CS667 **Lecture 7: Generalizing Radiosity** 17 February 2004

Jon Moon Lecturer: Steve Marschner

1 Beyond Constant Basis Functions

The radiosity work we've seen so far utilized piecewise constant basis functions for radiosity over surfaces. However, the Finite Element formulation doesn't require this, and later work in radiosity relaxed those constraints to allow more general basis sets. One simple example is linear elements where radiosities are associated with the vertices rather than the triangles of a triangle mesh.

1.1 Aside: Linear vs. Multilinear Functions

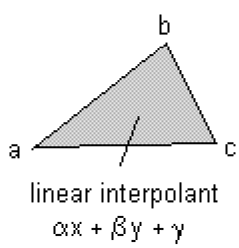
In dimensions higher than 1D, there is a difference between linear and multilinear functions. A linear function is of the form

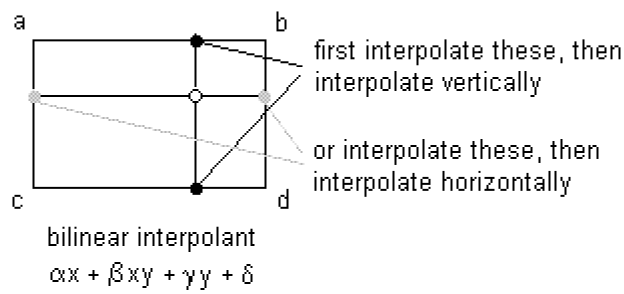
$$f(\mathbf{x}) = a_1x_1 + a_2x_2 + a_3x_3 + ... + a_nx_n = \langle \mathbf{a}, \mathbf{x} \rangle$$

This is often loosened to include a constant term, because it is so useful, although this is actually an affine function:

$$f(\mathbf{x}) = \langle \mathbf{a}, \mathbf{x} \rangle + c$$

A function of this form has $n+1$ degrees of freedom, and is therefore uniquely defined by the vertices of a simplex - the smallest polyhedron - in any dimension: a line segment in 1D, a triangle in 2D, a tetrahedron in 3D, etc. Thus a given simplex can be linearly interpolated, by interpolating values across. But often what we call "linear interpolation" is done on a grid, depending on 2^n vertices, not $n+1$. We do this by interpolating along each dimension in turn, which in 2D results in the bilinear interpolant $\alpha x + \beta xy + \gamma y + \delta$.

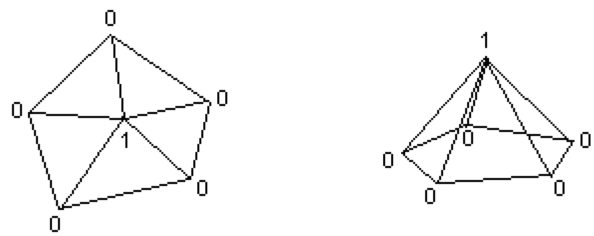




In general this is a multilinear interpolant: taken as a function of any one variable, it becomes an affine function. But it is NOT linear!

1.2 Back to Radiosity

This suggests that a triangle mesh is a natural setting for a linear basis. One basis function is associated with each vertex and its support is the star of that vertex:



There exists exactly one linear function on each triangle that has value 1 at the center and 0 at the other two vertices. Note that values along shared edges will be the same in both adjacent triangles' functions.

In any case, the first part of the discretization we used for constant basis functions remains unchanged in the more general case. We have the radiosity equation:

$$B(x) = B^0(x) + R(x) \int_{\mathcal{M}} B(y)\ G(x, y)\ dA(y)$$

Expanding B in the basis

$$\tilde{B}(x) = \sum_{k=1}^{n} b_k N_k(x)$$

where $\tilde{B}(x)$ is the computed, approximate radiosity yields

$$B(x) = B^0(x) + R(x) \sum_j b_j \int_{\mathcal{M}} N_j(y)\ G(x, y)\ dA(y)$$

What remains is to discretize the other instances of B. The first was simply substituting the definition, but the others involve comparing an approximate $\tilde{B}$ with the continuous result of the integral. We could write

$$\tilde{B}(x) \approx B^0(x) + ...$$

But "≈" isn't well-defined. One possible definition is the "Galerkin" formulation resulting from minimizing the residual by looking for it to be orthogonal to the approximation space. That is,

$$A \approx B \iff \forall i\ \langle A - B, N_i \rangle = 0$$

.

So we have

$$\begin{aligned}
\forall i,\ \langle \tilde{B}, N_i \rangle &= \langle B^0, N_i \rangle + \langle R(\cdot) \sum_j b_j \int_{\mathcal{M}} N_j(y)\ G(\cdot, y)\ dA(y), N_i \rangle \\
\sum_j b_j \langle N_j, N_i \rangle &= \langle B^0, N_i \rangle + \sum_j b_j \langle R(\cdot) \int_{\mathcal{M}} N_j(y)\ G(\cdot, y)\ dA(y), N_i \rangle \\
\sum_j b_j \langle N_j, N_i \rangle &= \langle B^0, N_i \rangle + \sum_j b_j \int_{\mathcal{M}} R(x) \int_{\mathcal{M}} N_j(y)\ G(x, y)\ dA(y)\ N_i(x) dA(x)
\end{aligned}$$

Letting $\Delta_{ij} = \langle N_j, N_i \rangle$ and $b_i^0 = \langle B^0, N_i \rangle$, we have

$$\forall i,\ \sum_j\ \Delta_{ij} b_j = b_i^0 + \sum_j \left[\int_{\mathcal{M}} R(x) N_i(x) \int_{\mathcal{M}} N_j(y)\ G(x, y)\ dA(y)\ dA(x) \right]\ b_j$$

The square-bracketed quantity is exactly the generalized form factor, $\hat{f}_{ij}$, and so this is in fact the same form that we have seen previously:

$$\mathbf{\Delta b} = \mathbf{b^0} + \mathbf{\hat{F}b}$$

And so the solver doesn't need to change at all with general basis functions, but the form factors do. It is straightforward to show that the old way is a special case of this formulation, wherein basis functions and reflectances are constant per patch.