CS664 Computer Vision

9. Camera Geometry

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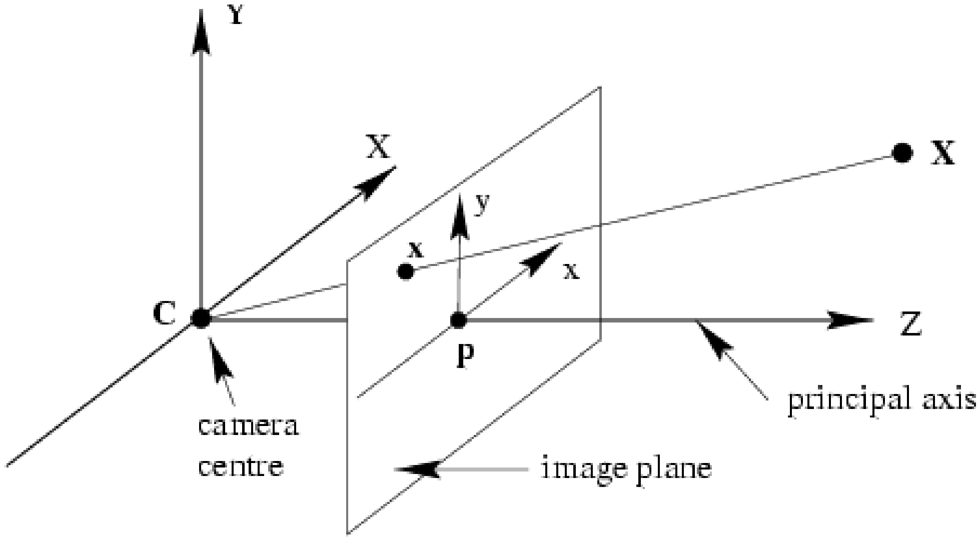
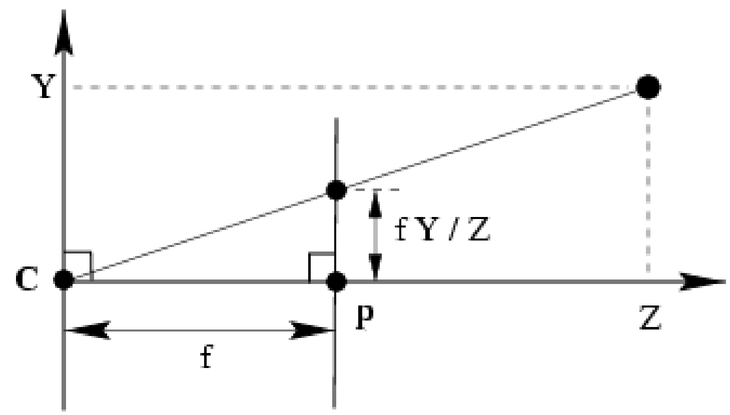
Pinhole Camera

- Geometric model of camera projection
  - Image plane I, which rays intersect
  - Camera center C, through which all rays pass
  - Focal length f, distance from I to C

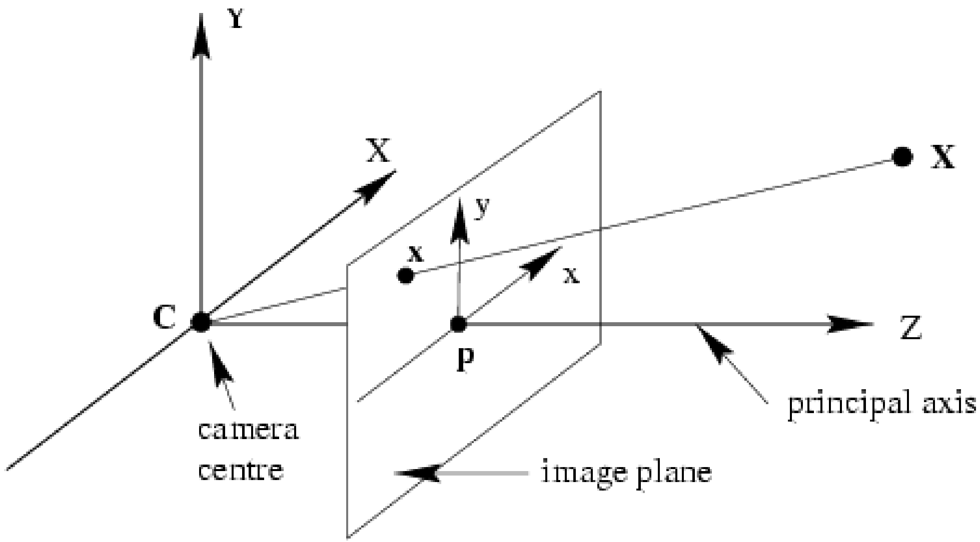


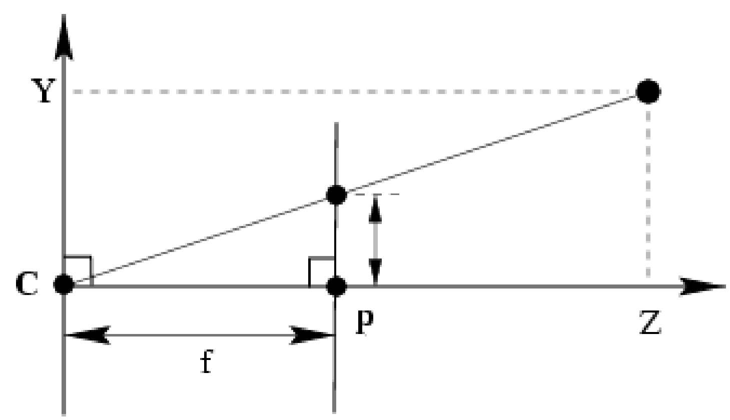


Pinhole Camera Projection

- Point (X,Y,Z) in space and image (x,y) in I
  - Simplified case
    - C at origin in space
    - I perpendicular to Z axis

x=fX/Z (x/f=X/Z)   y=fY/Z (y/f=Y/Z)





Homogeneous Coordinates

- Geometric intuition useful but not well suited to calculation
  - Projection not linear in Euclidean plane but is in projective plane (homogeneous coords)
- For a point (x,y) in the plane
  - Homogeneous coordinates are $(\alpha x, \alpha y, \alpha)$ for any nonzero α (generally use $\alpha=1$)
    - Overall scaling unimportant

      $(X,Y,W) = (\alpha X, \alpha Y, \alpha W)$
  - Convert back to Euclidean plane

    $(x,y) = (X/W, Y/W)$

Lines in Homogeneous Coordinates

- Consider line in Euclidean plane

  $ax+by+c = 0$

- Equation unaffected by scaling so

  $aX+bY+cW = 0$

  $u^T p = p^T u = 0$ (point on line test, dot product)

  – Where $u = (a,b,c)^T$ is the line

  – And $p = (X,Y,W)^T$ is a point on the line u

  – So points and lines have same representation in projective plane (i.e., in homog. coords.)

