CS 667 Problem 4.

Let a "two-sided BRDF" $f_s(\omega_i, \omega_e)$ be a BRDF that is not constrained to have the incident and exitant directions on one side of the surface.

1. Derive the two-sided BRDF for a layer of absorbing–scattering medium with thickness d, scattering coefficient σ_s, absorption coefficient σ_a, and phase function $p(\cos\alpha)$, using a single-scattering approximation. You will want to split your equation into two cases, one for when the two directions are on the same side of the surface and one for when they are on opposite sides. Report coherent transmission separately (i.e. give an expression for the fraction $k_c(\omega)$ that is transmitted coherently and two more expressions for the scattering $f_s(\omega_i, \omega_e)$ to the two sides).

2. Plot your model (on a polar plot) for incidence angles 0 and 60°, σ_s = 0.1 and 0.01 cm^{-1}, $\sigma_a = 0$ and 0.1 cm^{-1}, $d = 1$ cm, and isotropic scattering. That's 8 plots; they can be small and fit on one page.

3. Do a sanity check by integrating your model numerically (Monte Carlo is fine) over the exitant sphere at $\theta_i = 60°$ for the case of zero absorption and isotropic scattering. Since there is no absorption, it should integrate to $1 - k_c$ for low σ_s, and you should see it decrease as you increase σ_s (because multiply scattered energy is lost by the single-scattering approximation). Approximately how small does σ_s need to be to lose less than 10% of the energy?

Here are a couple of examples to get you going.

- For a layer of purely absorbing medium with absorption coefficient σ_a and thickness d, there is only coherent transmission, and

$$k_c(\theta) = e^{-\sigma_a d/\cos\theta}$$

  where θ is the angle of incidence.

- For a layer of emitting–absorbing medium with emission coefficient ϵ, absorption coefficient σ_a, and thickness d, the exitant radiance is

$$L_e(\omega) = \int_0^d e^{-\sigma_a t/\mu} \epsilon \, dt/\mu = \frac{\epsilon}{\sigma_a}\left(1 - e^{\sigma_a d/\mu}\right)$$

  where $\mu = \cos\theta$. k_c remains the same as in the previous example.