CS664 Lecture #14: Mutual information, multi-camera stereo

Some material taken from:

- **Steve Seitz, University of Washington**

http://www.cs.washington.edu/homes/seitz/

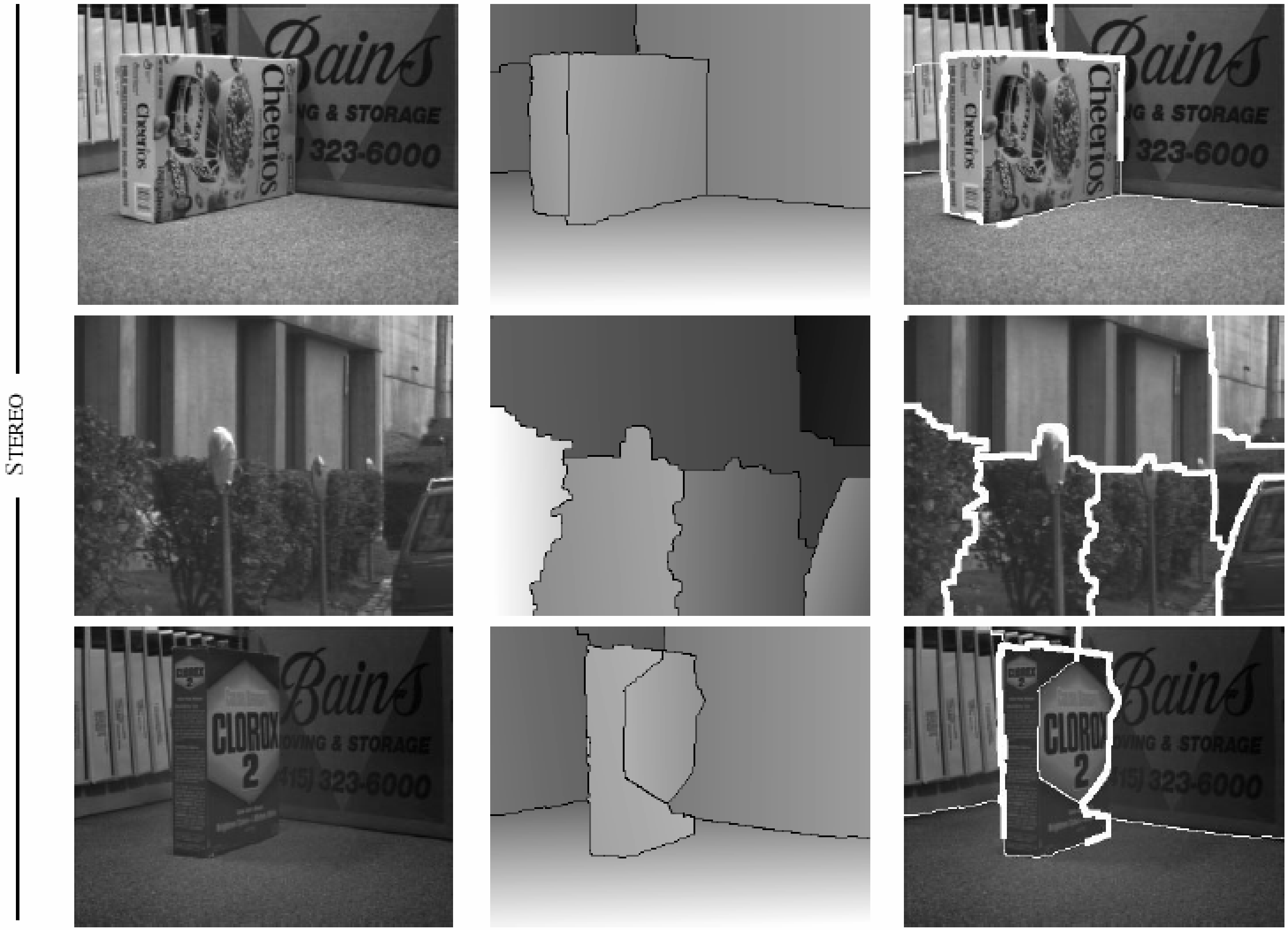
Announcements

- PS2 will be out next week
  - Energy minimization with graph cuts and ICM

Recap

- Expectation-maximization allows you to solve many “chicken and egg” problems
- EM plus graph cuts can solve slanted surface stereo quite well
  - Easy to generalize to curved surfaces