  – Parameters of line

    • Slope –a/b, x-intercept –c/a, y-intercept –c/b

Lines and Points

- Consider two lines

  $a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$

  – Can calculate their intersection as $(b_1c_2-b_2c_1/a_1b_2-a_2b_1,\ a_2c_1-a_1c_2/a_1b_2-a_2b_1)$
- In homogeneous coordinates

  $u_1=(a_1,b_1,c_1)$ and $u_2=(a_2,b_2,c_2)$

  – Simply cross product $p = u_1 \times u_2$

    • Parallel lines yield point not in Euclidean plane
- Similarly given two points

  $p_1=(X_1,Y_1,W_1)$ and $p_2=(X_2,Y_2,W_2)$

  – Line through the points is simply $u = p_1 \times p_2$

Collinearity and Coincidence

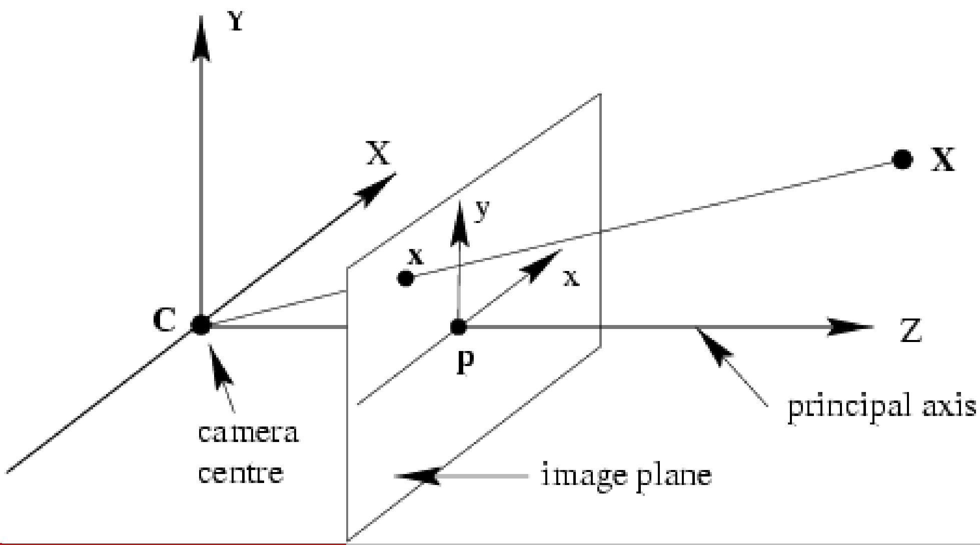
- Three points collinear (lie on same line)
  - Line through first two is $p_1 \times p_2$
  - Third point lies on this line if $p_3^T(p_1 \times p_2) = 0$
  - Equivalently if $\det[p_1\ p_2\ p_3] = 0$
- Three lines coincident (intersect at one point)
  - Similarly $\det[u_1\ u_2\ u_3] = 0$
  - Note relation of determinant to cross product $u_1 \times u_2 = (b_1c_2 - b_2c_1,\ a_2c_1 - a_1c_2,\ a_1b_2 - a_2b_1)$
- Compare to geometric calculations

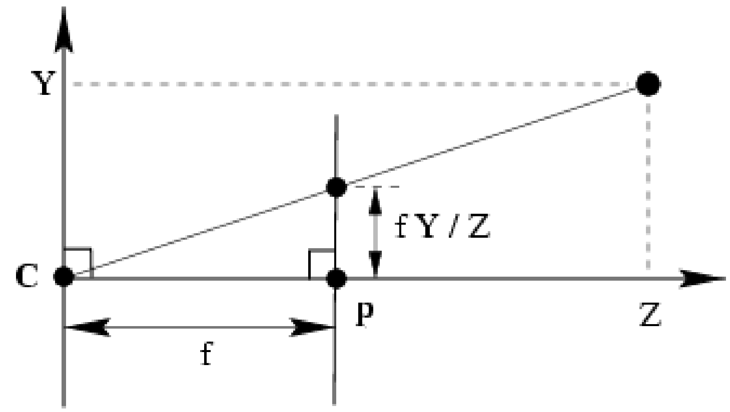
Back to Simplified Pinhole Camera

- Geometrically saw x=fX/Z, y=fY/Z

$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

3x4 Projection Matrix

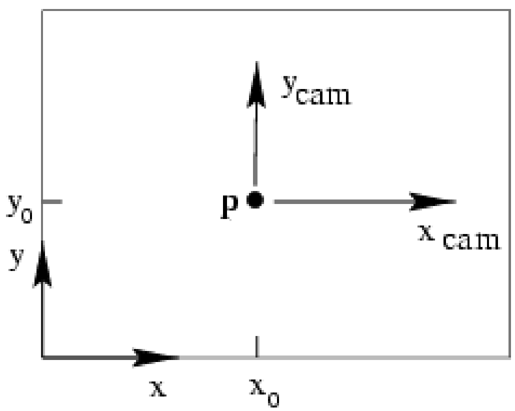




Principal Point Calibration

- Intersection of principal axis with image plane often not at image origin

$$\begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$



$$K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$

(Intrinsic) Calibration matrix

CCD Camera Calibration

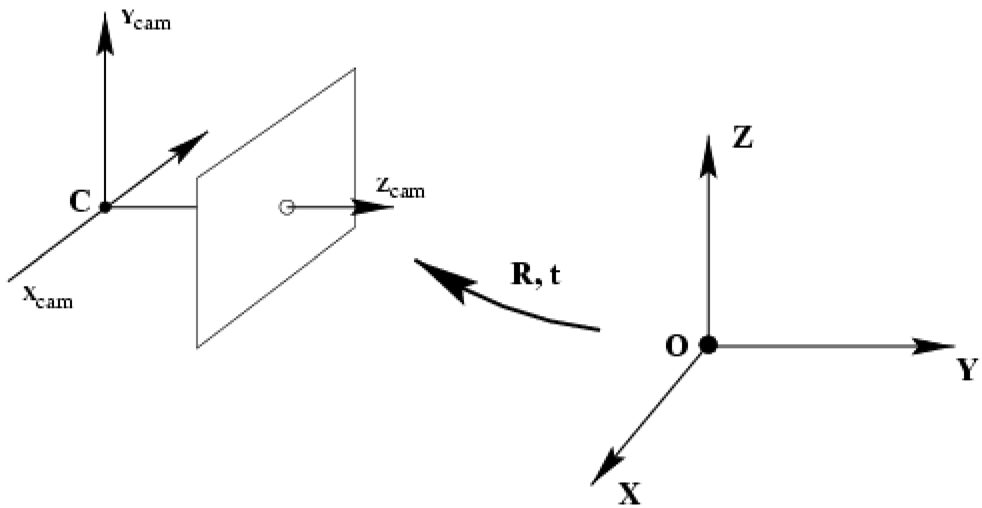
- Spacing of grid points
  - Effectively separate scale factors along each axis composing focal length and pixel spacing

$$K = \begin{bmatrix} m_{\mathbf{x}}f & & p_x \\ & m_{\mathbf{y}}f & p_y \\ & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha & & p_x \\ & \beta & p_y \\ & & 1 \end{bmatrix}$$

