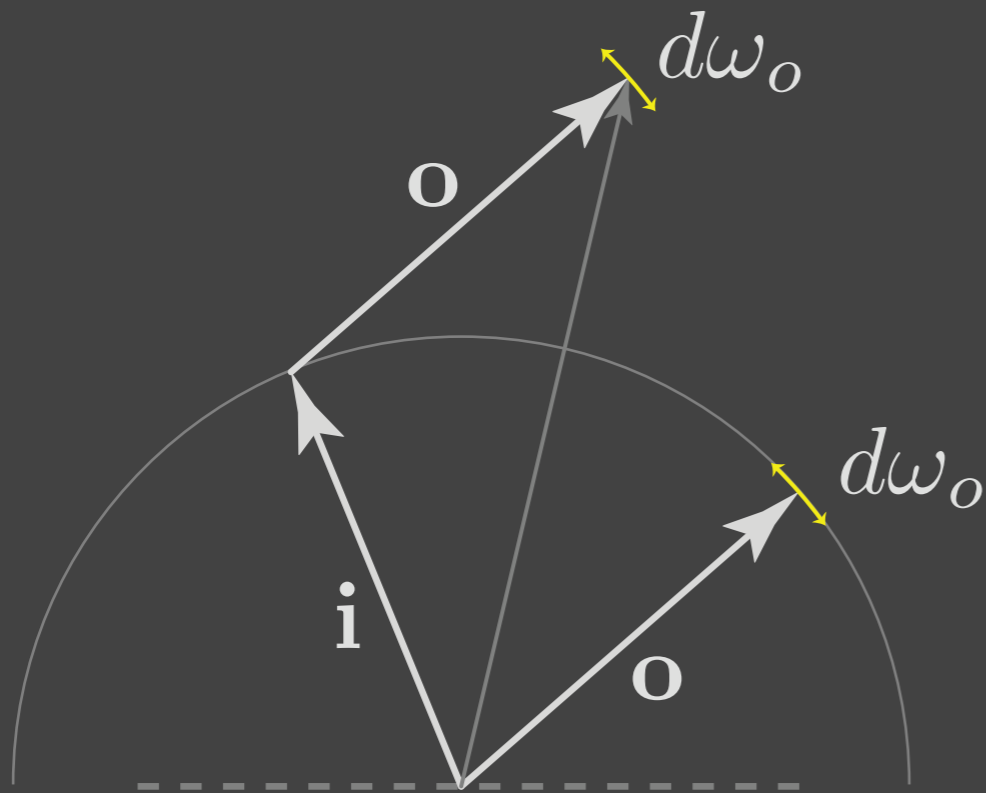
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Construction of half-vector solid angle

reflection

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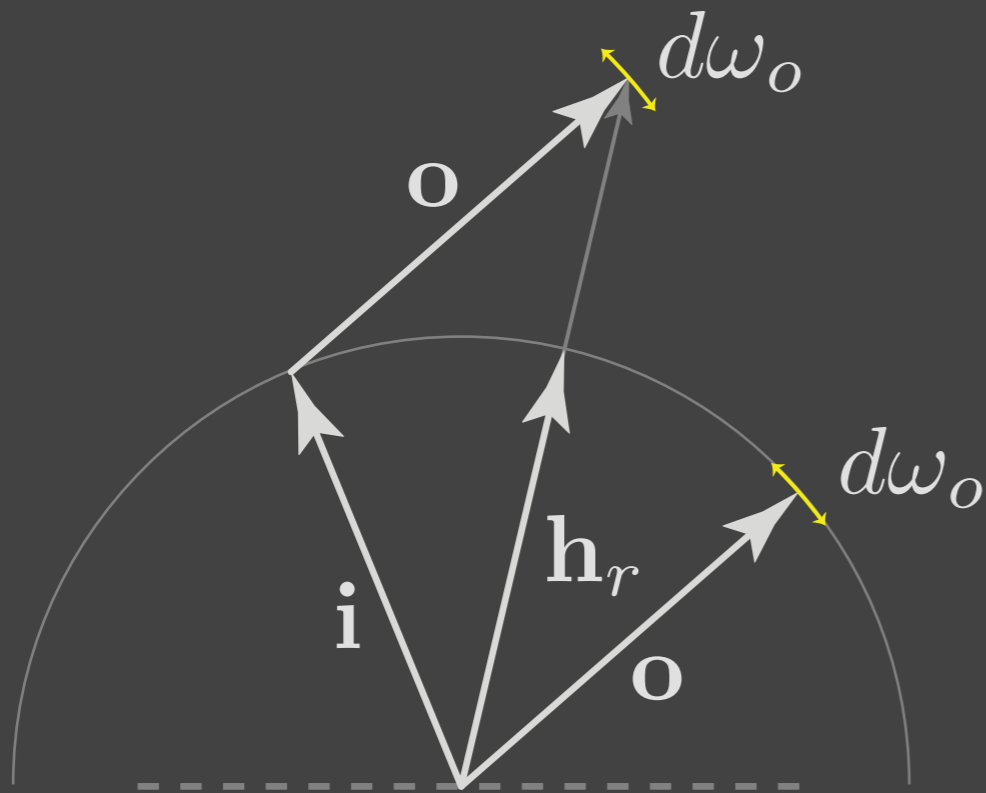
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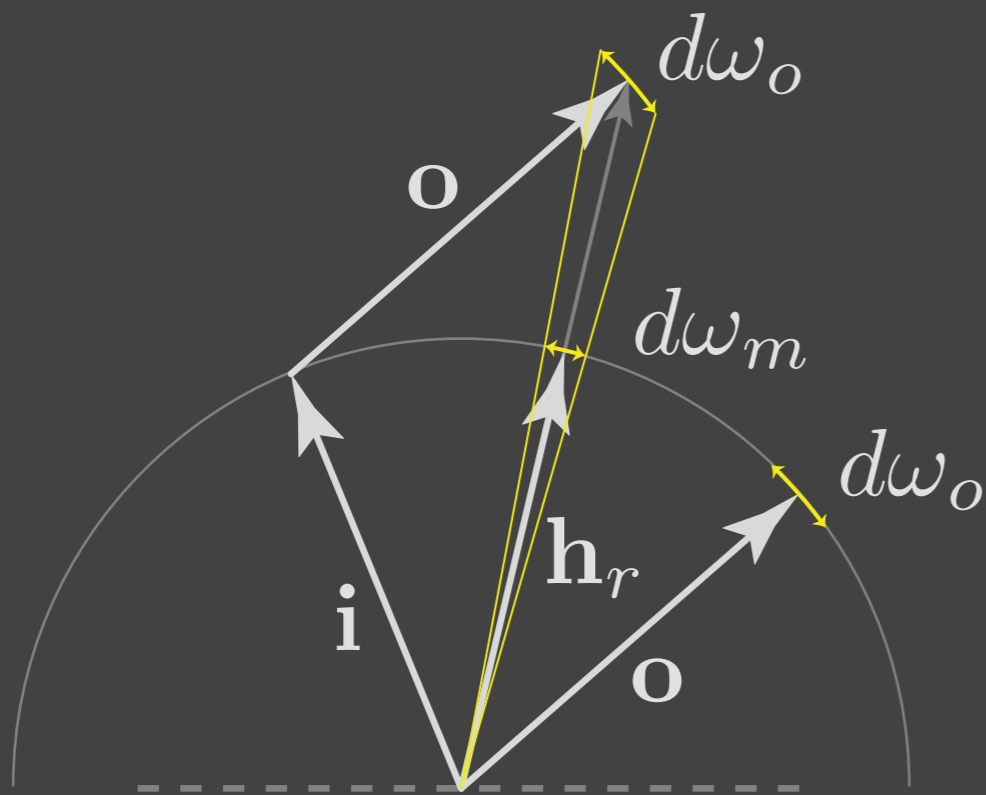
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Construction of half-vector solid angle

