CS667 **Lecture 4: Reflectance Models** 2 February 2006

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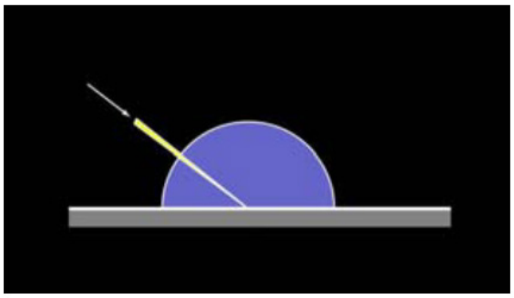
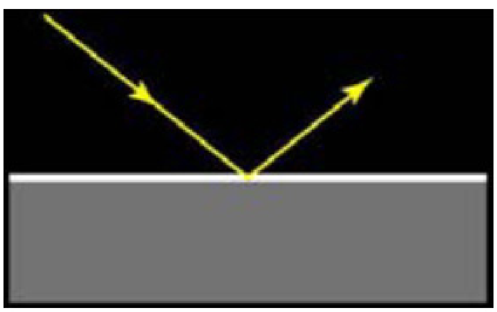
1 Introduction

Several different kinds of behaviors exist for Reflectance Models. They are categorized as follows:

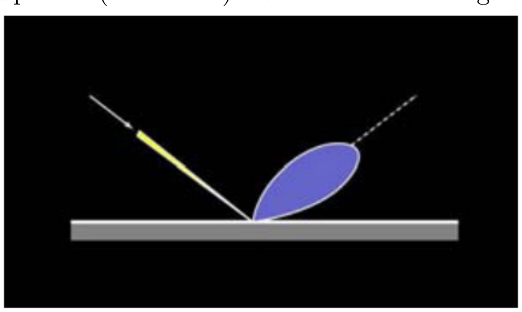
- Diffuse: Has a constant value of reflectance on the polar plot.



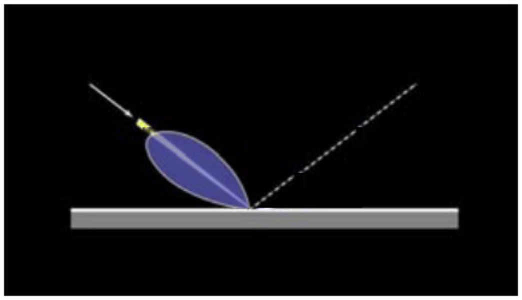
- Specular(Ideal): Reflects light at equal angles.



- Specular(Non-Ideal): Reflects the most light near the equal angle.



- retroreflection: For example, a field of grass.



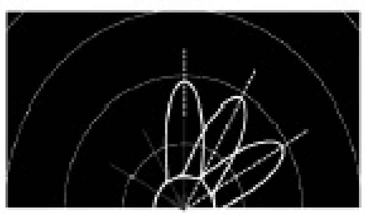
- Other: velvet

There is also a distinction between Isotropic and Anisotropic Materials. Isotropic materials are invariant to rotation about the normal to the surface, while Anisotropic materials change when rotated about the normal.

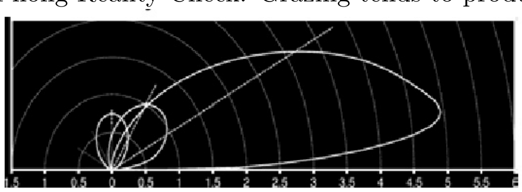
- Isotropic: $f_r(\omega_i, \omega_r) = f_r(\theta_i, \theta_r, \phi_r - \phi_r)$ (Material without oriented structure)
- Anisotropic: $f_r(\omega_i, \omega_r) = f_r(\theta_i, \phi_i, \theta_r, \phi_r)$ (Material with oriented structure)

1.1 Specular Behavior

Peak near mirror direction.



Phong Reality Check: Grazing tends to produce sharper reflections.



1.2 Empirical VS Physics Based Models

Empirical: Models invented to have the right behavior. (some based on experimental data)

- Lambertian
- Phong, Blinn-Phong
- Ward

- Kajiya: Anisotropic model.