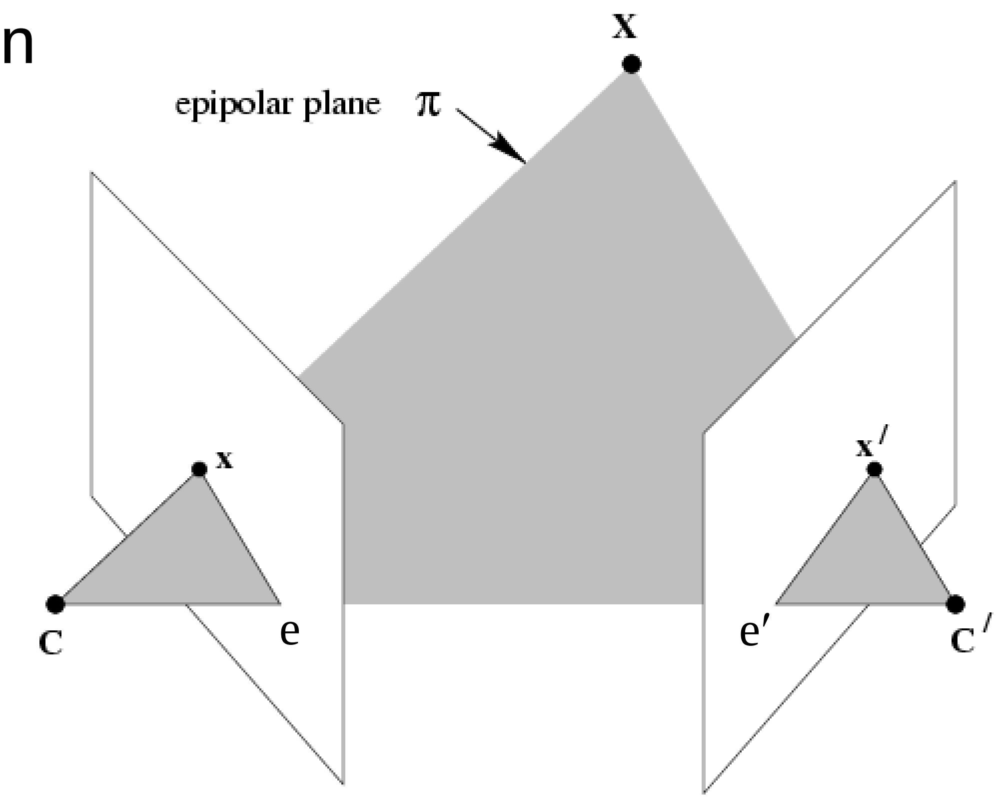
Camera Rigid Motion

- Projection P=K[R|t]
  - Camera motion: alignment of 3D coordinate systems
  - Full extrinsic parameters beyond scope of this course, see “Multiple View Geometry” by Hartley and Zisserman



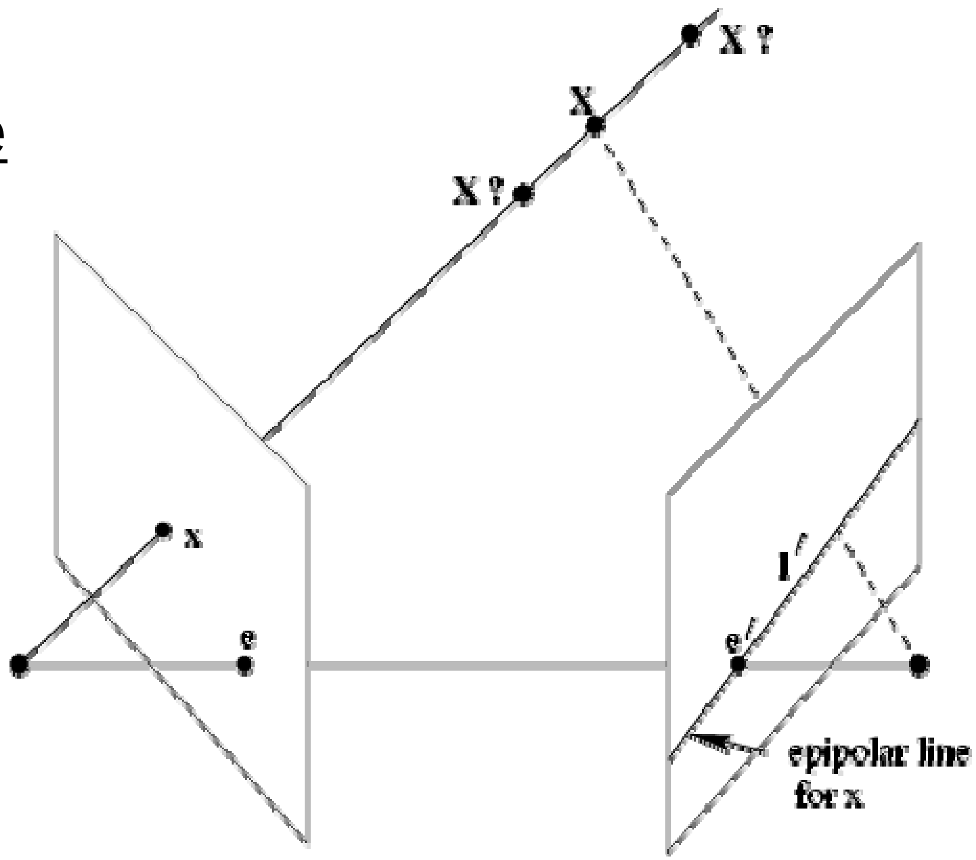
Two View Geometry

- Point X in world and two camera centers C, C' define the epipolar plane
  - Images x,x' of X in two image planes lie on this plane
  - Intersection of line CC' with image planes define special points called epipoles, e,e'



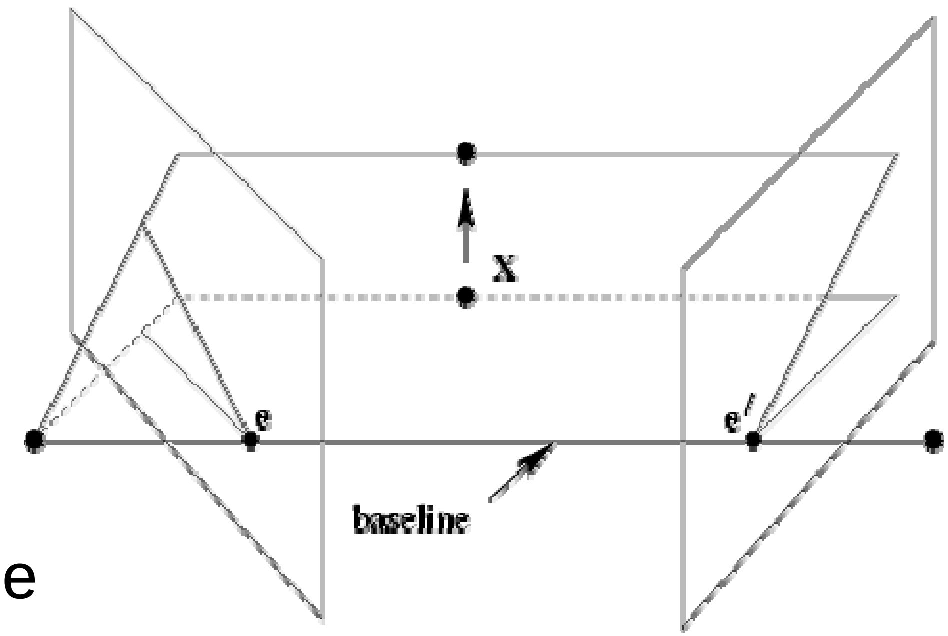
Epipolar Lines

- Set of points that project to x in I define line ℓ' in I'
  - Called <u>epipolar line</u>
  - Goes through epipole e'
  - A point x in I thus maps to a point on ℓ' in I'
    - Rather than to a point anywhere in I