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$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



refraction

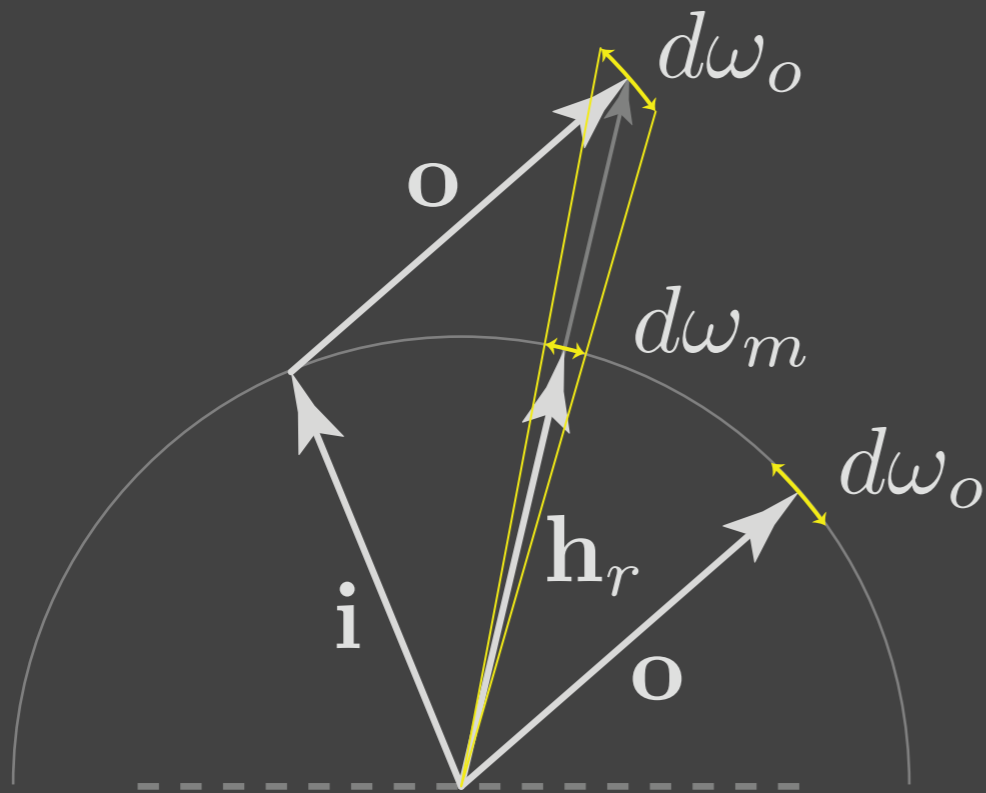
$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$

$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} d\omega_o$$

Construction of half-vector solid angle

reflection

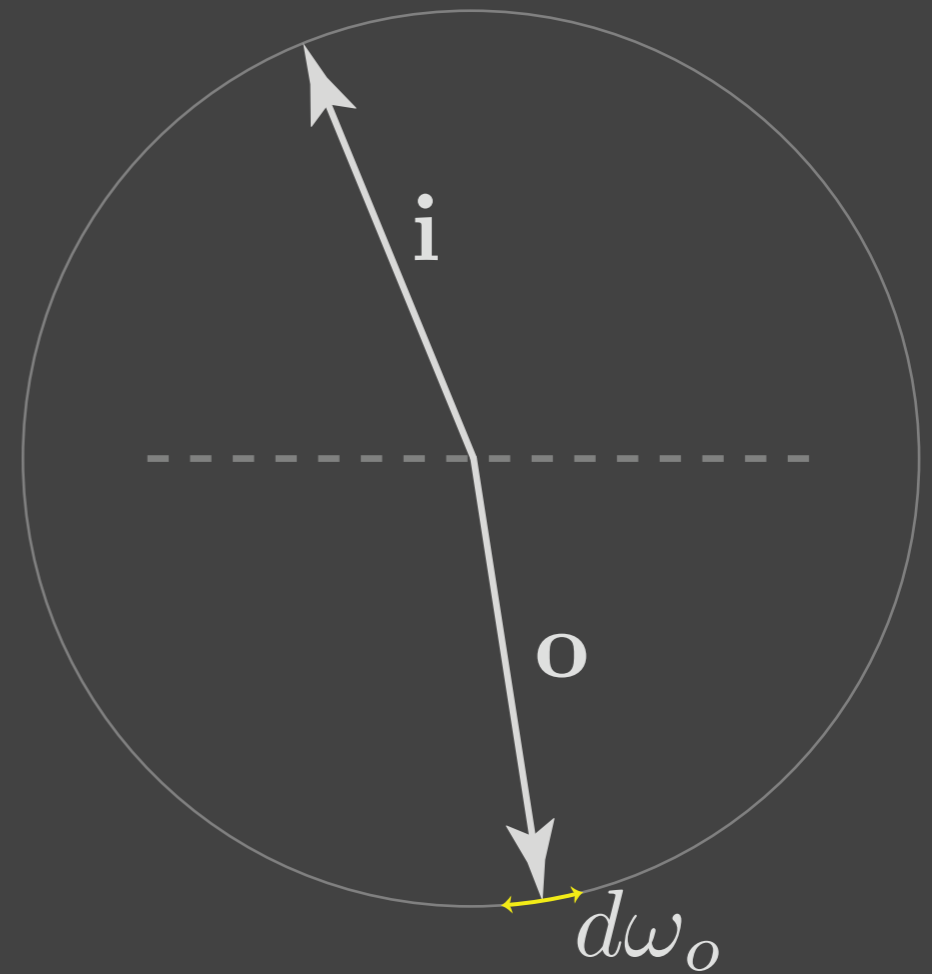
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refraction

