CS667 **Lecture 13: Photon Mapping with Participating Media** 9 March 2004

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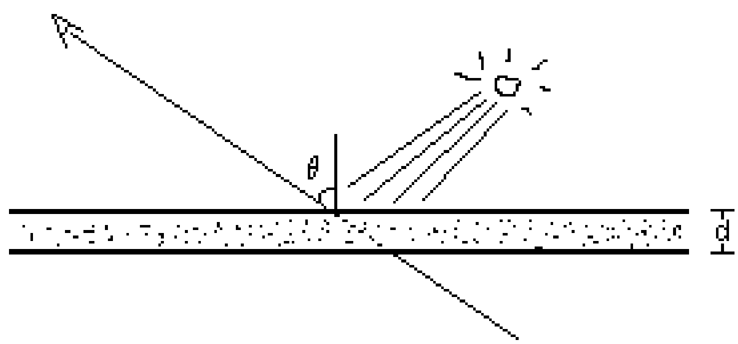
1 Aside: Problem 4 discussion

1.1 Reflection and Transmission From a Layer of Optical Medium

We can think of modeling a surface as a thin layer of medium. Depending on the layer's thickness, this will result in a BRDF or a two-sided BRDF. Note that we assume the layer is much smaller than the scene scale, so that we treat the layer as a surface, and so that there is no translucency (in the sense that light incident on a point of the surface exits from the same point).



This leads to some particularly simple results in scattering theory.

1.2 Absorbing Medium

Consider a thin layer that strictly absorbs, of thickness d and absorption coefficient σ_a. Then the distance that a beam of light incident at an angle θ travels within the layer is $d/cos(\theta)$, which we will call d/μ. Thus the coefficient of coherent transmission is

$$k_c = e^{-\sigma_a d/\mu}$$

This result causes a gel filter to look darker at oblique angles.

1.3 Emitting Medium

Now consider a thin layer that strictly emits light uniformly with emission coefficient ϵ. In this case, we collect light over the path through the layer, which still has length $d/cos(\theta)$, so we have

$$L_e = \epsilon d/\mu$$

If we further consider absorption at the same time, we have

$$L_e(\omega) = \int_0^d \alpha(0, t/\mu)\ \epsilon\ dt/\mu = \int_0^d e^{-\sigma_a t/\mu}\ \epsilon\ dt/\mu = -\mu/\alpha_a[e^{-\sigma_a t/\mu}]_0^d \epsilon/\mu = \epsilon/\alpha_a(1 - e^{-\sigma_a d/\mu})$$

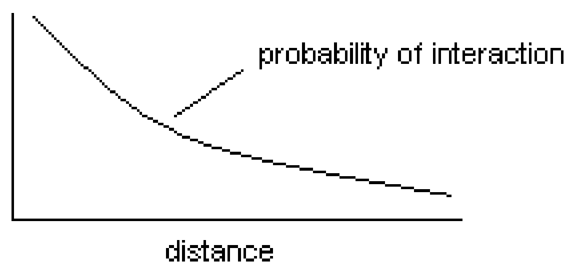
Note that this prevents bright rays at grazing angles, even with very small absorptions. L'Hopital lets us see equivalence to the strictly emitting layer case when α_a goes to zero:

$$\lim_{a\to 0} \epsilon(1 - e^{-\sigma_a d/\mu})/\sigma_a = \epsilon(d/\mu)e^{-\sigma_a d/\mu}/1 = \epsilon d/\mu$$

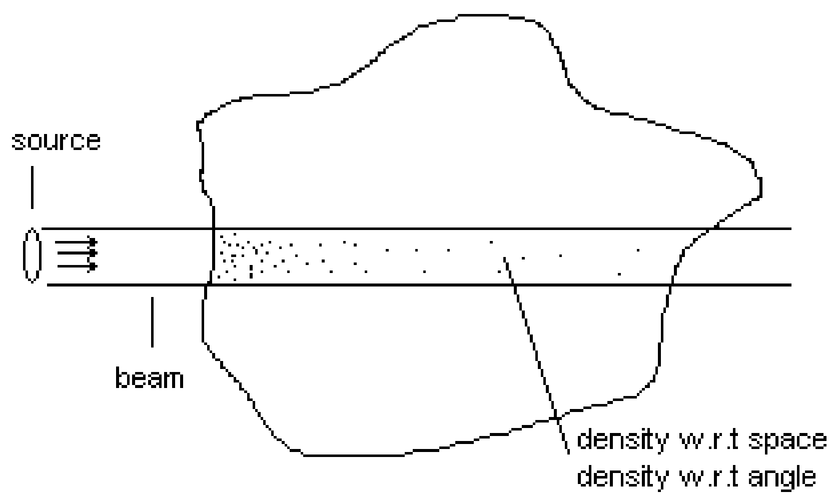
2 Photon Mapping with Participating Media

In order to avoid recursive path tracing, we'd like an immediate estimate of indirect lighting. As in the surfaces in a vacuum case, simulating particle transport leads to particle interactions that are proportional to the solution of the transport equation.

The procedure in the presence of participating media is to simulate interaction of photons by choosing a random lifetime proportional to an exponential:



At every interaction, record the photon position and direction. At this point we have the proper distribution of first order photons:



After each of these interactions, we choose to continue with probability σ_s/σ_t and choose directions proportional to phase function $p(\alpha)$.

Thus the scattered beam is a secondary source for the volume.

The interaction positions of the second interactions are distributed according to single-scattered radiance. We have:

$$L(x,\omega) = \int_{line} \alpha(x,x') \int_{S^2} p(x',\omega,\omega')\ L(x',\omega')\ dx'\ d\omega'$$

The α term represents the probability that a photon scattered at x' makes it to x. The p term represents the probability of a photon at x' being scattered to direction ω. The L term represents the density of photons at x'.

By extension, this is true for all scattering orders: the scattered first-order radiance acts as a secondary source for the second-order radiance, and so on. In practice, the first order argument is left out for the photon estimate.