Birchfield-Tomasi Stereo results



Beyond simple stereo

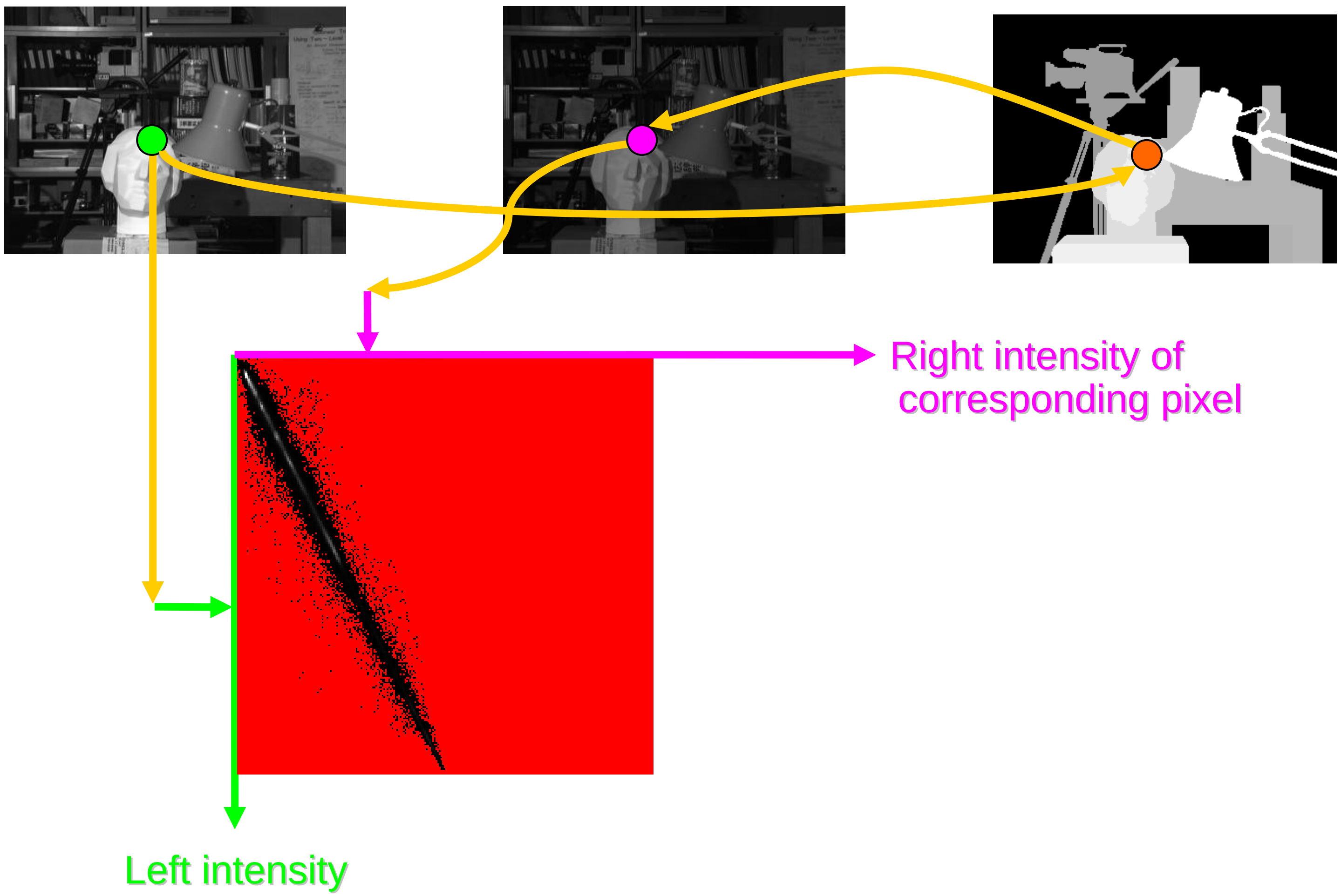
- Graph cuts work incredibly well for stereo
  - Original successful example
- We write the energy function as

$$\sum_{p \in \mathcal{P}} D_p(f_p) + \sum_{p,q \in \mathcal{N}} T(f_p \neq f_q)$$

$$D_p(f_p) = (I(p) - I'(p + f_p))^2$$

- What assumptions are we making?
  - Frontoparallel lambertian surfaces

Joint intensity histogram



Joint histogram properties

- For lambertian surfaces at the correct disparity, most entries are on the line of unit slope
  - What determines how wide the “band” is?
  - What are the dimensions? The size of the largest bucket?
- What happens if the disparity map is random?
  - What is the sum of the squared distances to the line with unit slope?
- Assignment costs show up in histogram!

