CS667 **Volume Path Tracing** 16 March 2006

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1 Introduction

Ray tracing is an adequate rendering approximation in the case of non participating media. In this case, all reflection, absorption and emittance occurs at and between surfaces. Many scenes, however, contain media that scatter, absorb, and even emit light. To render these scenes in a physically accurate manner, the ray tracing rendering equation must be extended to include nonlinear light pathways through the medium. This is known as path tracing.

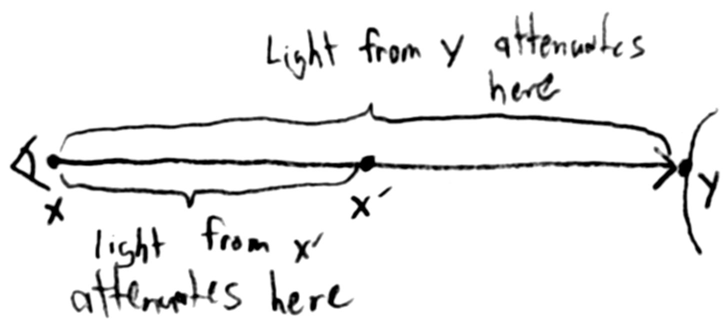

Figure 1: Light transport along a ray

In path tracing, the amount of light flowing along a ray is altered as it travels through the medium. In figure one, we see that there are two cases of light that must be handled:

1. End of Ray Light
2. Inscattered/Emitted Light

Each source of light must be computed, and then attenuated as it travels through the medium. This is given in the following ray transport equation:

$$L(x,\omega) = \int_y^x (\epsilon(x') + \sigma_s(x') \int_{4\pi} f_p(x',\omega,\omega')L(x',\omega')d\omega')e^{-\int_{x'}^{x}\sigma_t}dx' + e^{-\int_y^x \sigma_t} L_e(y,\omega)$$

There are two simultaneous double integrals here - one for the phase function for inscattering along the ray, and the other for attenuation along the ray.

2 Ray Marching Approximation

Because of the structure of the integrals, we can use dynamic programming to accelerate this computation, by re-using portions of the attenuation term. This suggests a very simple finite-step approximation, as seen in figure 2.

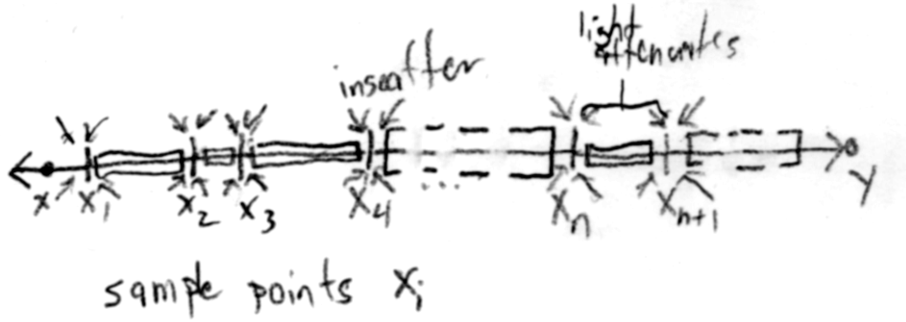

Figure 2: Computing light transport via ray-marching

We sample the integral along the ray by marching away from the eye. The attenuation term is accumulated and applied to the light at each sample point - the special case of which is the incoming light at the the termination of the ray (from a surface intersection, or infinitely distant source). Sampling is possible at any intervals we wish, however the answer is only guaranteed to converge to correctness as the intervals become very small. Importance analysis, however, suggests that the largest acceptable step at any given point is proportional to the attenuation interval, and the local magnitude and rate of change of absorption, emission, and inscattering. In fact, in optically "thick" media, it is often possible to terminate the ray march very early as the attenuation overwhelms all sources of light.