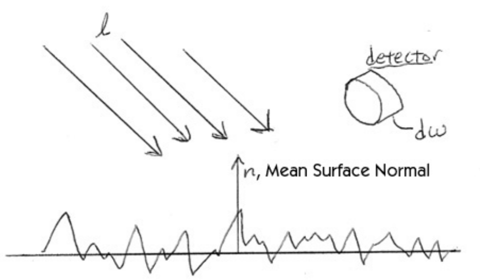
Physics Based: Models based on physics, behaves more realistically.

- Microfacet Models: based on "micro-bumps" on the surface.
- Oren-Nayer: based on distribution of normals of microfacets
- Kirchoff-based: wave optics models
- Koenderink: "thoroughly pitted" e.g. He et al.

2 Microfacet Model: Cook Torrence Model

2.1 Idea

Rough surface is really a smooth surface, with a bumpy geometry.



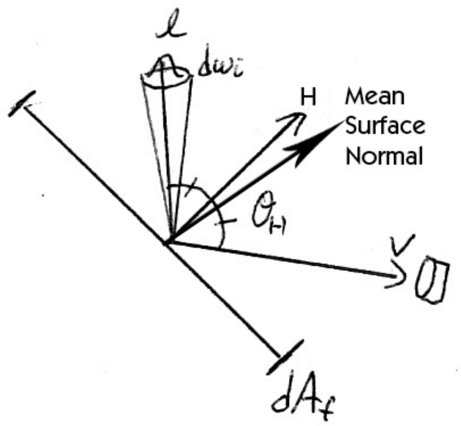
Any ray that comes in sees one microscale surface normal and reflects, mirror-wise. To get the BRDF of the whole thing, we need to compute the average behavior of the rays.
Assumptions (to start off):

- Ideal Micromirrors (No Fresnel term)
- Complete Visibility (No self shadowing)
- Mean Surface (With mean surface normal)

Mission: Find the fraction of rays that ends up in our detector.

2.2 Single Facet Reflection

Let's start by thinking about a single facet at some tilt to the Mean Surface normal.



Incoming light is from a solid angle dw_i, radiance $L_i(\omega_i)$.
Power falling on the facet is

$$L_i d\omega_i dA_f \cos\theta_H = L_i (H \cdot L) dA_f d\omega_i = d^2\Phi_i$$

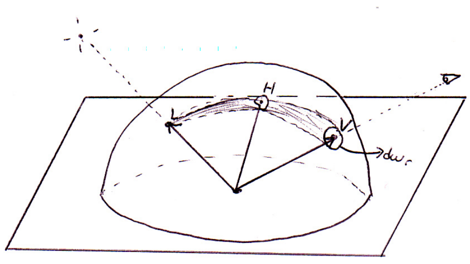
The reflected power from this facet is equal to the incident power:

$$d^2\Phi_r = d^2\Phi_i$$

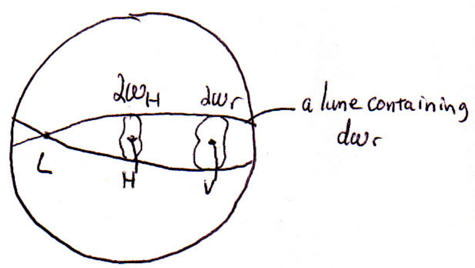
To know the BRDF we need to know the power that goes to a little solid angle $d\omega_r$. We'll do this by figuring out which facet normals will reflect into that solid angle.

2.3 Geometry Subproblem

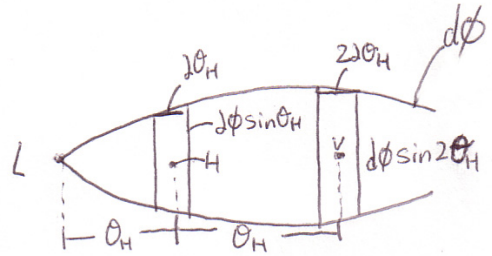
This is actually a nifty little geometry problem. We know that the set of facet normals that will reflect into $d\omega_r$ is going to be centered around H, the bisetctor of L and V. But how big is it?