More properties

- Suppose that one image is twice as bright as the other
  – What happens in the joint histogram?
- What about an arbitrary mapping between intensities?
  – Think about a CT versus MR image
- One place non-lambertian scenes arise is in multi-camera stereo
  – Next topic

Measuring sparseness

- Obvious (unprincipled) method: count the number of empty buckets
- More generally, consider any measure of the form $\sum_{x,y} h(|J(x,y)|)$
- What does this do in terms of counting sparseness if:
  - $h(b) = b$ (linear)?
  - $h(b) > b$ (superlinear)?
  - $h(b) < b$ (sublinear)?

Mutual information

- Related to joint histogram "sparseness"
  - Suppose we have a vector-valued random variable $\langle I_l, I_r \rangle$
  - There is a density associated with this, $p(i_l, i_r)$
  - We can estimate p from our data (how?)
- Joint entropy is $H(I_l, I_r) \equiv \sum_{i_l, i_r} -p(i_l, i_r) \log p(i_l, i_r)$
  - For small values, -b log b increases.
  - It is sub-linear, so H is low if sparse
  - Minimizing H can maximize mutual information

Multimodal Stereo with Graph Cuts

- There is a nice application that combines graph cuts, joint histograms and EM
  - I.e., almost everything in today's lecture!

Multimodal stereo

- Suppose the two cameras are different
  - Internal parameters, or modalities
  - There is some consistent mapping of intensities μ between them
  - At the right disparity, $I_1(p) \cong \mu(I_2(p+d))$

$$\mu(i) = 2i$$

Just do it?

- Problem input has no assignment cost
  - How can we tell how much p likes d?