The only computation left is that of the inscattering - this is almost identical to integration of incident light on a surface. The key differences are that it is a spherical integration, rather than hemispherical, and that it is integrated over solid angles, rather than projected solid angles. This means that we can use many sampling methods with minor modifications, such as importance sampling, and even photon maps/diffusion approximations. The simplest method is recursive path tracing, however even with "Russian roulette" style termination criteria, this is computationally intractable for media with high rates of diffusion.

For path tracing importance sampling, given a probability distribution $p(\omega_i)$, the integration sum is computed as:

$$L_n = e^{-\sigma_t(x_n)(x_{n+1}-x_n)} L_{n+1} + (x_{n+1} - x_n)\left(\frac{\sigma_s(x_n)}{K} \sum_{k=1}^{K} \frac{f_p(x_n, \omega, \omega_i) L_i(x_n, \omega_i)}{p(\omega_i)} + \epsilon(x_n)\right)$$

Computation of the incident light (L_i) involves yet another ray march. For optically thin media, this only needs to be evaluated along rays that terminate on a significant source of light, which terminates the algorithm. Figure 3 illustrates the additional ray march to a source of illumination.

To make this a full path tracer (necessary for optically thick media), this ray-march can be converted into a fully recursive call in all directions. As mentioned before, this is computationally intractable in many circumstances.

Luckily, there are approximations that can be used to reduce the complexity of ray-marching:

1. Isotropic scattering - $p(\omega, \omega') = \frac{1}{4*\pi}$
2. Homogeneous medium - $\sigma_t(x) = \sigma_t, \sigma_t(x) = \sigma_t$ All attenuation integrals can be replaced by $L_i(x_n, \omega_i) = L_s e^{-\sigma_t ||x'-x||^2}$. Step size is also no longer constrained by inhomogenuity.

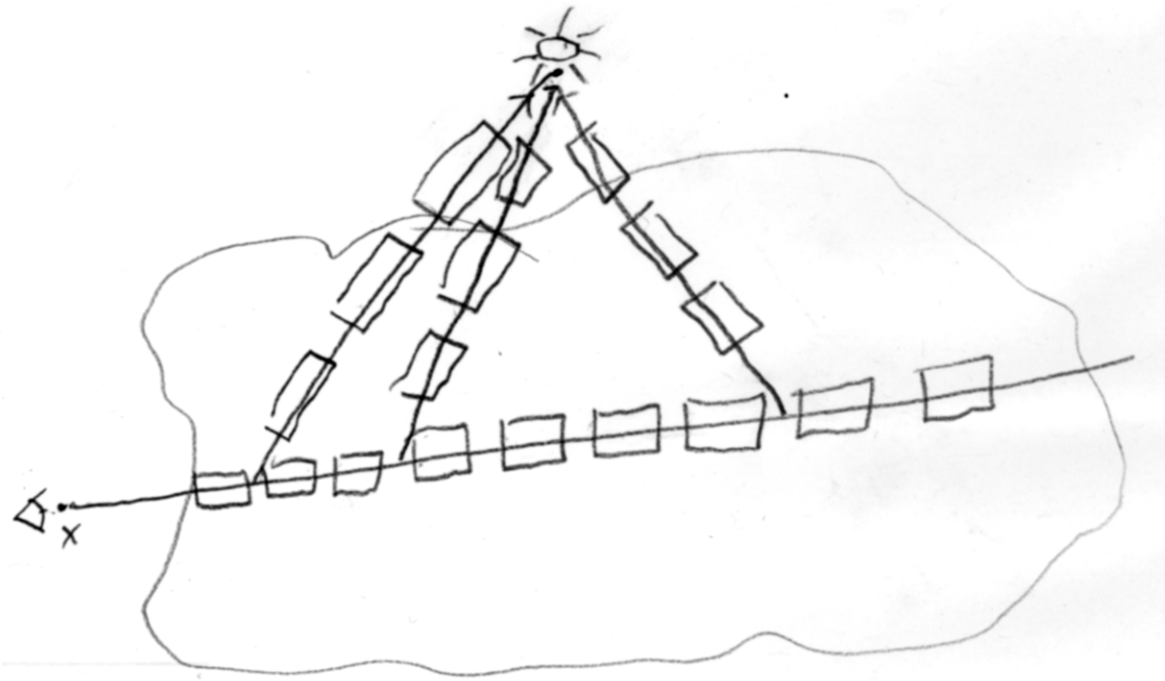

Figure 3: Double integral ray-marching (single scattering approximation)

