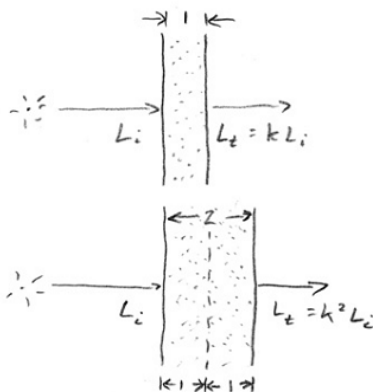


If we add participating media into the scene, our equations become somewhat more complicated. The radiance $L(x, \omega)$ is now defined for all x in 3D, not just for surface points, and the equation describing light transport becomes integro-differential.

Absorption

One thing that can happen to a light particle traveling through a medium is that it gets absorbed. The amount of radiance that passes through a given piece of medium is proportional to the initial amount. Let's denote the coefficient by k . Now if we double the thickness of the piece, the new coefficient should be k^2 , from which we might conclude that there is an exponential decay of radiance with respect to the length of path traveled (in a homogeneous medium). This can be seen on the picture:



So we can conclude that:

$$L_t = L_i k^d = L_i e^{-\sigma_a d}$$

where L_i is the initial radiance, L_t is the transmitted radiance, d is the path length in medium and σ_a is the absorption coefficient. To get rid of the assumption of homogeneous medium, we might rewrite this into a differential equation. We can say that the rate of change of the radiance at a particular point in the medium is proportional to the radiance at that point times the absorption coefficient at that point. On a line parametrized by u , this would be:

$$\frac{d}{du} L(u) = -\sigma_a(u) L(u)$$

The general 3D formulation of the last equation is that the directional derivative of radiance is proportional to radiance:

$$(\omega \cdot \nabla) L(x, \omega) = -\sigma_a(x) L(x, \omega)$$

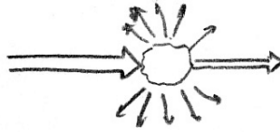
Examples of purely absorbing materials include clear liquids, gemstones and (colored) glass. The following image was rendered with absorption:



(Josh Will, 2003 UCSD Rendering Competition)

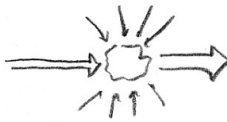
Out-scattering

Apart from being absorbed in the medium, a light particle can also get scattered away. The effect of out-scattering is essentially identical to that of absorption, just the scattering coefficient σ_s might be different. The total extinction coefficient, $\sigma_t = \sigma_a + \sigma_s$, describes the loss of radiance due to both absorption and out-scattering.



In-scattering

The light that gets scattered away from a ray might show up in other rays, increasing their radiance. This process is called in-scattering:



Isotropic in-scattering means that a particle may be scattered into a given direction from any other direction with equal probability. Anisotropic in-scattering can be modeled by a *phase function*, which describes which directions are more likely to be scattered into the given direction. (The meaning of “anisotropic” is different from the one for BRDF.) The following equation describes the combined effect of absorption, in-scattering and out-scattering:

$$(\omega \cdot \nabla)L(x, \omega) = -\sigma_t(x)L(x, \omega) + \sigma_s(x) \int_{S^2} p(x, \omega, \omega')L(x, \omega')d\sigma(\omega')$$

Phase Functions

The phase function, $p(x, \omega, \omega')$, describes the distribution of scattering in the medium. The integral of the phase function over the entire sphere must be equal to one. Usually, the spatial orientation of the medium

does not matter, so the phase function only depends on θ , the angle between ω and ω' . The simplest phase function is the one for isotropic scattering:

$$p(\theta) = \frac{1}{4\pi}$$

The Heyney-Greenstein phase function was formerly used for scattering in intergalactic dust, but it has no particular physical relevance. Similarly to Phong BRDF, it is used because of its simple form.

$$p(\theta) = \frac{1 - g^2}{4\pi(1 + g^2 - 2g \cos \theta)^{1.5}}$$

The parameter g is the average cosine of the scattering direction. Positive g leads to forward scattering, negative g to backward scattering, $g = 0$ to isotropic scattering.