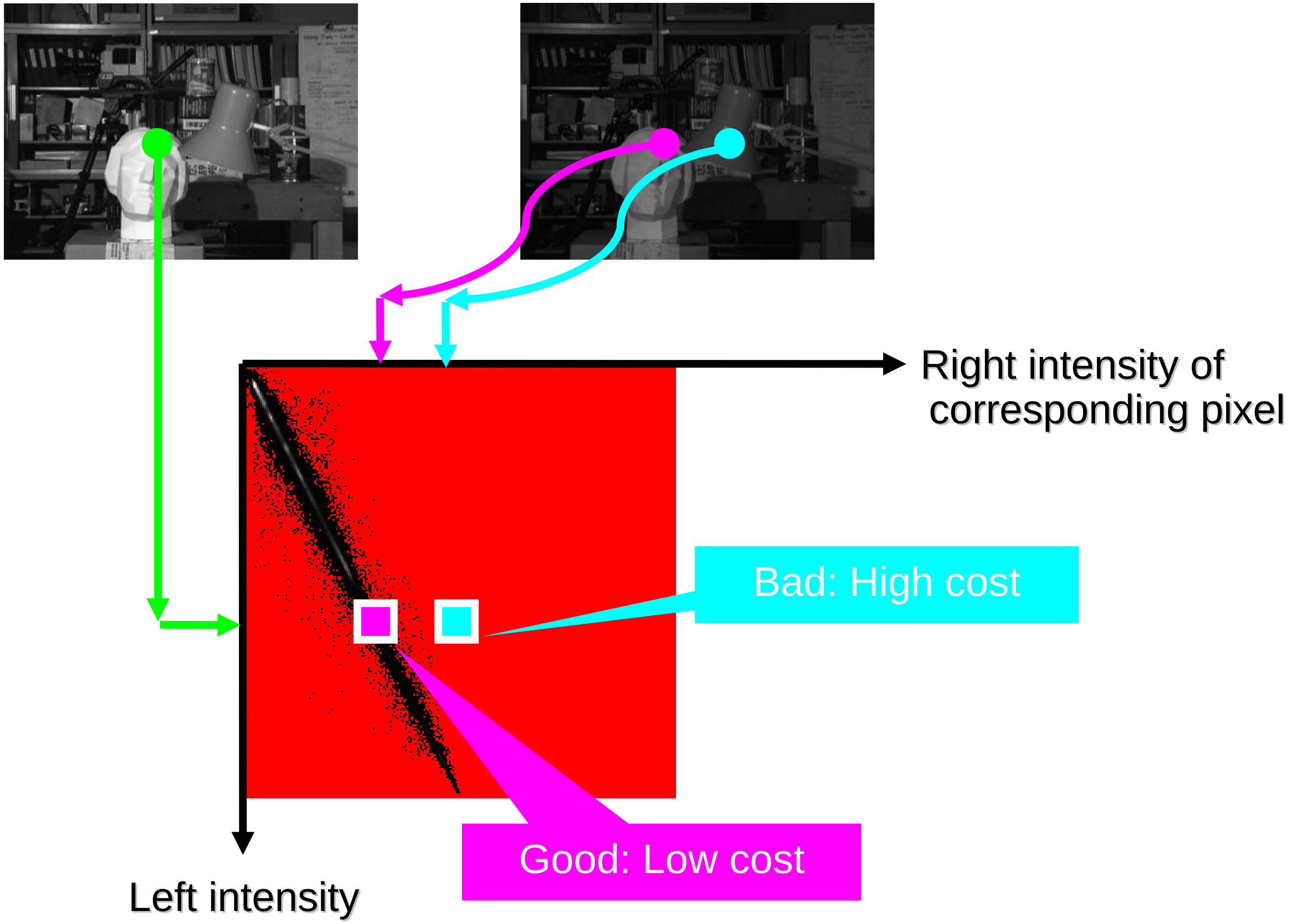
$$D(p,d) = (I_1(p) - \mu(I_2(p+d)))^2$$

  - Depends, obviously, on μ
  - We could compute μ from right f
- Suggests an iterative (EM) approach
  - Alternate between estimating the assignment costs D and the labeling f

EM-style approach

- When f is correct, the joint histogram will be highly “concentrated”
  - And vice-versa
- For a given f, we can construct an assignment cost that tends to make the joint histogram more concentrated
- Iterative EM-style algorithm
  - Find f given the assignments costs
  - Find the assignment costs given f

Assignment costs



Formalizing this

$$\arg\min_{(\cdots f_p, f_q \ldots)} \sum_p D(p, f_p) + \sum_{p,q \in N} V(f_p, f_q)$$