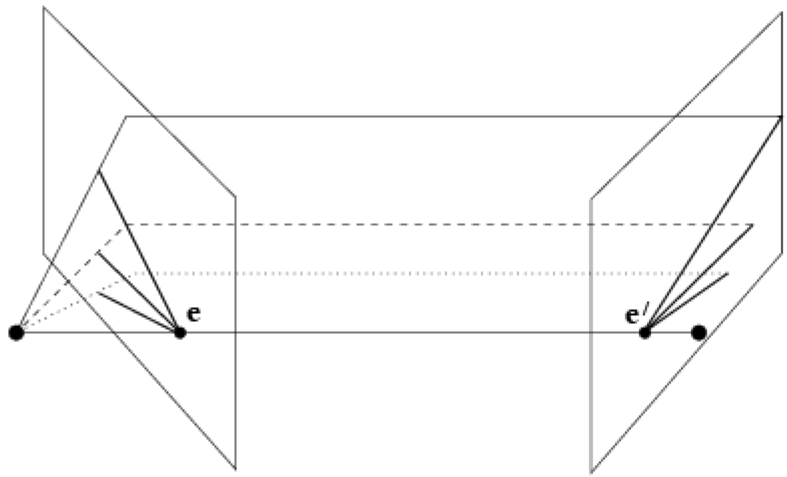


Epipolar Geometry

- Two-camera system defines one parameter family (pencil) of planes through baseline CC'
  - Each such plane defines matching epipolar lines in two image planes
  - One parameter family of lines through each epipole
  - Correspondence between images

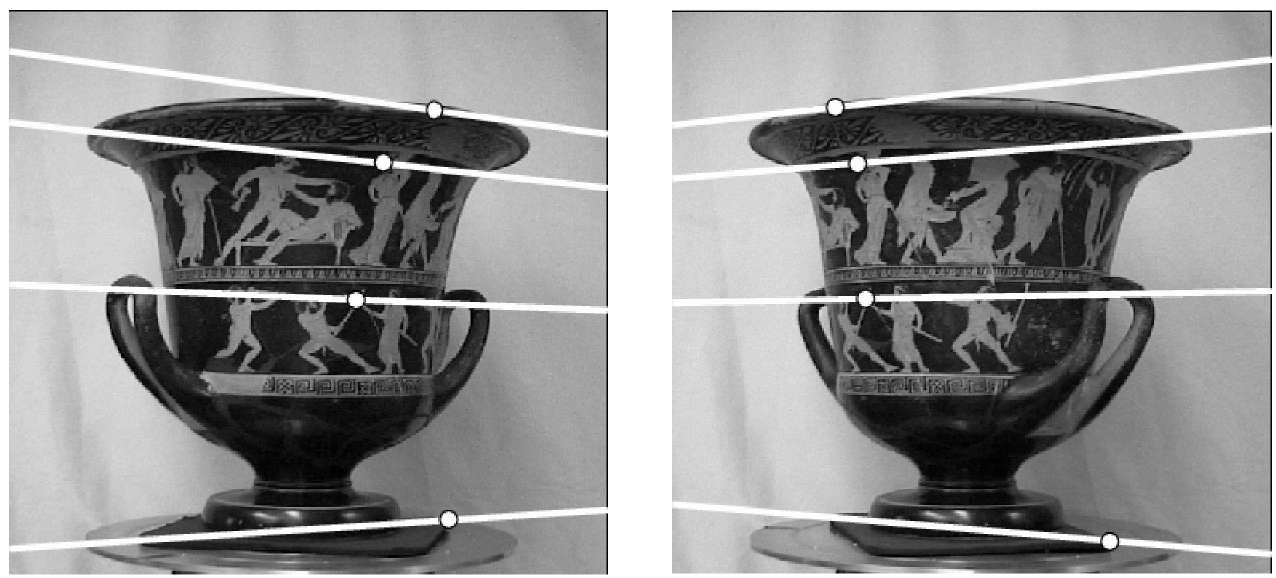


Converging Stereo Cameras



Corresponding points lie on corresponding epipolar lines

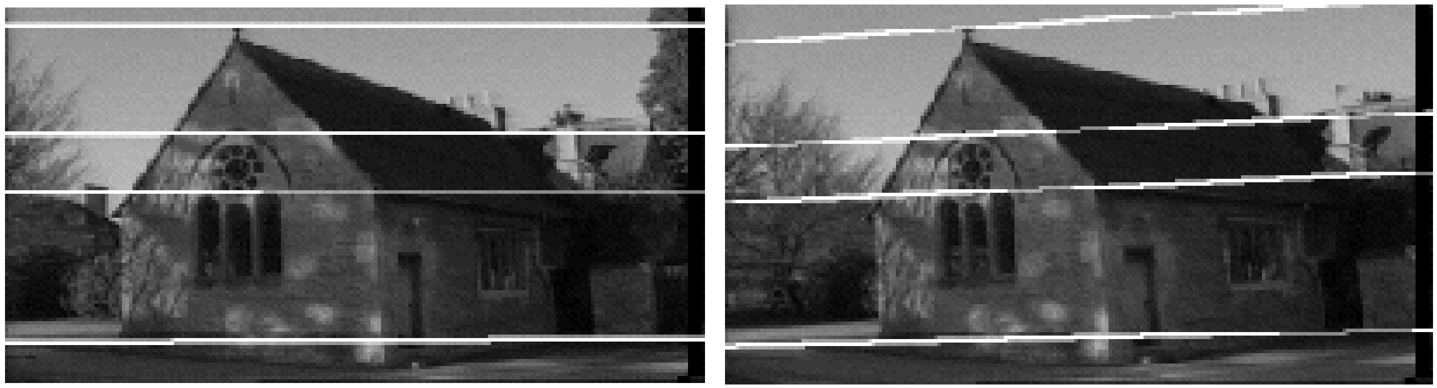
Known camera geometry so 1D not 2D search!



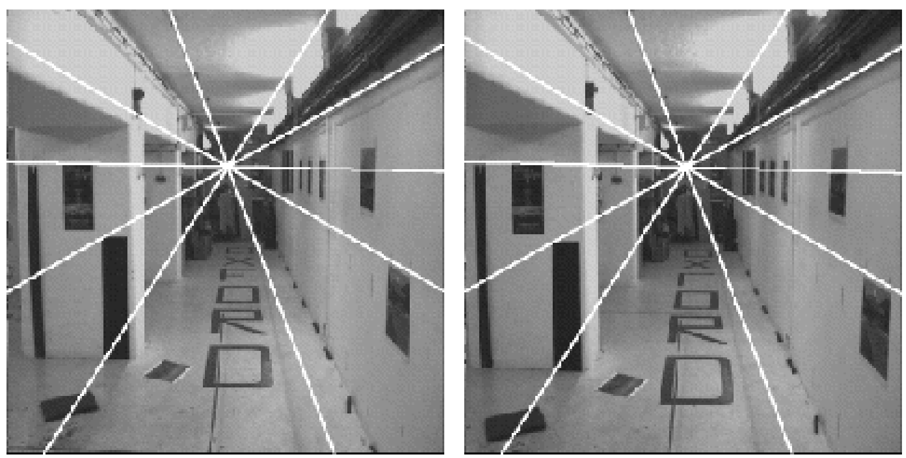
Motion Examples

- Epipoles in direction of motion

Parallel to Image Plane



Forward



Fundamental and Essential Matrix

- Linear algebra formulation of the epipolar geometry
- Fundamental matrix, F, maps point x in I to corresponding epipolar line ℓ' in I'

$$\ell' = Fx$$

  - Determined for particular camera geometry
    - For stereo cameras only changes if cameras move with respect to one another
- Essential matrix, E, when camera calibration (intrinsic parameters) known
  - See slides 9 and 10