The solid angle $d\omega_H$ contains the bisectors between L and every point in $d\omega_r$. Drawing the geometry out of context, we have:



Depending on where to V is, the size of $d\omega_H$ changes. Approximating the areas as rectangles, we have the following areas:



$$\begin{aligned}
d\omega_H &= d\theta_H d\phi \sin\theta_H \\
d\omega_r &= 2d\omega_H d\phi \sin 2\theta_H \\
&= 4d\theta_H d\phi \sin\theta_H \cos\theta_H
\end{aligned}$$

and so...

$$\begin{aligned}
\frac{d\omega_H}{d\omega_r} &= \frac{d\theta_H d\phi \sin\theta_H}{4d\theta_H d\phi \sin\theta_H \cos\theta_H} \\
&= \frac{1}{4\cos\theta_H} \\
&= \frac{1}{4(H \cdot L)}
\end{aligned}$$

2.4 Microfacet Normal Distribution $D(\omega)$

The area of the microscale surface that has its normal in $d\omega_H$ is determined by the surface's **Microfacet Normal Distribution**

$$\begin{aligned}
D(\omega)d\omega &= \text{The ratio of the microsurface area that has normals in d}\omega\text{to the macrosurface area. } \omega? \\
D(\omega) &= \frac{d}{d\omega}\frac{dA_f}{dA}
\end{aligned}$$

Even though the bits of surface that have their normals in $d\omega_H$ are scattered around, they reflect the same amount of light as a single facet of the same total area.

So if we talk about the amount of flux reflected from dA into the receiving solid angle $d\omega_r$:

$$\begin{aligned} d^2\Phi_r &= L_i(H \cdot L)dA_f d\omega_i \\ &= L_i(H \cdot L)D(H)d\omega_H dA d\omega_i \\ &= L_i(H \cdot L)\frac{D(H)d\omega_r}{4(H \cdot L)}dA d\omega_i \\ &= L_i \frac{D(H)d\omega_r}{4}dA d\omega_i \end{aligned}$$

2.5 Putting it together

Now to interpret this power as a BRDF. (The exitant radiance divided by incident irradiance)
Exitant Radiance is:

$$\begin{aligned} dL(\omega_r) &= \frac{d^2\Phi_r}{dA d\omega_r \cos\theta_r} \\ &= L_i(H \cdot L)\frac{D(H)}{4\cos\theta_H}\frac{d\omega_i}{\cos\theta_r} \\ &= L_i \frac{D(H)}{4}\frac{d\omega_i}{\cos\theta_r} \end{aligned}$$

Incident Irradiance is:

$$dE(\omega_i) = L_i d\omega_i \cos\theta_i$$

BRDF is:

$$\begin{aligned} f_r(\omega_i, \omega_r) &= \frac{dL_r}{dE_i} \\ &= \frac{L_i D(H) d\omega_i}{L_i d\omega_i \cos\theta_i cos\theta_r} \\ &= \frac{D(H)}{(V \cdot N)(L \cdot N)} \end{aligned}$$

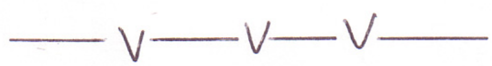
Note that this is reciprocal, a good property.

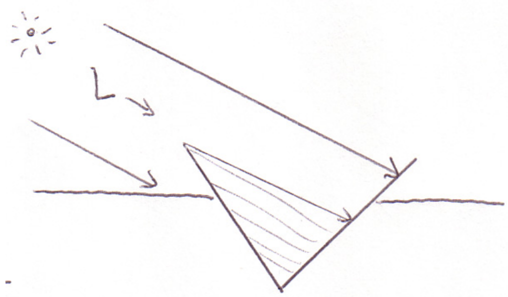
2.6 Self-Shadowing Term

We will now remove one of the assumptions about our model, that there is no self-shadowing.

We will assume that the edges of the microfacet can be modelled by a v shaped indentation, which is straight, and makes calculating how much of the facet can be seen easy. The cook torrence paper works out the details, resulting in:

$$\begin{aligned} G(V, L) &= \text{Shadowing/Masking Term} \\ &= Min\{1, \frac{2(N \cdot H)(N \cdot V)}{(V \cdot H)}, \frac{2(N \cdot H)(N \cdot L)}{(V \cdot H)}\} \end{aligned}$$





We call our SelfShadowing Term, G

Note that now, our BRDF will not blow up at grazing angles.

2.7 Fresnel Term

We further remove another assumption that all reflections on the microfacets are perfect. We add the fresnel term to remove the perfect mirror assumption.

$$F(V,H) = \text{Fresnel Term}$$

2.8 Final BRDF (no, really. it's final)

$$f_r = \frac{F(V,H)G(V,L)D(H)}{4(V \cdot N)(L \cdot N)}$$