3 Choosing sample points

In a highly inhomogeneous medium, the variation of density (and its affects on scattering and attenuation) are more important to sample at a high rate than the incoming light integral. Also as noted before, materials that absorb strongly reduce the need for sampling as the ray penetrates further into the material, as contributions have a negligible effect on the final result.

Let us examine the simpler case for sampling is that of a single, homogeneous medium. This allows arbitrarily long attenuation integration lengths. If the step sizes are chosen randomly, this can be thought of as a Monte Carlo integral:

$$L(x,\omega) = \frac{\sigma_s}{4\pi} \int_y^x \int_{4\pi} L(x',\omega_i) d\omega_i e^{-\sigma_t ||x-x'||} dx'$$

Because the attenuation term is only dependent on the properties of the medium, and it is monotonic, it is easily used as a sampling distribution.

$$P(t) = \int_0^t e^{\sigma_t x} dx = \frac{-1}{\sigma_t} + C$$

We have c as an undetermined constant, yet we also know that probability density functions must integrate to one. At $\lim_{t\to\infty} p(t) = 0$, and $p(0) = \frac{-1}{\sigma_t}$ so we must normalize the function to $1 - e^{-\sigma_t t}$. Simple algebra gives:

$$\xi = 1 - e^{-\sigma_t t}$$

$$e^{-\sigma_t t} = 1 - \xi$$
$$\sigma_t t = \ln 1 - \xi$$
$$t = \frac{-\ln \xi}{\sigma_t}$$

In the last line, we can substitute ξ for $1 - \xi$, because they have the same probability distribution. As we have constructed it, choosing samples proportional to $\frac{-1}{\sigma_t} e^{-\sigma_t t}$, we are able to cancel out the attenuation term completely, leaving:

$$g(x') = \frac{f(x')}{p(x')} = \frac{\sigma_s}{4\pi\sigma_t} \int_{4\pi} L(x', \omega_i) d\omega_i$$

If we choose a single sample and recurse, this corresponds strongly to a physical simulation of single-photon scattering through a medium, as seen in Figure 4.

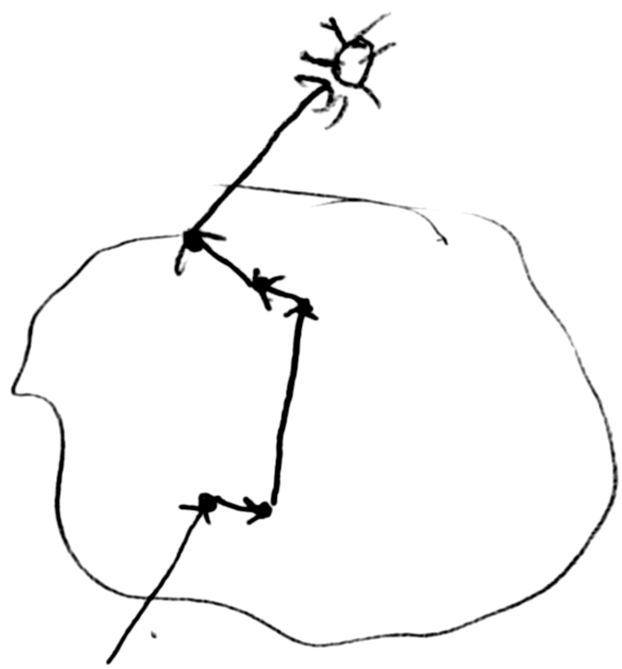

Figure 4: Single photon diffusion simulation