Fundamental Matrix

- Epipolar constraint

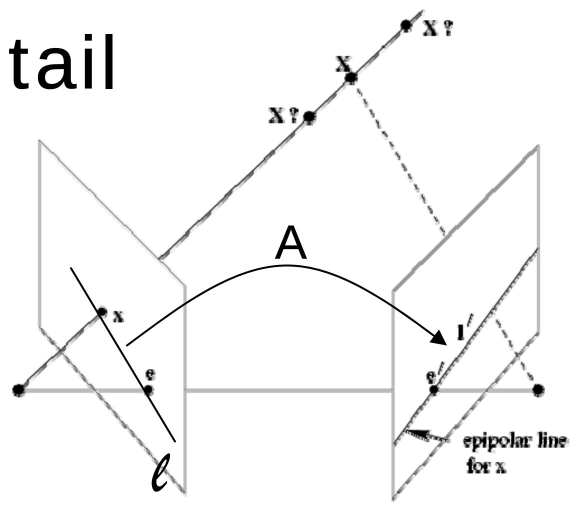
$$x'^T F x = x'^T \ell' = 0$$

  – Thus from enough corresponding pairs of points in the two images can solve for F
    - However not as simple as least squares minimization because F not fully general matrix

- Consider form of F in more detail

$$x \xrightarrow{L} \ell \xrightarrow{A} \ell'$$

$$F = AL$$



Form of Fundamental Matrix

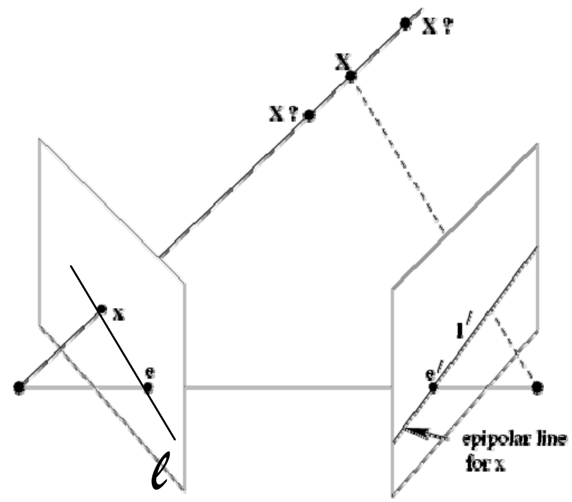
- L: $x \rightarrow \ell$
  - Epipolar line ℓ goes through x and epipole e
  - Epipole determines L

$$\ell = x \times e$$

$$\ell = Lx \qquad \text{(rewriting cross product)}$$

  - If e=(u,v,w)

$$L = \begin{bmatrix} 0 & w & -v \\ -w & 0 & u \\ v & -u & 0 \end{bmatrix}$$



  - L is rank 2 and has 2 d.o.f.

Form of Fundamental Matrix

- A: $\ell \rightarrow \ell'$
  - Constrained by 3 pairs of epipolar lines

    $$\ell'_i = A\,\ell_i$$

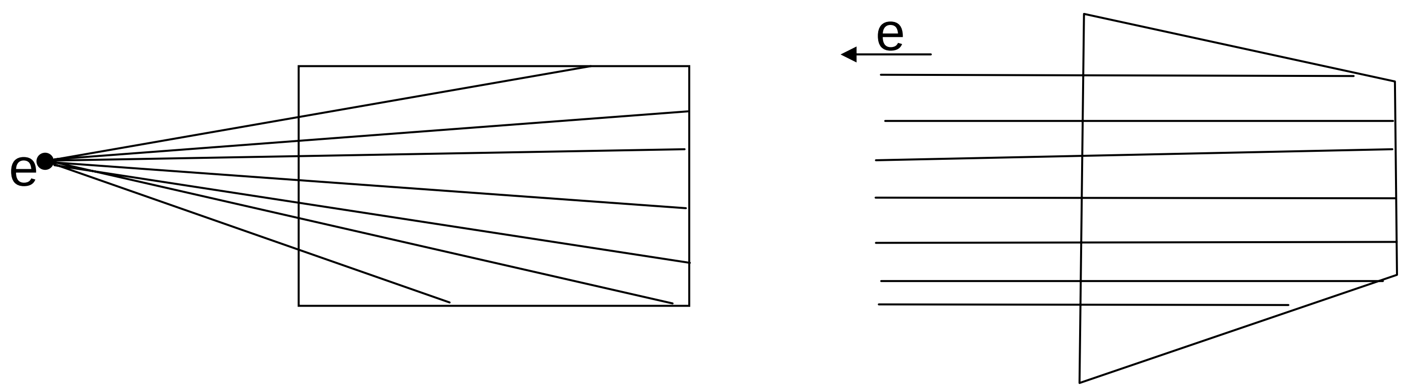
  - Note only 5 d.o.f.
    - First two line correspondences each provide two constraints
    - Third provides only one constraint as lines must go through intersection of first two
- F=AL rank 2 matrix with 7 d.o.f.
  - As opposed to 8 d.o.f. in 3x3 homogeneous system

Properties of F

- Unique 3x3 rank 2 matrix satisfying $x'^T F x = 0$ for all pairs x,x'
  - Constrained minimization techniques can be used to solve for F given point pairs
- F has 7 d.o.f.
  - 3x3 homogeneous (9-1=8), rank 2 (8-1=7)
- Epipolar lines $\ell' = Fx$ and reverse map $\ell = F^T x'$
  - Because also $(Fx)^T x' = 0$ but then $x^T (F^T x') = 0$
- Epipoles $e'^T F = 0$ and $Fe = 0$
  - Because $e'^T \ell' = 0$ for any ℓ'; Le=0 by construction

Stereo (Epipolar) Rectification

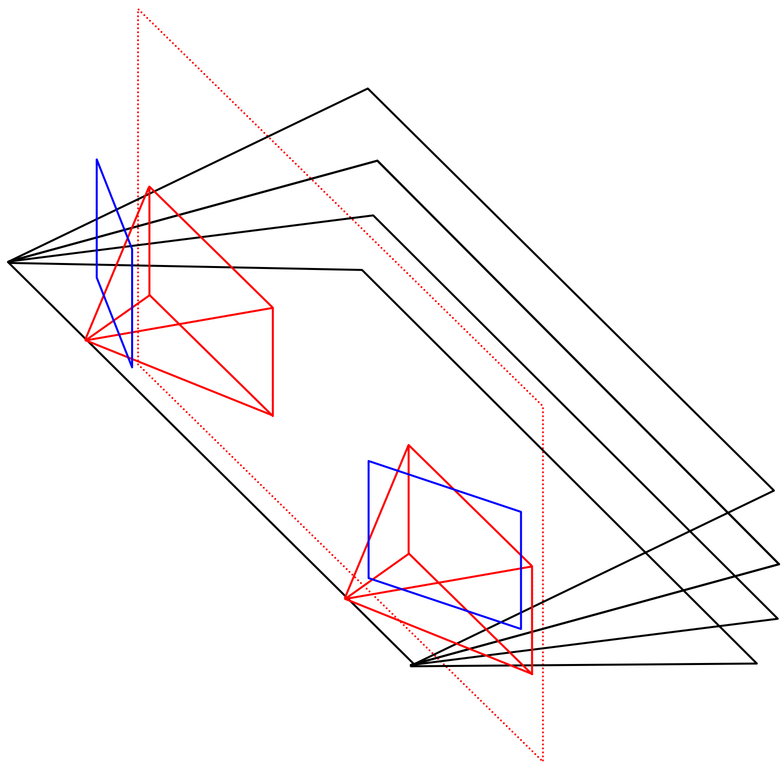
- Given F, simplify stereo matching problem by warping images
  - Shared image plane for two cameras
  - Epipolar lines parallel to x-axis
    - Epipole at (1,0,0)
    - Corresponding scan lines of two images