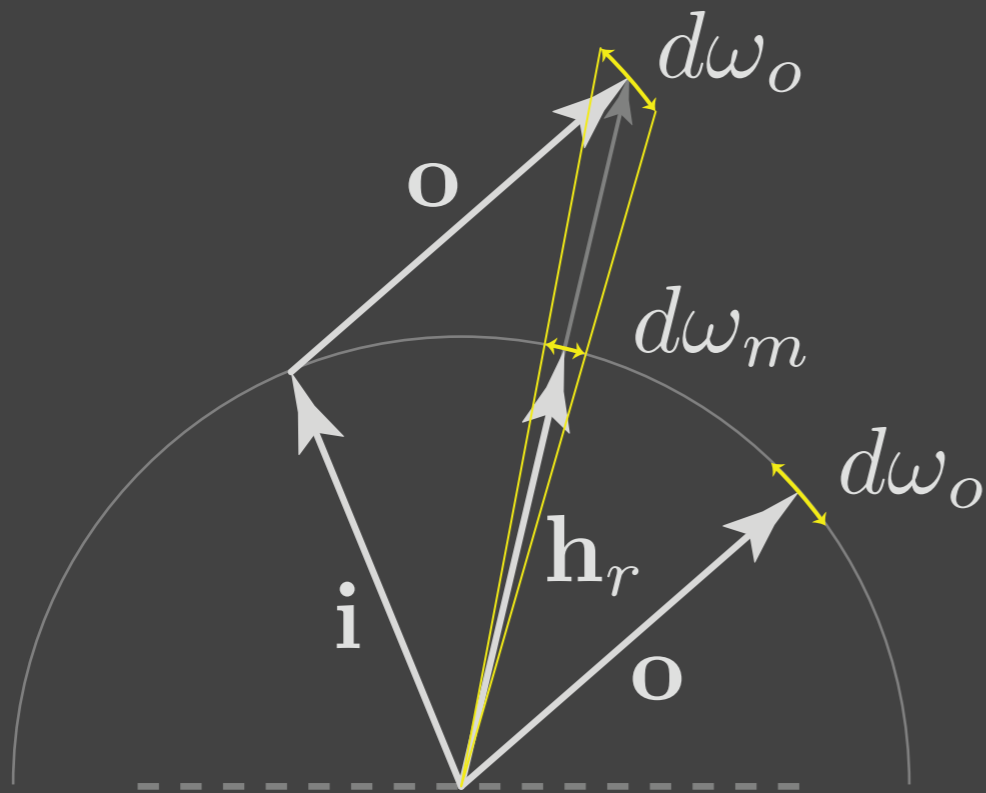
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reflection

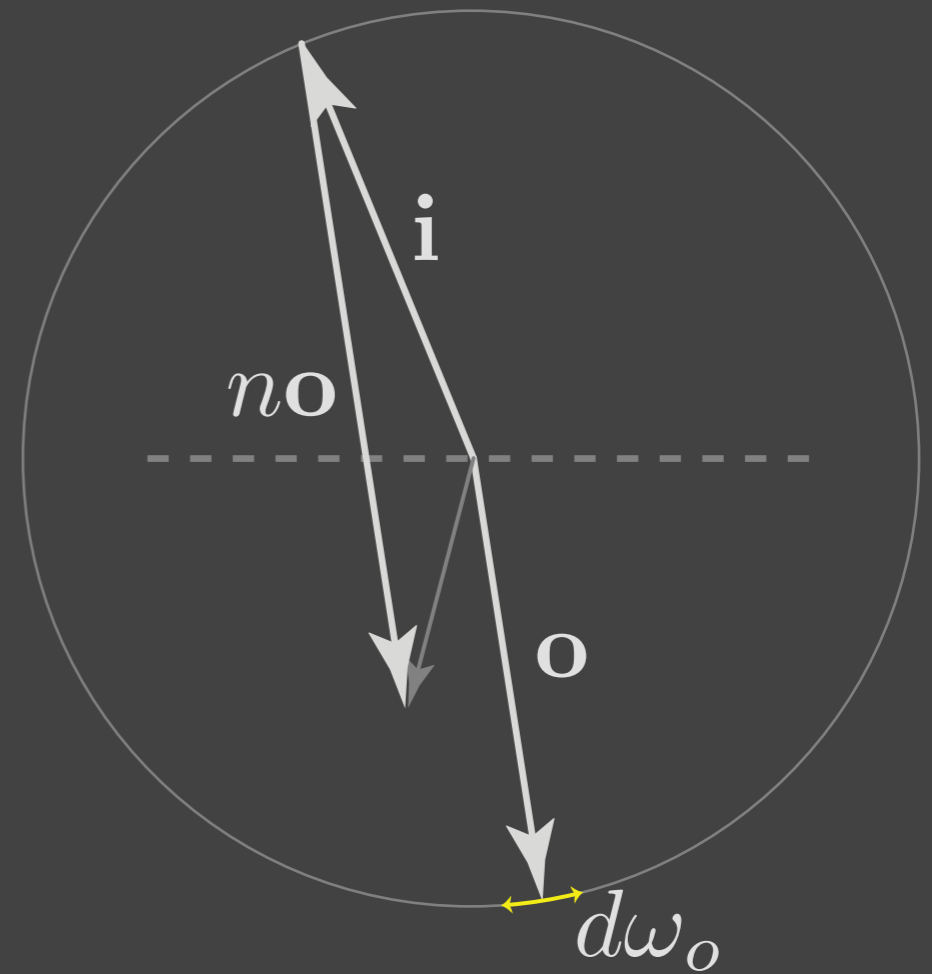
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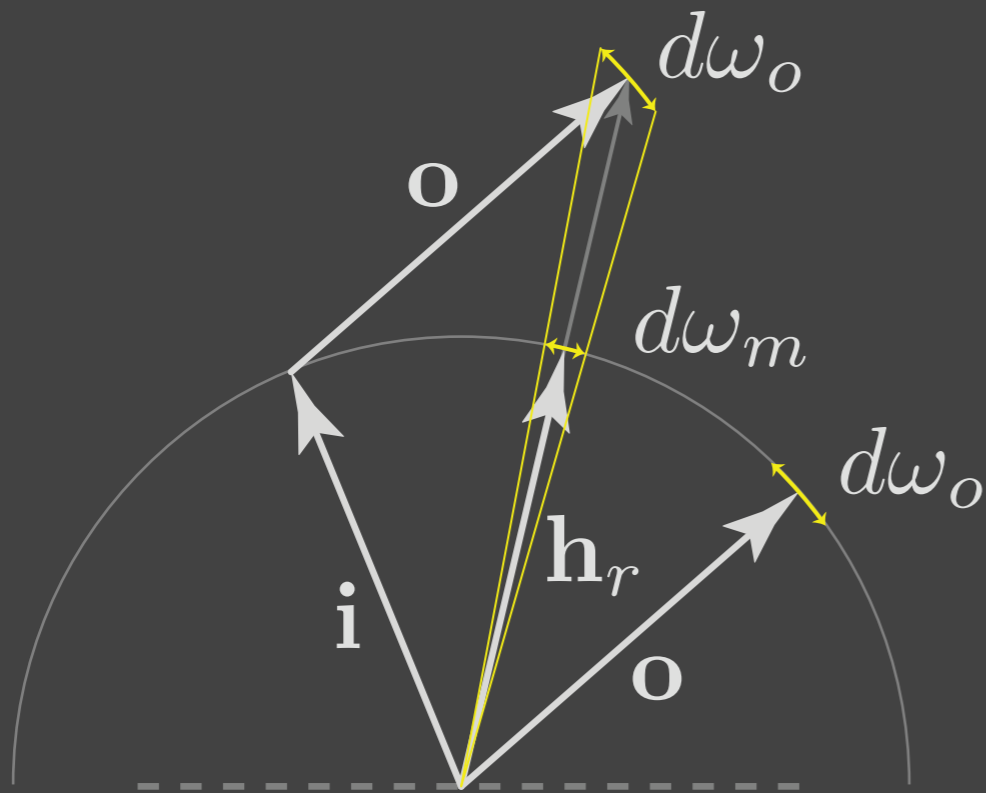
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Construction of half-vector solid angle