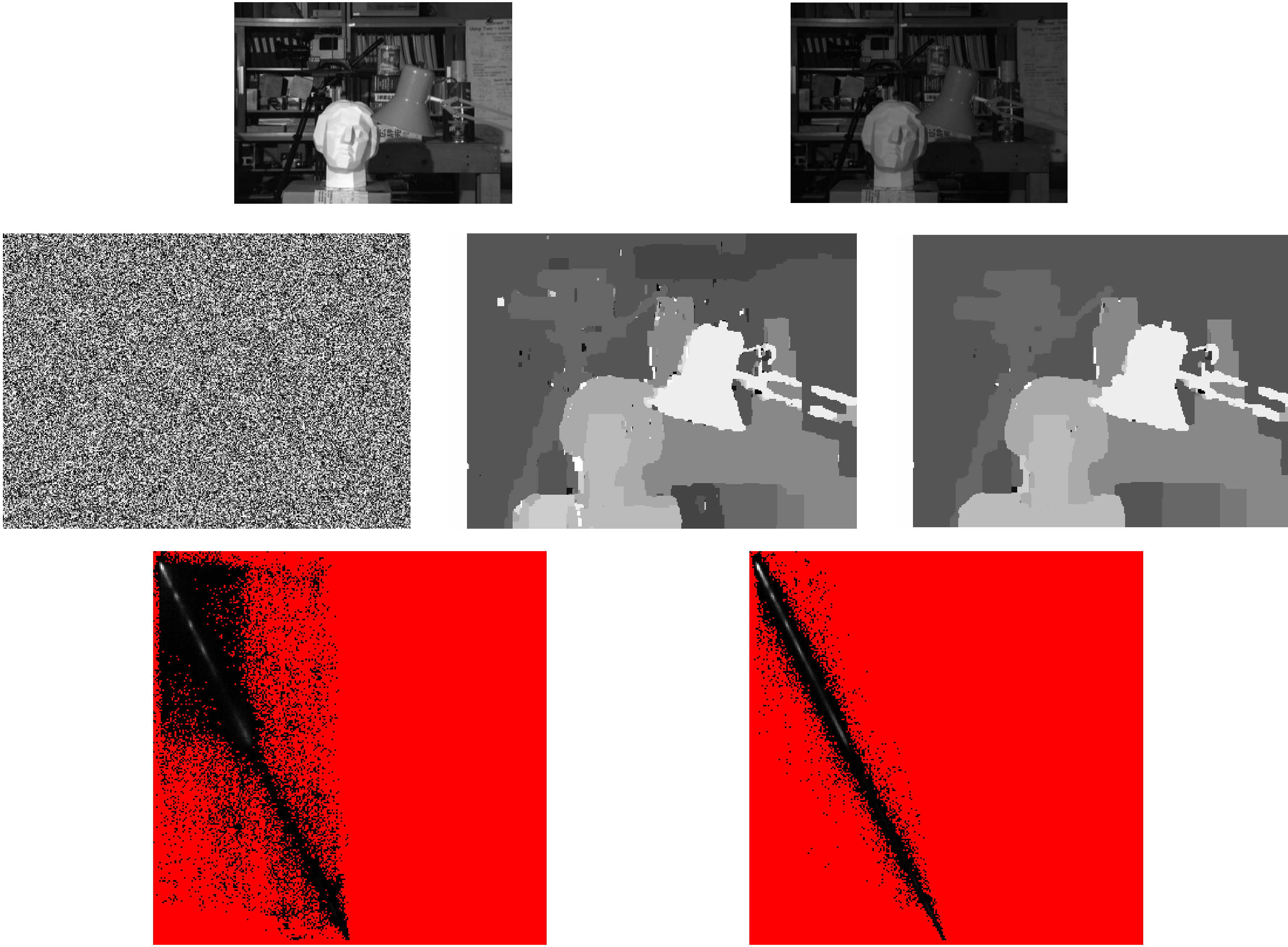
Depends on labeling

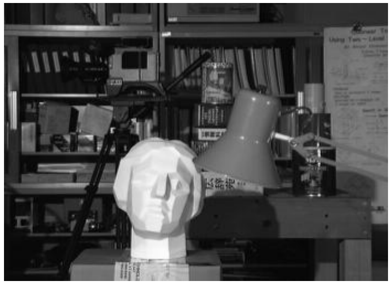
$$f^n = \arg\min_{(\cdots f_p, f_q \ldots)} \sum_p D^n(p, f_p) + \sum_{p,q \in N} V(f_p, f_q)$$

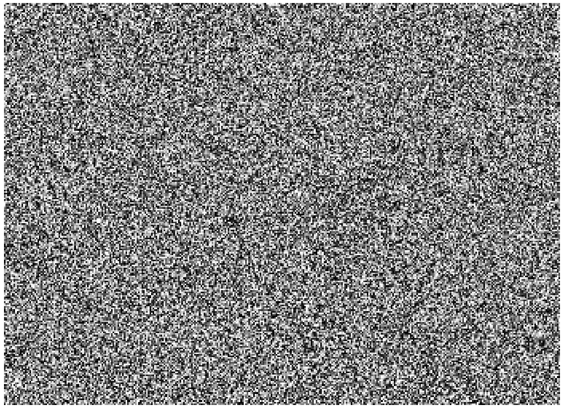
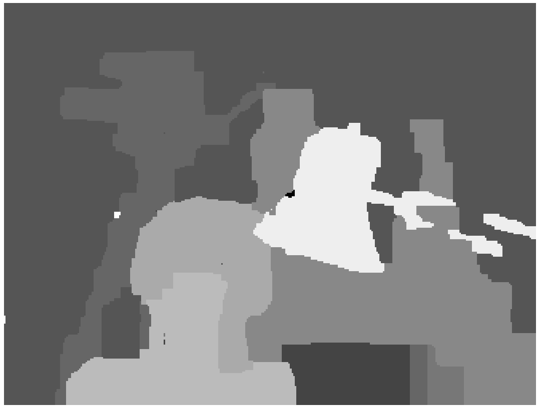
Compute D^{n+1} from f^n

