

# Microfacet scattering models

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CS 667 Spring 2007, 4–6 September

[These notes are meant to supplement the slides, so I leave out details here that are clear in the slides and concentrate on filling in what is not clear in the slides.]

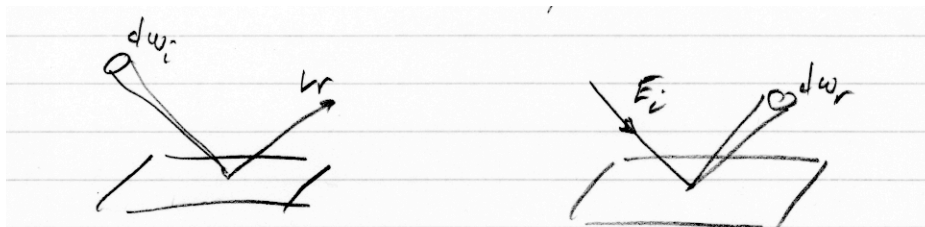
The idea of a microfacet model is to think of a surface that is rough at a scale small relative to the size of the pixels in an image but still large relative to the wavelengths of light. The roughness is supposed to be random, so that there is nothing special about one part of the surface or another: they will have different surface geometry but large areas will have the same statistics. This kind of surface behaves like a mirror locally, but when we observe it at the large scale we can't see any of the surface features—they are all averaged out spatially, and we just see a surface that gets less glossy as it gets more rough.

We call the detailed surface that the rays reflect from the *microsurface*, and its surface normal is the *microsurface normal* or sometimes *micronormal*. The large-scale surface with respect to which we define the average BRDF is the *macrosurface*. We can think of this as being flat for the purposes of deriving the microfacet model, because we're interested in just a differential area of that big surface (which we assume contains a large amount of representative microgeometry). The macrosurface has its normal, the *macrosurface normal* or just *macronormal*, which I will call **n**.

## Geometric setup

The mathematical question is, if we consider the surface at the large scale, what is the BRDF—that is, if we illuminated such a surface uniformly from a particular direction, what would be the average distribution of light reflected? In the microfacet framework, this amounts to counting up reflected rays as they come out and keeping track of their density with respect to solid angle.

This is using a slightly backward definition of BRDF geometry from what we saw in the last lecture, but it is equivalent:



(On the left, we think about how  $L_r$  depends on the distribution of incident radiance; on the right we think about how irradiance  $E_i$  produces a distribution of reflected radiance.)

So the idea is, if we send in irradiance from direction  $\mathbf{i}$  (think of flooding the whole surface with photons, all arriving from the same direction but landing at different points on the surface), how many of those photons end up in a solid angle  $d\omega_o$  around the direction  $\mathbf{o}$ ?

In this lecture I will derive microfacet BRDF (which is traditional) and microfacet BTDF (less usual; we just worked this out and tested it in a 2007 paper) at the same time by simply allowing  $\mathbf{o}$  to be on either side of the surface.

The idea of the microfacet model is to work backwards to find the rays that will end up reflecting into  $d\omega_o$ . Given a particular vector  $\mathbf{o}$ , there is exactly one microsurface normal that will reflect from  $\mathbf{i}$  to  $\mathbf{o}$ . Call it  $\mathbf{m}$  for “microsurface normal”—it is a function of  $\mathbf{i}$  and  $\mathbf{o}$ ; I’ll call the function  $\mathbf{h}$  for “half vector.” So  $\mathbf{m} = \mathbf{h}(\mathbf{i}, \mathbf{o})$ .

Remember that we care about light going to  $d\omega_o$ , not exactly to the direction  $\mathbf{o}$ . This changes the question about a *single*  $\mathbf{o}$  producing a *single*  $\mathbf{m}$  into a question about a *range* of outgoing directions  $d\omega_o$  producing a range of microsurface normals  $d\omega_m$ . The relationship between  $d\omega_o$  and  $d\omega_m$  is determined by the derivative of the function  $\mathbf{h}$ , and we’ll get back to computing this later.

Once we know the set of relevant microsurface normals (by relevant I mean that they are microsurface normals that would reflect incident rays from direction  $\mathbf{i}$  to outgoing rays that fall into  $d\omega_o$ ), the question is how many rays will encounter normals in that set. Once we know that, we have solved most of the problem, because we then know how many rays will end up in  $d\omega_o$ .

### Normal distribution

For this purpose we need to know something about the geometry of the reflecting surface, since obviously the geometry determines how often different normals will occur. For nearly flat, barely rough surfaces, we will very often find the microsurface normal close to the macrosurface normal, whereas for very rough surfaces the microsurface normals will be all over the place. The distribution of microsurface normals is a *statistical property* of a random surface: when we look at different areas of the surface we expect to see the same distribution of micronormals. This statistical description of a rough surface is called the *normal distribution* (or *microfacet distribution* or *slope distribution*). I will call it  $D(\mathbf{m})$  and it measures the density of microsurface area with respect to microsurface normal:

$$\frac{dA_m}{dA} = D(\mathbf{m}) d\omega_m$$

If you look for micronormals in a particular set  $d\omega_m$  you will find some area of microsurface where the normal is in that set; obviously the larger the set of micronormals you allow the larger area of surface you will find—and  $D$  tells you how large this area will be *per unit solid angle of micronormal*. To be more precise,  $D$  tells you the ratio between the *fraction* of relevant surface area and the size of  $d\omega_m$ .

