CS667 **Lecture 8: Light Transport in Medium** 17 February 2005

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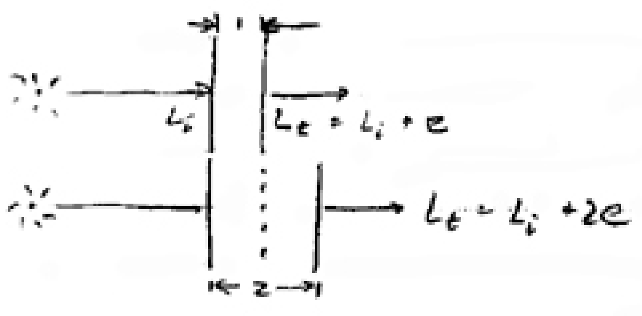
1 Introduction

Once we relax the light transport in vacuum assumption, we must take into account the medium in between surfaces in the scene. Now radiance is no longer invariant along a straight line, but instead, there can be an addition or subtraction of photons along rays between surfaces. This changes the surface solid-angle formulation of radiance $L_e : \mathcal{M} \times H^2 \rightarrow \mathbb{R}$ to a 3D formulation of radiance $L_e : \mathbb{R}^3 \times H^2 \rightarrow \mathbb{R}$.

There are four main events than can occur along a ray: a photon can be *emitted* from a particle, a photon can hit a particle and be *absorbed*, a photon can hit a particle and be *deflected out* of the path of the ray, or a photon from another ray can hit a particle and be *deflected in* to the path of the ray. The terms used for these are respectively emission, absorbtion, out-scattering, and in-scattering.

2 Emission

The simplest case is a medium of particles that simply emit light photons. Consider a ray that travels through 1 or 2 units of the medium:



The generalization is that for a ray that travels through d units of a constant emissive medium, the resulting radiance $L_t = L_i + de$, a linear growth with distance:

$$\frac{d}{dt}L(t) = \epsilon(t)$$

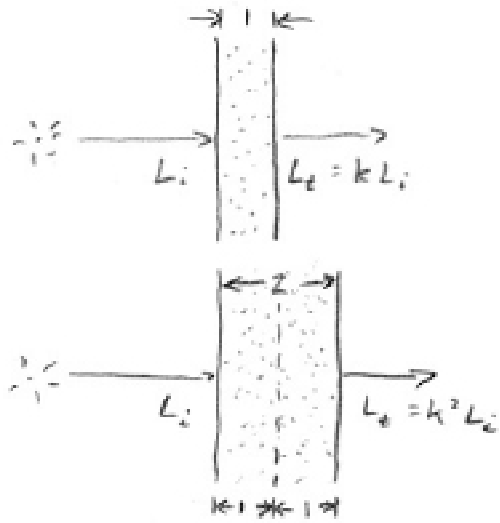
where the emission $\epsilon(t)$ can be different at different places on the ray. For a generalized position x with a ray traveling in the direction ω, the derivative along the line is the directional derivative of $L(x, \omega)$, or $(\omega \cdot \nabla)L(x, \omega)$, and

$$(\omega \cdot \nabla)L(x, \omega) = \epsilon(x, \omega)$$

Usually, emission is independent of direction, or *isotropic*, and $\epsilon(x, \omega) = \epsilon(x)$. If this is not then case, then the emission is said to be *anisotropic*.

3 Absorbtion

When photons travel through an abosorbing medium, a certain percentage of the photons will hit a particle and be absorbed, and the amount that exits is proportional to the amount that enters. Thus, instead of absorbtion being additive like emission, it is an exponential decay.



And in general:

$$L_t = L_i k^d = L_i e^{-\sigma_a d}$$

where σ_a is the absorbtion coefficient. If we don't assume that the medium is homogenous, it becomes easier to write it as a differential equation, with the rate of change of radiance at a point proportional to the absorbtion coefficient and radiance at the point. On a given line parametrized by t, this is

$$\frac{d}{dt}L(t) = -\sigma_a(t)L(t)$$

and the general 3D directional derivative formulation is

$$(\omega \cdot \nabla)L(x, \omega) = -\sigma_a(x)L(x, \omega)$$

4 Out-Scattering

Out-scattering is the loss of photons that are deflected away in other directions. Since we are only concerned with the radiance along a ray, the direction of the scattering is unimportant, and the effect of out-scattering behaves identically to that of absorbtion, with a scattering coefficient of σ_s.