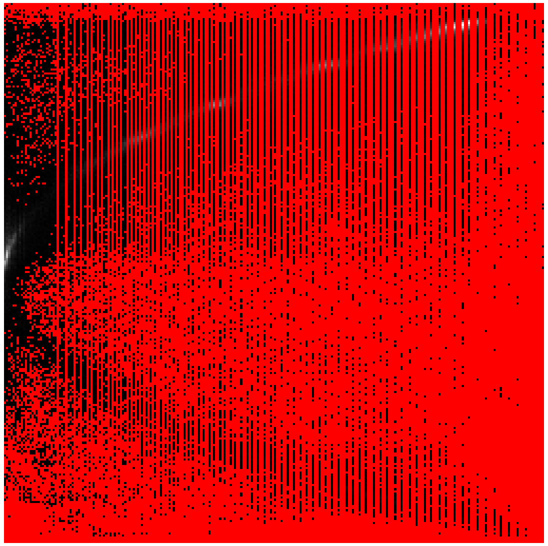
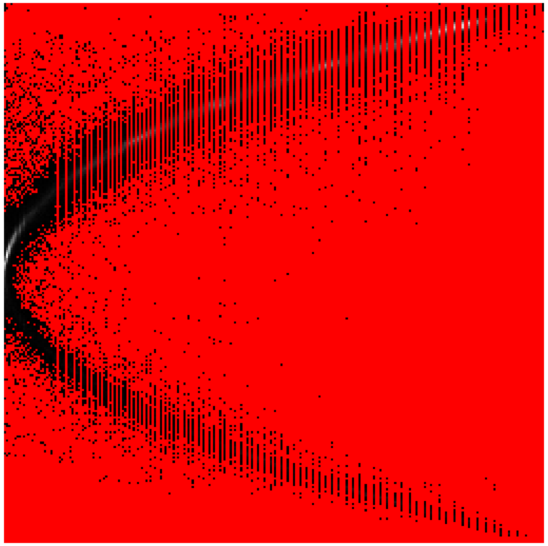
Properties

- Usable with other matching algorithms
- Can handle spatially varying μ
- Need decent D^0, and $D^{n+1} \simeq D^n$
  - For correspondence, easily true (why?)
- Related to Mutual Information
  - With right formula for D

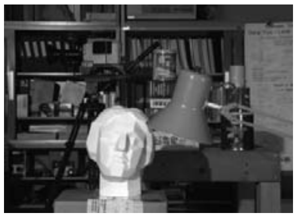
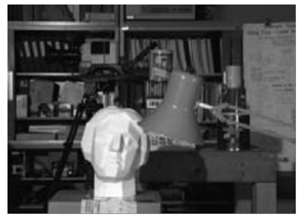


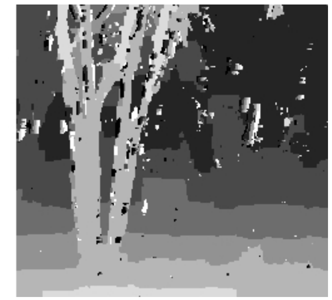
 

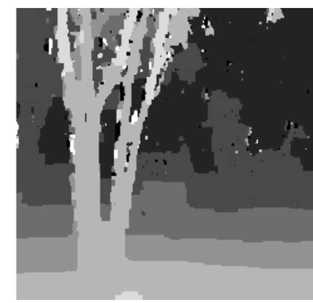
Results with synthetic distortions



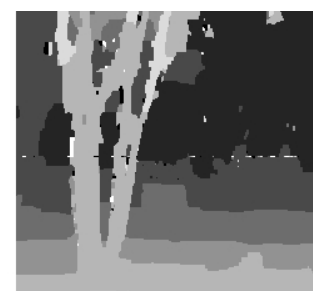
I^0_{Left} $\quad$ I^0_{Right}



$I_{Left} = I^0_{Left} / 2$ $\quad$ $I_{Right} = I^0_{Right}$



$$I_{Left} = I^0_{Left} \qquad I_{Right} = \begin{cases} I^0_{Right} / 2 \\ \left(I^0_{Right} + 1\right) / 2 \end{cases}$$