### Shadowing–masking

Once we’ve narrowed down the reflecting area by its surface normal we are most of the way to measuring the relevant area, but there is another detail to contend with. Not all points that have the right normal will be illuminated (they could be shadowed by another part of the surface) and

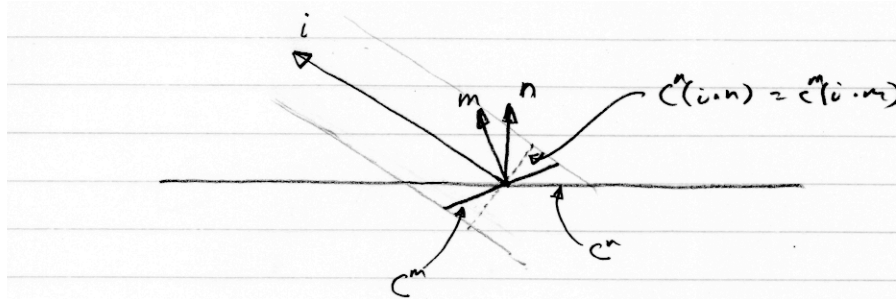
even if they are not shadowed the light reflected there may not get out in the direction  $\mathbf{o}$  (it could be occluded, or *masked* by another part of the surface). To account for this we invent another statistical surface property known as the *shadowing–masking function*  $G(\mathbf{i}, \mathbf{o}, \mathbf{m})$ . The value of  $G$  is between 0 and 1, and it tells us what fraction of the points on the surface with normal  $\mathbf{m}$  will be visible to the outside in both directions  $\mathbf{i}$  and  $\mathbf{o}$ . Thus the average *relevant* reflecting area is  $dA_m = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m$ .

### Attenuation

The final factor to account for is the fraction of light that is actually scattered to the side of the surface where  $\mathbf{o}$  is. That is, if we're asking about reflection we need to know what fraction of light incident on the microsurface is reflected (rather than absorbed or transmitted) and for transmission we need to know what fraction is transmitted. This will just be determined by the Fresnel factor (in this lecture I'm tacitly assuming a dielectric material, which means no light is lost, but as Cook and Torrance show, this can all be made to go through for metals that have absorption. To stay agnostic about whether we are reflecting or transmitting I'll just call this the "attenuation factor" and name it  $\rho$ .

### Irradiance and reflected radiance

Now back to the question of how many rays fall on the surface area  $dA_m$ . These rays are uniformly distributed across the surface from the direction  $\mathbf{i}$ . To define a BRDF we need to relate reflected radiance *measured with respect to the macrosurface* to incident irradiance *measured with respect to the macrosurface*. This means the incident irradiance  $E_i$  we start with is specified as a density with respect to the macrosurface (i.e. perpendicular to  $\mathbf{n}$ ); to convert it to a density of rays with respect to the differently inclined microsurface (perpendicular to  $\mathbf{m}$ ) we need to divide by  $\mathbf{i} \cdot \mathbf{n}$  and multiply by  $\mathbf{i} \cdot \mathbf{m}$ . The following illustration shows why this is so:



So the irradiance on the reflecting area is

$$dE_i^m = \frac{\mathbf{i} \cdot \mathbf{m}}{\mathbf{i} \cdot \mathbf{n}} dE_i$$

again because the irradiance was measured with respect to  $\mathbf{n}$  but we need it measured with respect to  $\mathbf{m}$ .

The incident power is

$$dE_i^m dA^m = d\Phi_i^m$$

and the reflected power is

$$d\Phi_o^m = d\Phi_i^m \rho(\mathbf{i}, \mathbf{o}) = \frac{\mathbf{i} \cdot \mathbf{m}}{\mathbf{i} \cdot \mathbf{n}} \rho(\mathbf{i}, \mathbf{o}) dA^m dE_i$$

This reflected power is larger if there is more irradiance, more reflecting area, or a larger attenuation factor.

We can interpret this power as a radiance reflected from the macrosurface area  $dA$  into the solid angle  $d\omega_o$ :

$$dL_o = \frac{d\Phi_o^m}{dA d\omega_o |\mathbf{o} \cdot \mathbf{n}|}$$

Remember the  $\mathbf{o} \cdot \mathbf{n}$  in the denominator is required because we're measuring radiance using an area that's not perpendicular to the solid angle. Finally, the BRDF is the ratio of  $dL_o$  to  $dE_i$ :

$$\frac{dL_o}{dE_i} = \frac{\frac{|\mathbf{i} \cdot \mathbf{n}|}{|\mathbf{i} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{c}) dA^m d\mathbf{E}_i}{dA d\omega_o |\mathbf{o} \cdot \mathbf{n}| d\mathbf{E}_i} \quad \frac{dA^m}{dA} = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m$$

$$= \frac{|\mathbf{i} \cdot \mathbf{n}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{c}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$

and this last formula is the general form of a microfacet BRDF. We need to fill in values for  $\rho$  (which will just be the Fresnel factor),  $D$  and  $G$  (which have to come from some model of surface geometry), and the solid angle ratio  $d\omega_m/d\omega_o$  (which is the Jacobian of  $\mathbf{h}$ ). In the slides I show a geometric derivation of the solid angle ratio by geometrically differentiating the familiar rule of computing the half vector by summing  $\mathbf{i}$  and  $\mathbf{o}$  and then normalizing.