The total extinction coefficient, $\sigma_t = \sigma_a + \sigma_s$, describes the loss of radiance due to both absorbtion and out-scattering, with the net effect

$$(\omega \cdot \nabla)L(x, \omega) = -\sigma_t(x)L(x, \omega)$$

As seen in the directional derivative, the scattering coefficient, as well as the absorbtion coefficient, is independent of the direction of the ray, ω.

5 In-Scattering

In-scattering takes into account photons that are deflected, or scattered, from another ray into the direction of the ray we are looking at. The effect is similar to emission, but depends on the radiance in the medium.

In the isotropic case, the in-scattering depends only on the total amount of light at a point, independent of the direction from which it arrived:

$$\text{total light, or fluence} \quad \phi(x) = \int_{4\pi} L(x,\omega)d\omega$$

$\phi(x)$ is the power per unit area that is scattered, uniformly distributed in the sphere. So the radiance from the point is then $\frac{1}{4\pi}\phi(x)$, and the directional derivative is

$$(\omega \cdot \nabla)L(x,\omega) = \frac{\sigma_s(x)}{4\pi} \int_{4\pi} L(x,\omega')d\sigma(\omega')$$

If the scattering is anisotropic, though, we need to replace the $\frac{1}{4\pi}$ with a probability distribution $p(x,\omega,\omega')$, the probability that a photon traveling in direction ω will scatter in the direction ω' at the point x, and is also known as the *phase function.*

Now the rate of change of the radiance due to in-scattering is

$$(\omega \cdot \nabla)L(x,\omega) = \sigma_s(x) \int_{4\pi} p(x,\omega,\omega')L(x,\omega')d\sigma(\omega')$$

6 Phase Functions

The phase function $p(x,\omega,\omega')$ describes the distribution of scattering, and its integral over the sphere must equal one. Usually, the phase function depends only on the angle between ω and ω', θ, and does not vary depending on the spatial orientation of the sphere, and can be written as $p(\theta)$. The isotropic phase function is

$$p(\theta) = \frac{1}{4\pi}$$

Another common phase function is the Heyney-Greenstein, which was proposed for the scattering from intergalactic dust. However, it is entirely empirical and is only used because of its simple form

$$p(\theta) = \frac{1-g^2}{4\pi(1+g^2-2g\cos\theta)^{\frac{3}{2}}}$$

where g is the average cosine, or the expected value of $\omega \cdot \omega'$

$$g = \int_{4\pi} p(\omega,\omega')\omega \cdot \omega' d\sigma(\omega')$$

And for values of:

$$g = 0 \Longrightarrow \text{isotropic scattering}$$
$$g > 0 \Longrightarrow \text{forward scattering}$$
$$g < 0 \Longrightarrow \text{backward scattering}$$

Some other examples include the Rayleigh phase function

$$p(\theta) = \frac{1+\cos^2\theta}{\lambda^4}$$

which relates the scattering to the wavelength and works well for smaller particles, and the Mie phase function, which is more complex and works better for larger particles.

7 Combined Equation and Evaluation

The combined equation describing emission, absorbtion, out-scattering, and in-scattering is

$$(\omega \cdot \nabla)L(x, \omega) = -\sigma_t(x)L(x, \omega) + \epsilon(x, \omega) + \sigma_s(x) \int_{4\pi} p(x, \omega, \omega')L(x, \omega')d\sigma(\omega')$$

For calculating the absorbtion, if the medium is homogenous, then the problem is trivial and can be directly evaluated. For inhomogenous mediums, the calculation of abosorbtion and single scattering, which only deals with the case where photons go through one deflection at most, can be done with a ray-marching algorithm with shadow rays to the light sources. Calculating multiple scattering, where a photon may be scattered multiple times and change its course more than once, is a much harder problem.