reflection

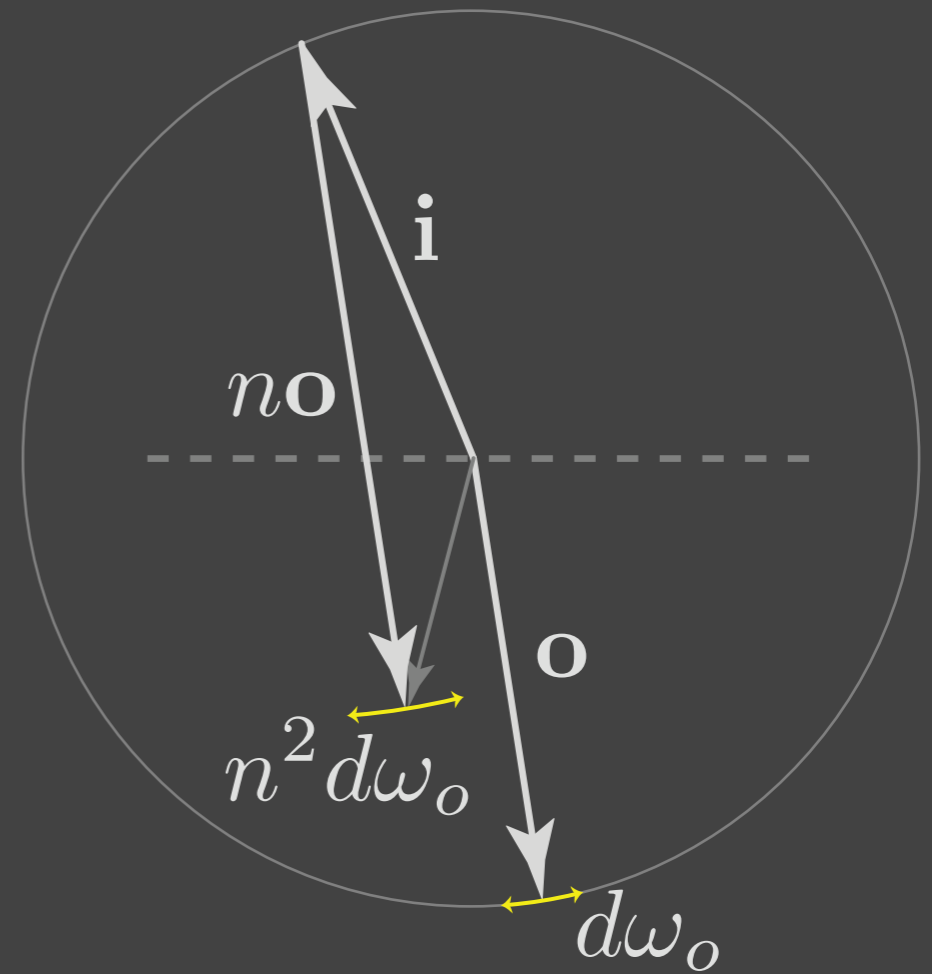
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} d\omega_o$$

refraction

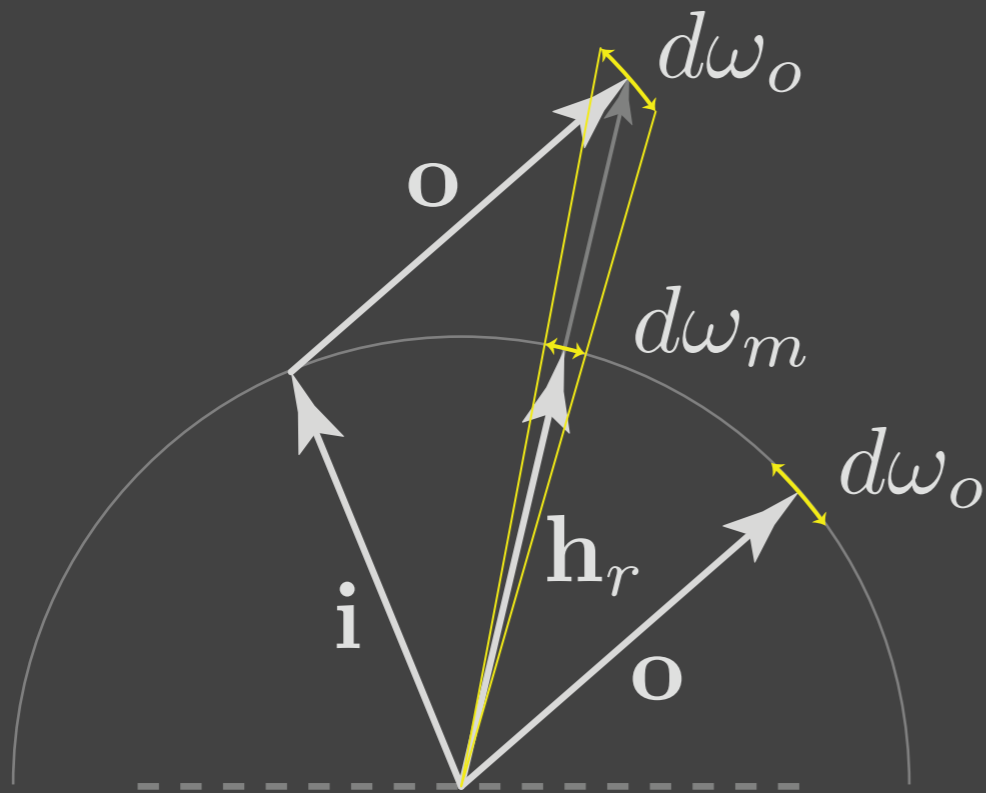
$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