  - OpenCV: calibration and rectification

Planar Rectification

- Move epipoles to infinity
  – Poor when epipoles near image

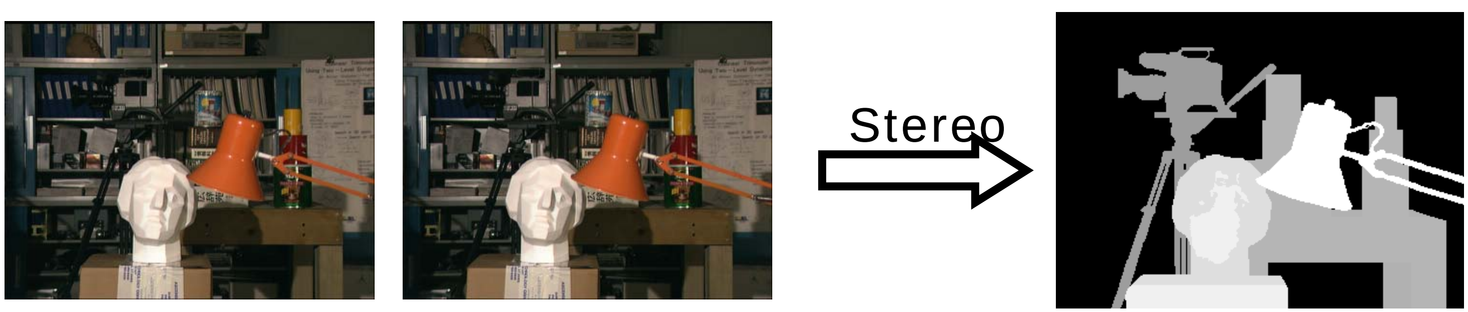


Stereo Matching

- Seek corresponding pixels in I, I'
  - Only along epipolar lines
- Rectified imaging geometry so just horizontal disparity D at each pixel

$$I'(x',y') = I(x + D(x,y), y)$$

- Best methods minimize energy based on matching (data) and discontinuity costs



Plane Homography

- Projective transformation mapping points in one plane to points in another
- In homogeneous coordinates

$$\begin{pmatrix} aX+bY+cW \\ dX+eY+fW \\ gX+hY+iW \end{pmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{pmatrix} X \\ Y \\ W \end{pmatrix}$$

- Maps four (coplanar) points to any four
  - Quadrilateral to quadrilateral
  - Does not preserve parallelism

Contrast with Affine

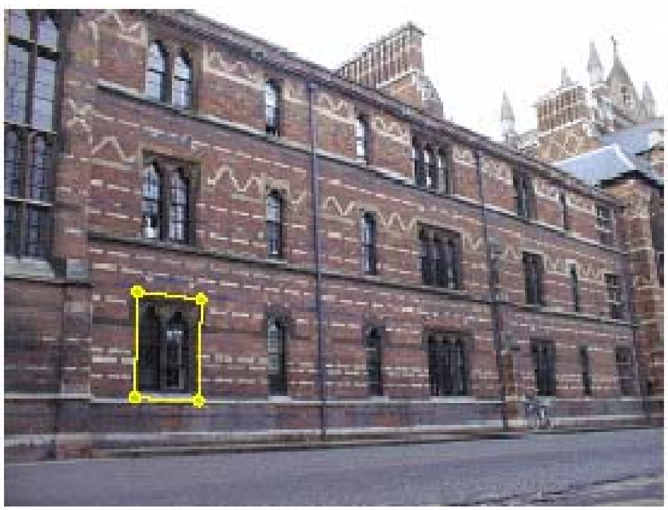
- Can represent in Euclidean plane x'=Lx+t
  - Arbitrary 2x2 matrix L and 2-vector t
  - In homogeneous coordinates

$$\begin{pmatrix} aX+bY+cW \\ dX+eY+fW \\ W \end{pmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & i \end{bmatrix} \begin{pmatrix} X \\ Y \\ W \end{pmatrix}$$

- Maps three points to any three
  - Maps triangles to triangles
  - Preserves parallelism

Homography Example

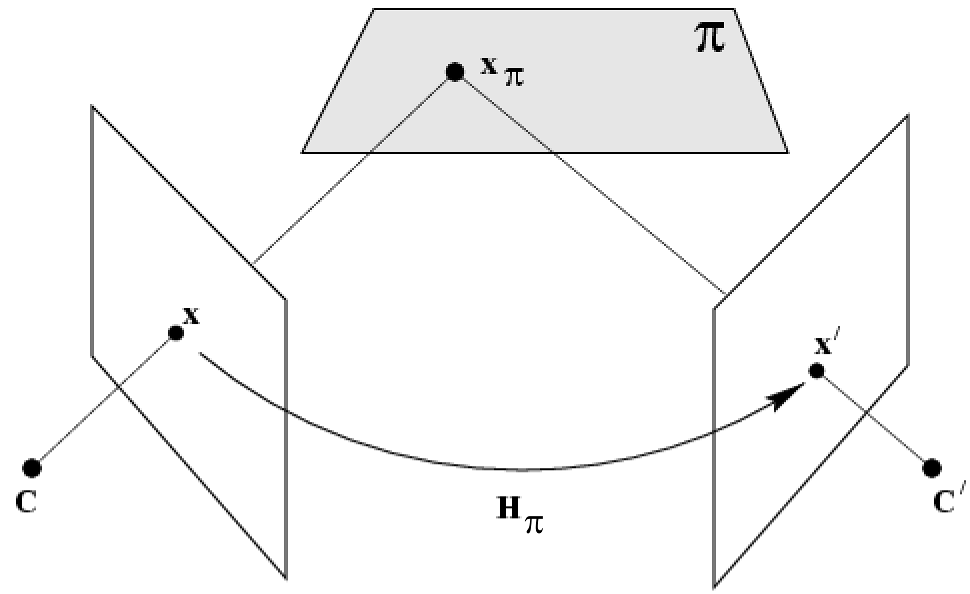
- Changing viewpoint of single view
  - Correspondences in observed and desired views
  - E.g., from 45 degree to frontal view
    - Quadrilaterals to rectangles
  - Variable resolution and non-planar artifacts



