CS 667 Problem 2.

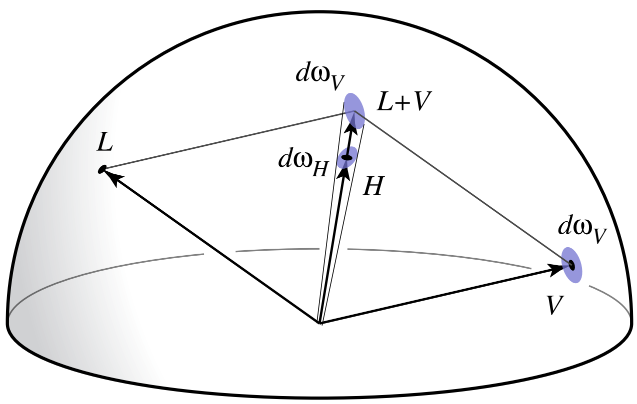
In class we derived a microfacet model for reflection from a rough surface. A key part of the derivation was to show that, for reflection from a single illumination direction L and a reflected solid angle $d\omega_V$ centered around the viewing direction V, the set of surface normals that will reflect the incident rays into the outgoing solid angle is centered around the half vector H and is of size:

$$d\omega_H = \frac{1}{4H \cdot V} d\omega_V.$$

We found this formula via an argument about spherical geometry. Another way to find it is via vector arithmetic. We know that the half vector can be found by adding L and V and projecting back onto the sphere:

$$H = \frac{L + V}{\|L + V\|}$$

If we follow the solid angle $d\omega_V$ through this process we can find $d\omega_H$ directly. Here is a picture of the idea.



The nice thing about this approach is that it can be extended to transmission fairly easily.

1. Use this geometric approach to re-derive the ratio of $d\omega_H$ to $d\omega_V$. *Hint*: Use two right triangles to get the length of $L + V$.

2. Now suppose that, instead of reflection, we are interested in transmission. Let F be the facet normal that leads to specular transmission from L to V (analogous to H for reflection). Use a similar approach to part 1 to derive $d\omega_F / d\omega_V$.

3. Write down a microfacet BTDF (bidirectional transmission distribution function) for a dielectric interface.