CS667 **Lecture 5: Duality of Light Transport, Radiosity Method** 10 February 2004

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Duality of Light Transport

The basic idea of duality is that light sources and light detectors are in some sense interchangeable.

Importance

The value of a given pixel depends on the radiance that reaches the camera aperture from a particular cone of directions. This radiance is equal to the radiance leaving the surface that is visible by the camera. That radiance is in turn determined by the incident radiance. The sensitivity to incoming radiance is given by the BRDF. This can be continued recursively.

The term "importance" is used to describe this process. We can think of a pixel as a source of importance. The value of importance for a given point and direction means, intuitively, how much does the radiance for that point and direction affect the radiance at the importance source. The importance reflects around the scene in exactly the same way as radiance does and it eventually reaches an equilibrium distribution.

Relations Between Radiance and Importance

W_e^0 – emitted (intitial) importance	L_e^0 – emitted radiance
$W_i^0 = \mathbf{G}W_e^0$ – direct importance	$L_i^0 = \mathbf{G}L_e^0$ – direct lighting
$W_e^0 = \mathbf{G}W_i^0$	$L_e^0 = \mathbf{G}L_i^0$
W_e – equilibrium importance	L_e – equilibrium radiance
$W_e = \mathbf{GK}W_e + W_e^0$ – r. eq. for importance	$L_e = \mathbf{GK}L_e + L_e^0$ – rendering equation

To express the interaction between radiance and importance, we will use inner products of functions:

$$\langle f, g\rangle \;=\; \int_{M\times H^2} f(x,\omega)g(x,\omega)dA(x)d\mu(\omega)$$

With this definition, we can express the flux that reaches a given pixel as

$$\Phi \;=\; \langle W_i^0, L_e\rangle$$

Intuition behind this formula: The direct importance from the pixel, W_e^0, expresses where to measure the global illumination solution (equilibrium radiance) L_e. Another way to express the same thing would be

$$\Phi \;=\; \langle W_e, L_i^0\rangle$$

Interpretation: The equilibrium importance W_e expresses where to measure the direct radiance L_i^0.

To get more algebraic insight, consider the following definition. We will say that two operators $\mathbf{A}$ and $\mathbf{B}$ are *adjoint* if

$$\forall f, g\ \langle \mathbf{A}f, g\rangle = \langle f, \mathbf{B}g\rangle$$

(In linear algebra, a transposed matrix is adjoint to a given matrix.) The two equivalent ways to compute flux (above) mean that the operator $\mathbf{RG}$ (where $\mathbf{R}$ is the rendering operator $\mathbf{R} = (\mathbf{I}-\mathbf{KG})^{-1}$) is self-adjoint:

$$\begin{aligned}
L_e &= \mathbf{R}L_e^0 \\
W_e &= \mathbf{R}W_e^0 \\
\Phi = \langle W_i^0, L_e\rangle &= \langle W_i^0, \mathbf{R}L_e^0\rangle = \langle W_i^0, \mathbf{RG}L_i^0\rangle \\
\Phi = \langle W_e, L_i^0\rangle &= \langle \mathbf{R}W_e^0, L_i^0\rangle = \langle \mathbf{RG}W_i^0, L_i^0\rangle
\end{aligned}$$

Radiosity Method

The general form of the rendering equation is

$$L_e \;=\; L_e^0 + \mathbf{KG}L_e$$

The radiosity method assumes that all reflectors are perfectly diffuse, so

$$L_e(x,\omega) \;=\; \frac{1}{\pi}B(x)$$

where $B(x)$ is the radiosity at point x. Moreover, the BRDF becomes just $\frac{R(x)}{\pi}$. With these assumptions, we can rewrite the rendering equation as:

$$\begin{aligned}
L_e(x,\omega_e) &= L_e^0(x,\omega_e) \;+\; \int_{H^2} f_r(x,\omega_e,\omega_i)L_e(\Psi(x,\omega_i),-\omega_i)d\mu(\omega_i) \\
&= \frac{1}{\pi}B^0(x) \;+\; \int_{H^2} \frac{R(x)}{\pi^2}B(\Psi(x,\omega_i))d\mu(\omega_i) \\
B(x) &= B^0(x) \;+\; \frac{R(x)}{\pi}\int_{H^2} B(\Psi(x,\omega_i))d\mu(\omega_i)
\end{aligned}$$

If we switch to area formulation, we get:

$$B(x) \;=\; B^0(x) \;+\; R(x)\int_M B(y)G(x,y)dA(y)$$

where the geometry term is defined as

$$G(x,y) \;=\; V(x,y)\,\frac{\cos\theta_x\cos\theta_y}{\pi \parallel x-y \parallel^2}$$

One approach to solving this integral equation is to discretize the functions $B(x)$, $B^0(x)$ and $R(x)$, i.e. make them piecewise constant. We define:

$$B(x) = \sum_k b_k P_k(x) \qquad b_i = \frac{1}{A_i}\int_{P_i} B(x)dA(x)$$

The function $P_k(x)$ is 1 for all x on patch k and 0 otherwise. So we get:

$$\begin{aligned}
b_i &= \frac{1}{A_i}\int_{P_i} B(x)dA(x) \\
&= \underbrace{\frac{1}{A_i}\int_{P_i} B^0(x)dA(x)}_{b_i^0} \;+\; \frac{1}{A_i}\int_{P_i} r_i \sum_j b_j \int_{P_j} G(x,y)dA(y)dA(x) \\
&= b_i^0 \;+\; r_i\sum_j b_j \underbrace{\frac{1}{A_i}\int_{P_i}\int_{P_j} G(x,y)dA(x)dA(y)}_{form\ factor\ f_{ij}} \\
&= b_i^0 \;+\; r_i \sum_j f_{ij}b_j
\end{aligned}$$

So we have a system of linear equations, which can be expressed in matrix form:

$$(I - rF)b = b^0$$

where b is the vector of b_i-s, b^0 is the vector of b_i^0-s, F is the matrix of form factors and I is the identity matrix.