Homography and Epipolar Geometry

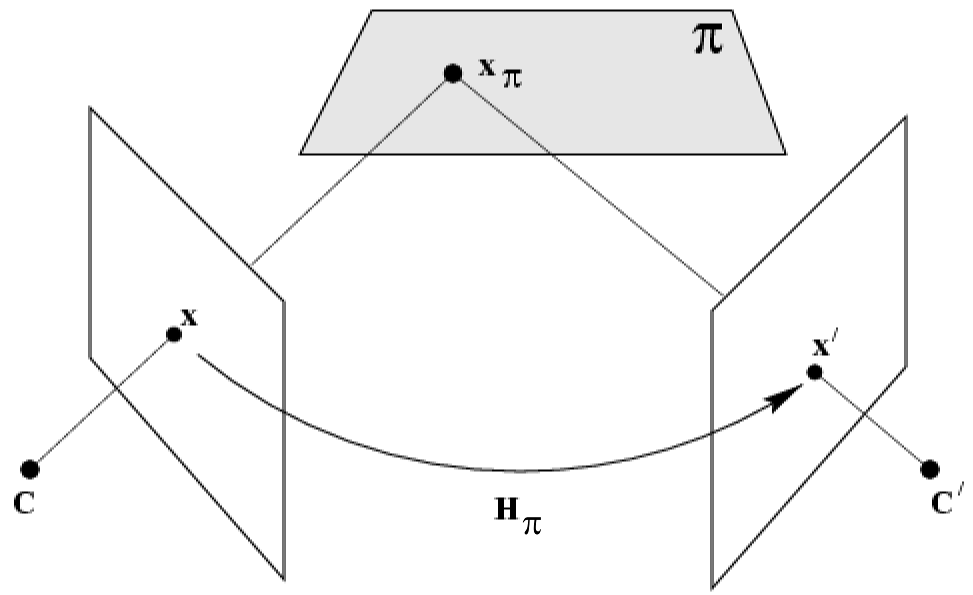
- Plane in space π induces homography H between image planes

  $x' = H_\pi x$ for point X on π, x on I, x' on I'



Obeys Epipolar Geometry

- Given F,H_π no search for x' (points on π)
- Maps epipoles, e'= H_πe

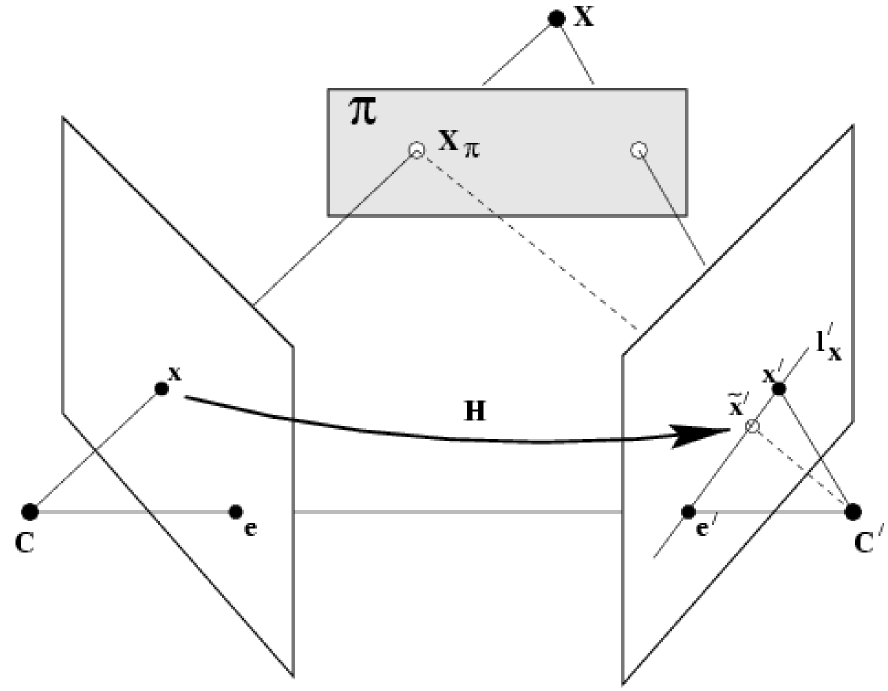


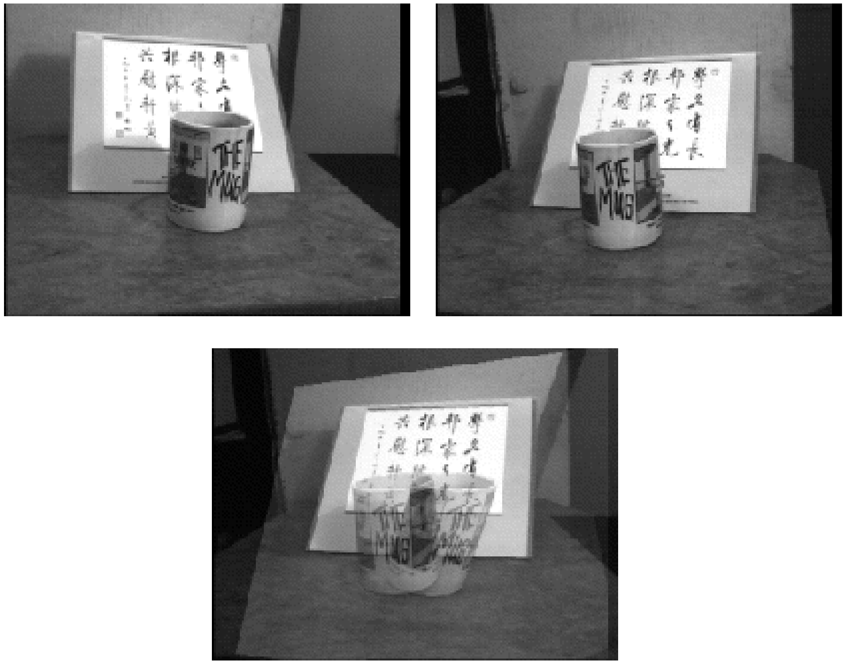
Computing Homography

- Correspondences of four points that are coplanar in world (no need for F)
  - Substantial error if not coplanar
- Fundamental matrix F and 3 point correspondences
  - Can think of pair e,e’ as providing fourth correspondence
- Fundamental matrix plus point and line correspondences
- Improvements
  - More correspondences and least squares
  - Correspondences farther apart