Construction of half-vector solid angle

reflection

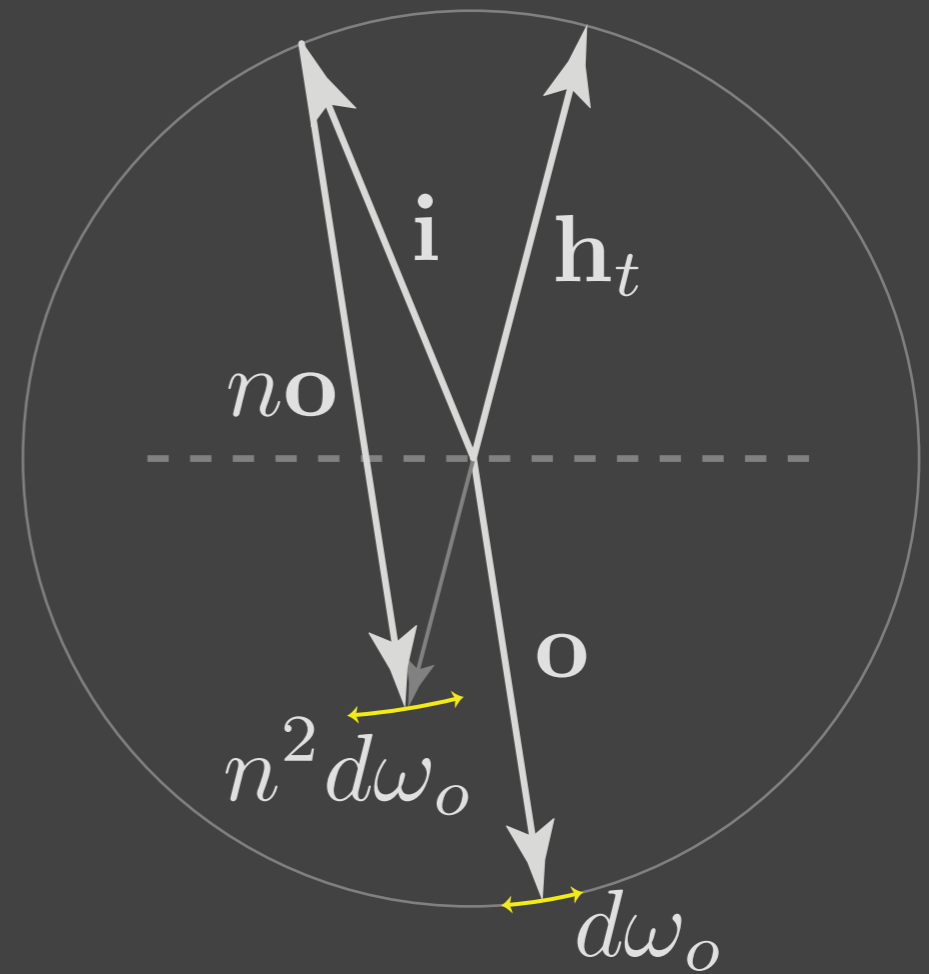
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



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refraction

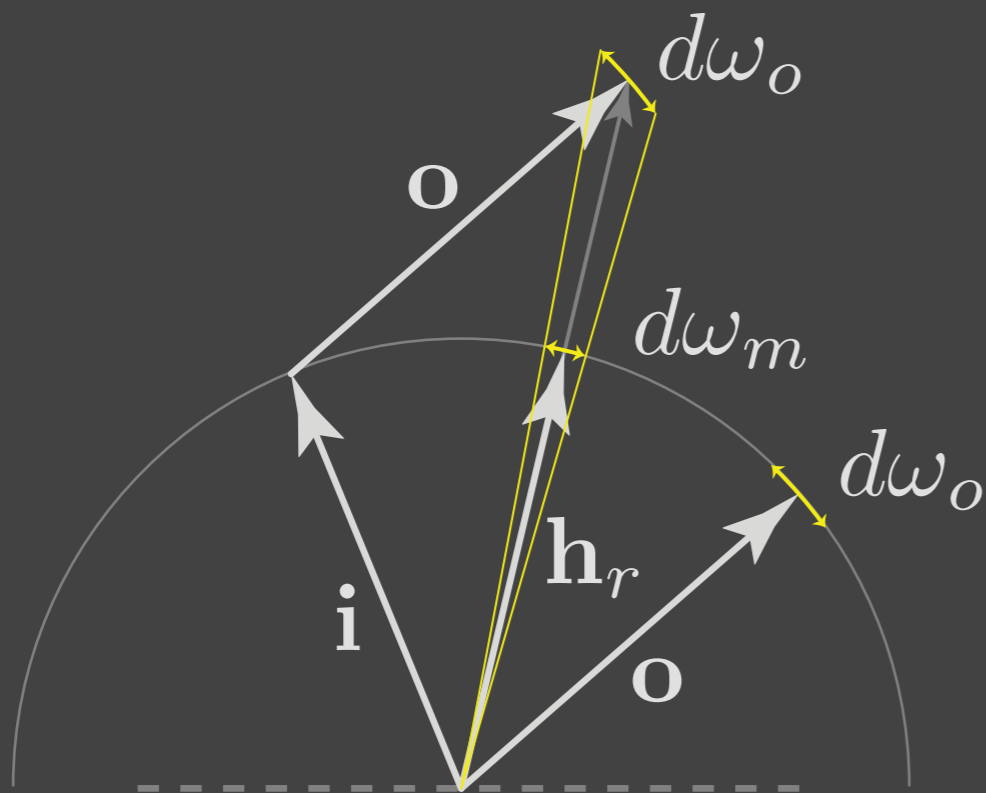
$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



Construction of half-vector solid angle

reflection

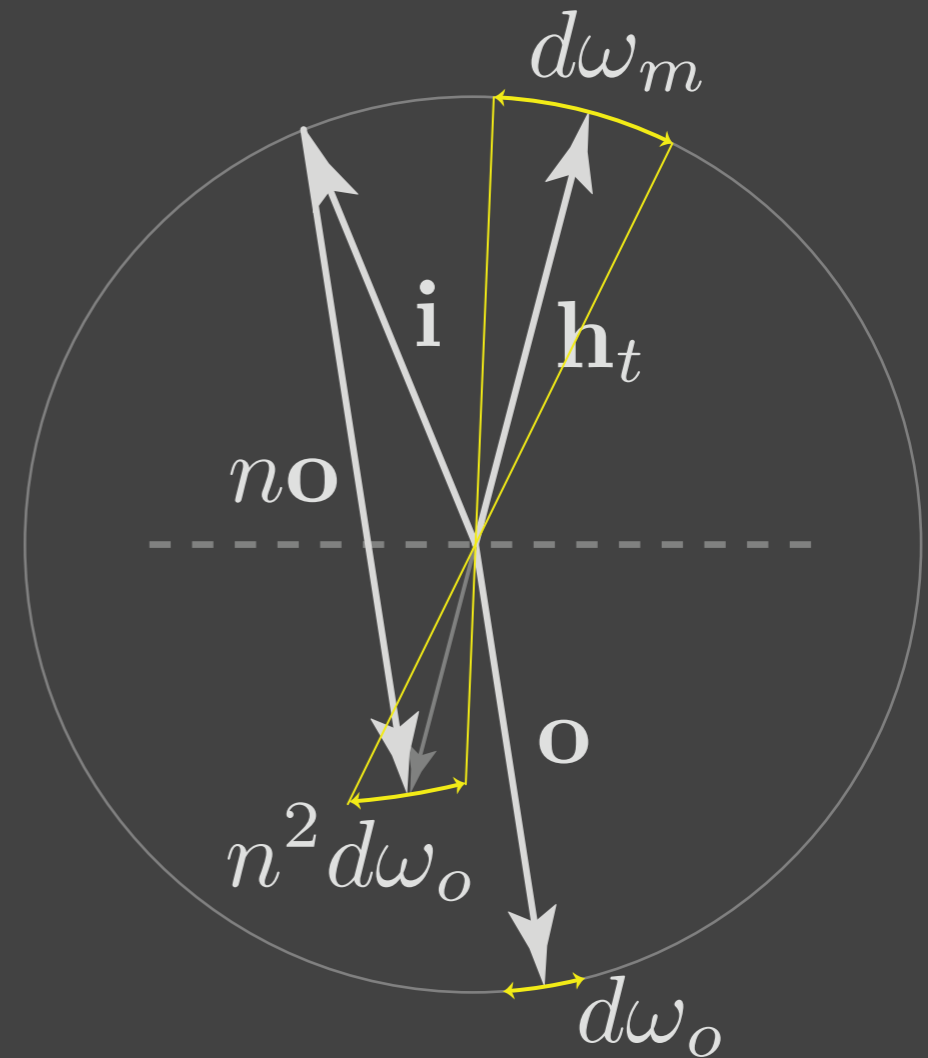
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} d\omega_o$$

refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_t|}{\|\mathbf{i} + n\mathbf{o}\|^2} n^2 d\omega_o$$