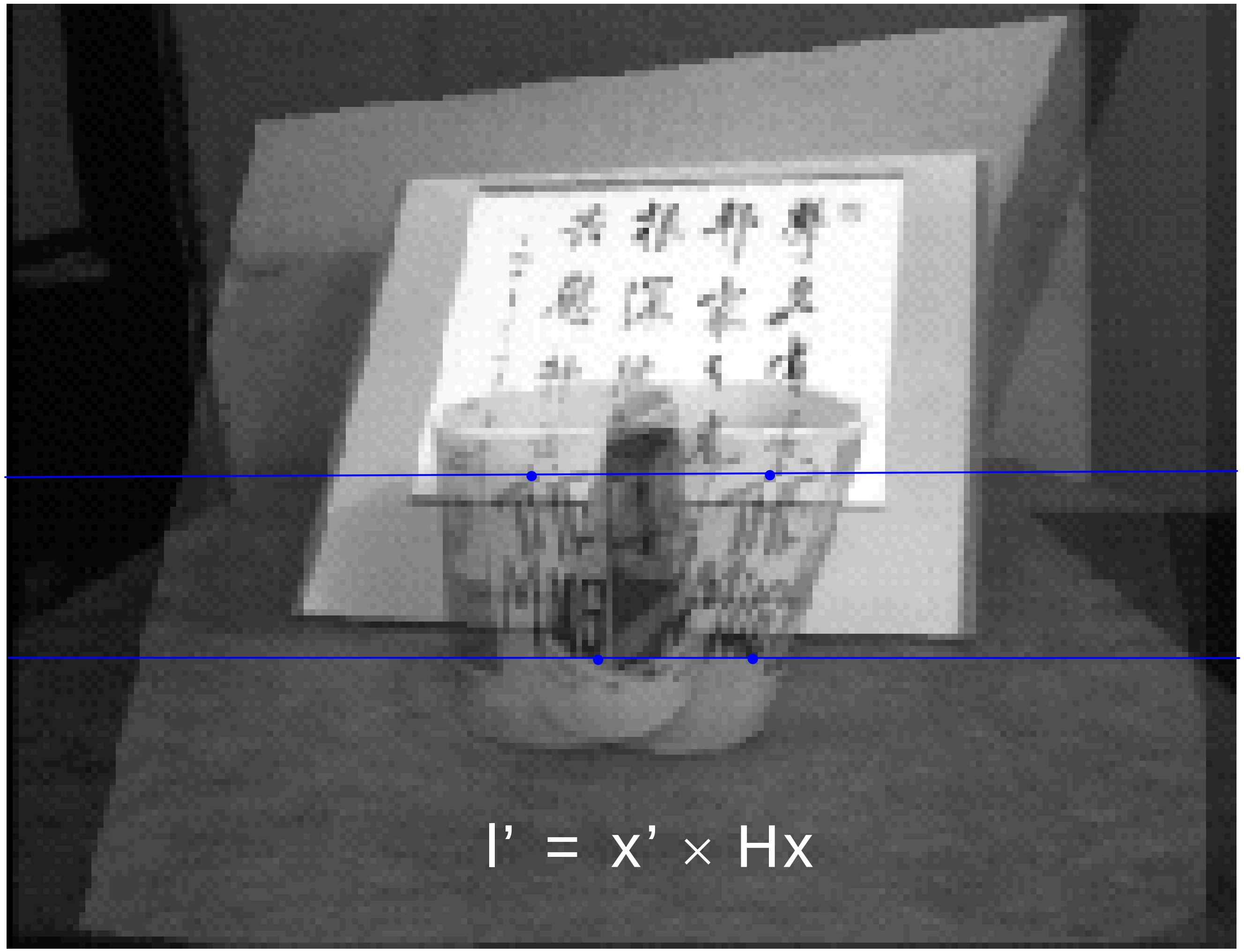
Plane Induced Parallax

- Determine homography of a plane
  - Remaining differences reflect depth from plane
  - Flat surfaces like in sporting events





Plane + Parallax Correspondences

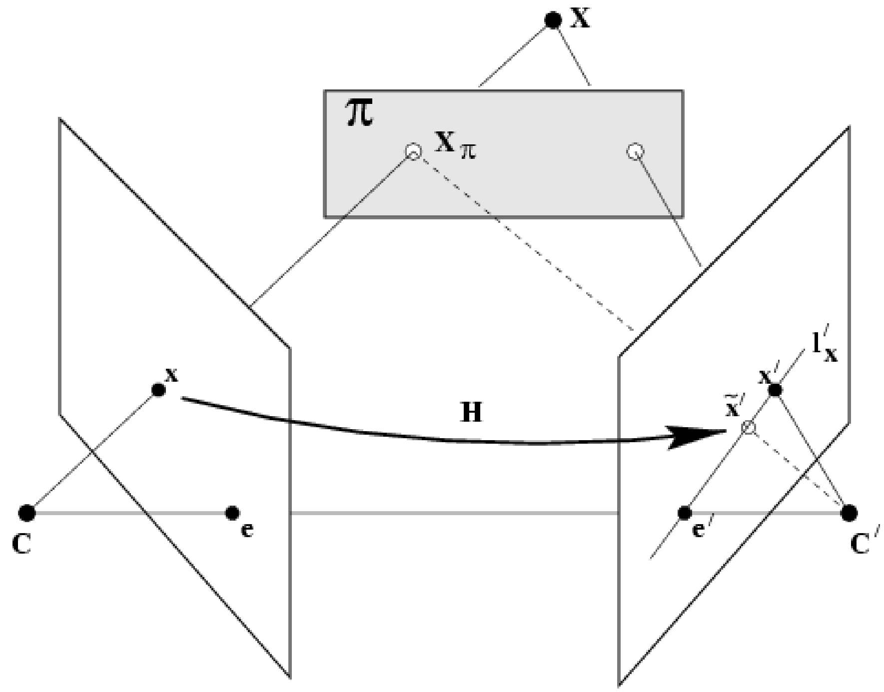


Plane + Parallax

Vaish et al CVPR04

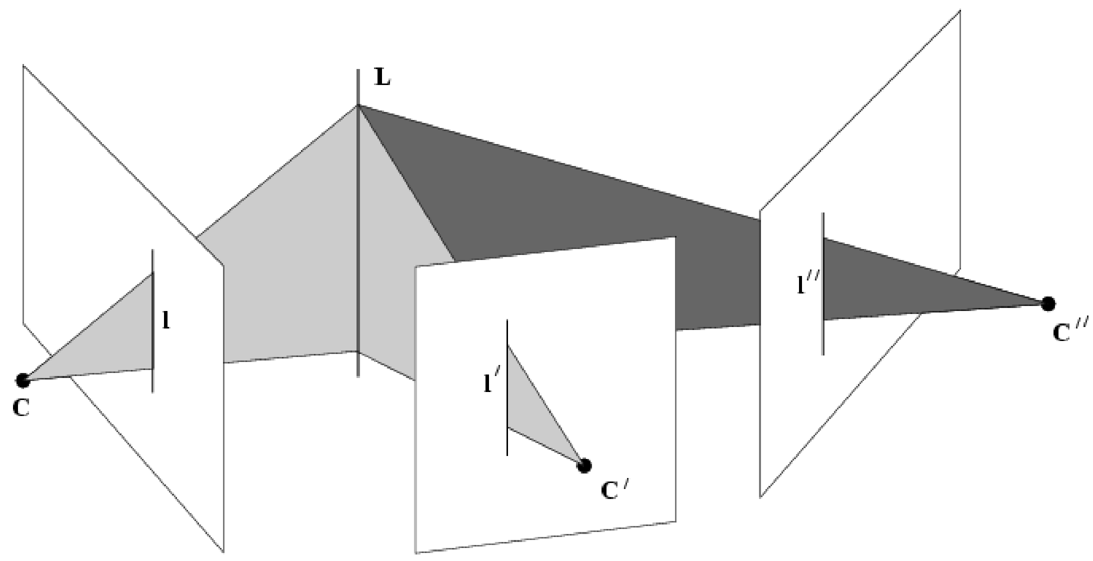
Projective Depth

- Distance between $H_{\pi}x$ and x' (along l') proportional to distance of X from plane π
  - Sign governs which side of plane



Multiple Cameras

- Similarly extensive geometry for three cameras
  - Known as tri-focal tensor
    - Beyond scope of this course



- Three lines
- Three points
- Line and 2 points
- Point and 2 lines