Result: scattering functions

reflection

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$

transmission

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} F(\mathbf{i}, \mathbf{m}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{|\mathbf{o} \cdot \mathbf{m}|}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} F(\mathbf{i}, \mathbf{m}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{|\mathbf{o} \cdot \mathbf{m}|}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} (1 - F(\mathbf{i}, \mathbf{m})) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{n^2 |\mathbf{o} \cdot \mathbf{m}|}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}| |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \frac{F(\mathbf{i}, \mathbf{m}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} (1 - F(\mathbf{i}, \mathbf{m})) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{n^2 |\mathbf{o} \cdot \mathbf{m}|}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}| |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \frac{F(\mathbf{i}, \mathbf{m}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}| |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \frac{n^2 (1 - F(\mathbf{i}, \mathbf{m})) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{1}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \frac{F(\mathbf{i}, \mathbf{m}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{4}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}| |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \frac{n^2 (1 - F(\mathbf{i}, \mathbf{m})) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{F(\mathbf{i}, \mathbf{m}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{4|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}| |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \frac{n^2 (1 - F(\mathbf{i}, \mathbf{m})) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

Fresnel reflectance

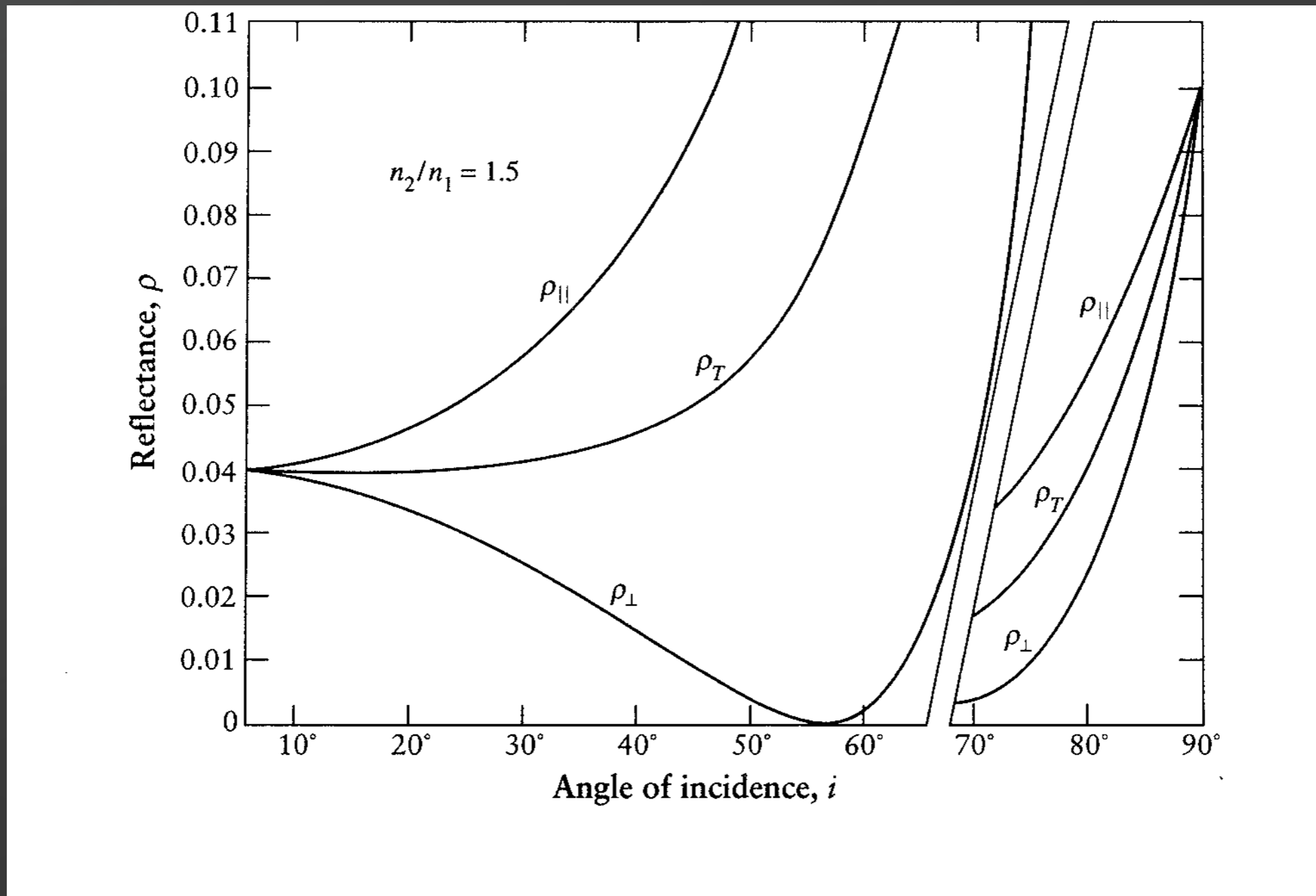


FIGURE 15.8

The Fresnel reflectance for an air-glass boundary with index of refraction 1.5. We show the two polarized components and the term for unpolarized light. Redrawn from Judd and Wyszecki, *Color in Business, Science and Industry*, fig. 3.2, p. 400.

Fresnel reflectance

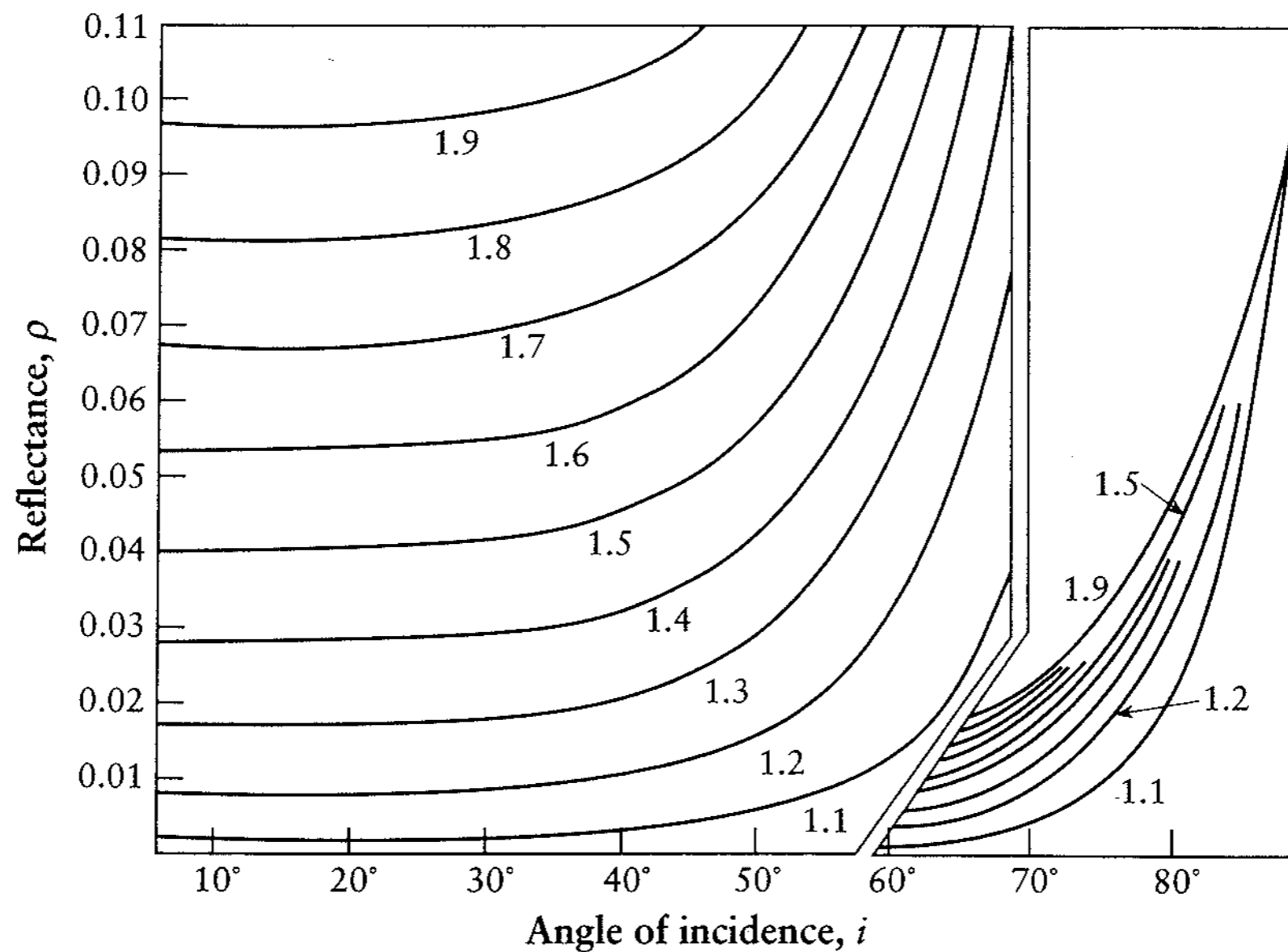


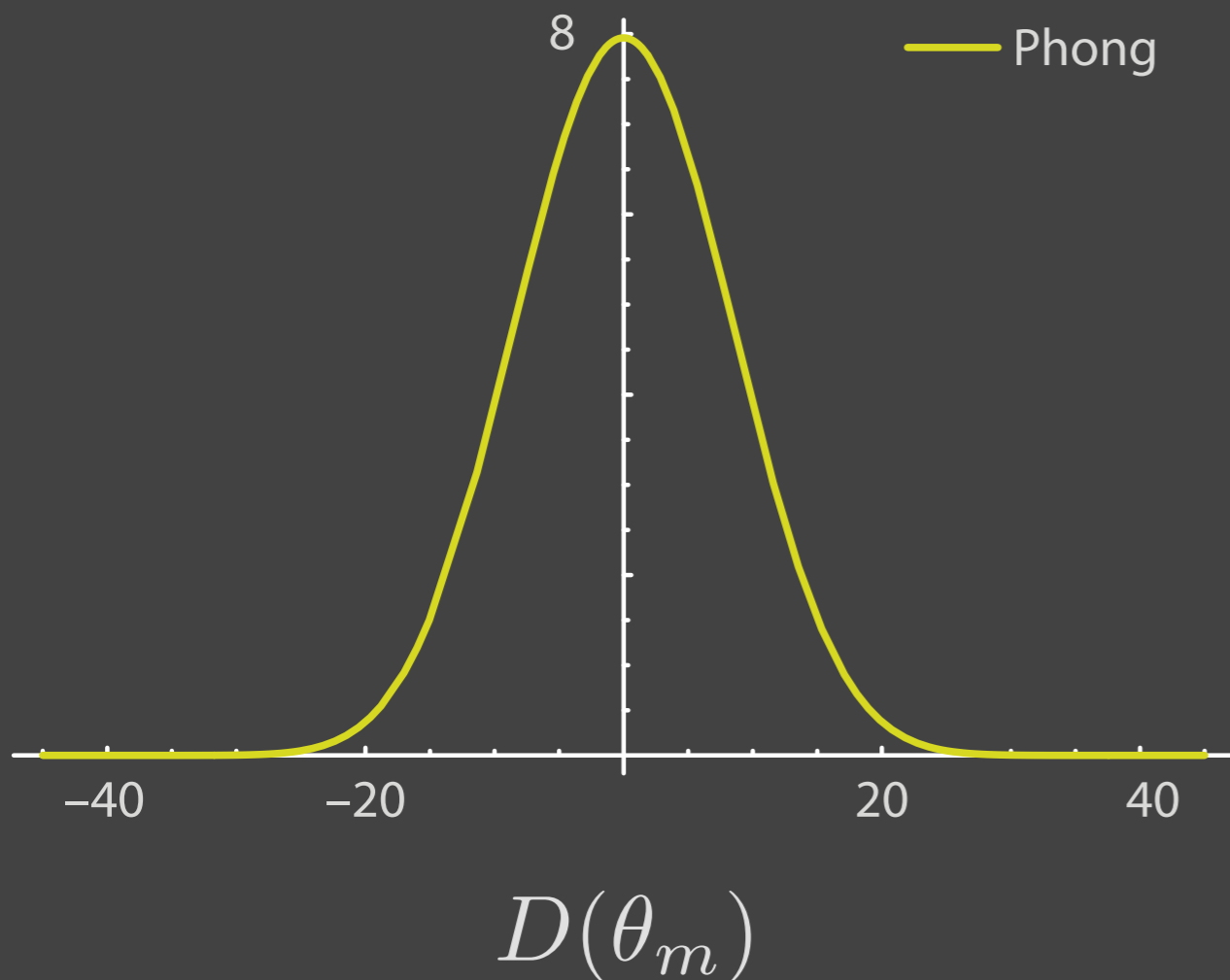
FIGURE 15.9

The Fresnel reflection for unpolarized light for different indices of refraction. Redrawn from Judd and Wyszecki, *Color in Business, Science and Industry*, fig. 3.3, p. 401.

Normal distributions

Choice of distribution is determined by surface

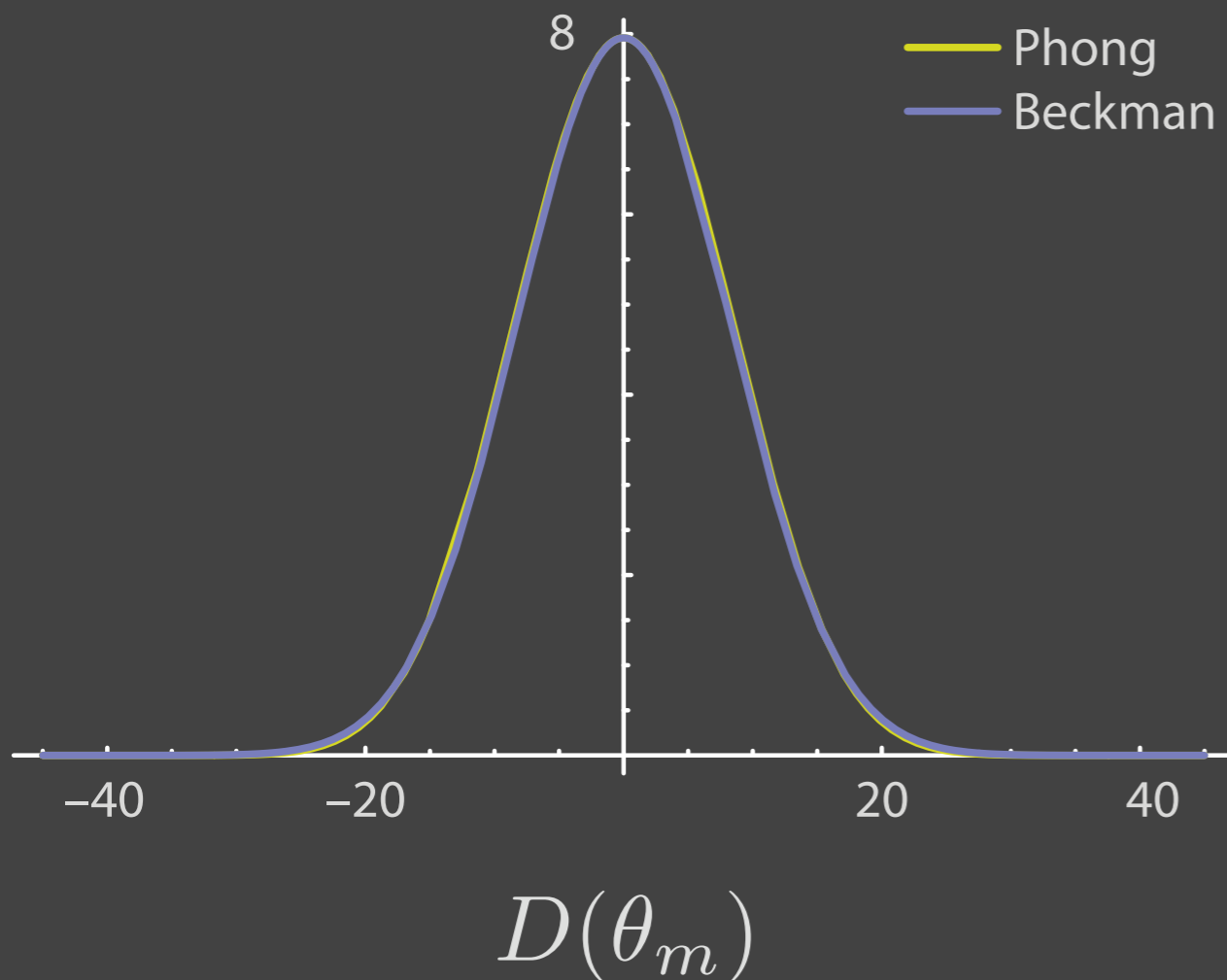
- Phong, Beckman are popular choices
- “GGX” distribution is another option
- [Smith 67] gives a way to produce smooth Gs



Normal distributions

Choice of distribution is determined by surface

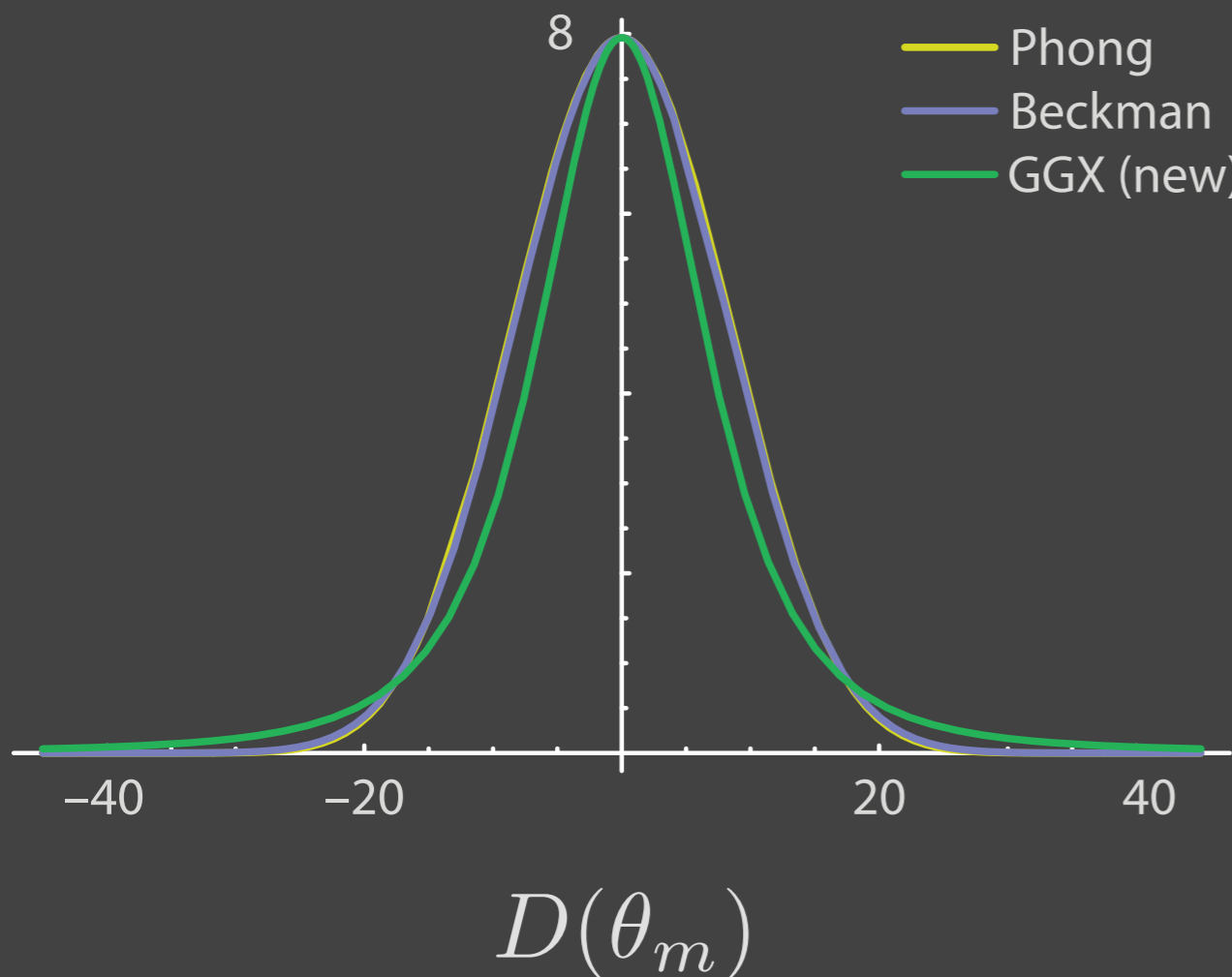
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