CS6630 Homework 1

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Consider a point source at the origin, radiating isotropically with power Φ. We saw already that the irradiance of this source at a location $\mathbf{r}$ for a surface with normal $\mathbf{n}$ is

$$E(\mathbf{r}, \mathbf{n}) = \begin{cases} \frac{\Phi}{4\pi} \frac{\cos\theta}{\|\mathbf{r}\|^2} & \text{when } \mathbf{r} \cdot \mathbf{n} \leq 0 \\ 0 & \text{when } \mathbf{r} \cdot \mathbf{n} \geq 0. \end{cases}$$

1. What is the net irradiance $\bar{E}(\mathbf{r}, \mathbf{n})$?
2. Derive from the definition that the scalar irradiance at $\mathbf{r}$ is

   $$\phi(\mathbf{r}) = \frac{\Phi}{4\pi} \frac{1}{\|\mathbf{r}\|^2}$$

   That is, it is the same as the irradiance on a surface facing the source.
3. Derive from the definition that the vector irradiance at $\mathbf{r}$ is

   $$\vec{E}(\mathbf{r}) = \frac{\Phi}{4\pi \|\mathbf{r}\|^3} \mathbf{r}$$

4. Plot a map of the vector irradiance field over a plane that passes through the source.
5. Plot the irradiance on a planar surface one unit away from the point source, as a function of position on the plane (it's radially symmetric so a 1D plot suffices).

Now replace the point source with a spherical emitting surface of radius R centered at the origin, with constant radiance L and total power Φ.

6. Give an expression for L in terms of Φ and R.
7. By integrating the incident light at $\mathbf{r}$, conclude that the irradiance $E(\mathbf{r}, \mathbf{n})$ is the same as the point source when the normal points directly at the origin.
8. Sketch the Nusselt analog of the incident hemisphere for the computation of irradiance when the normal does and does not point directly at the origin.

9. Show that the net irradiance $\bar{E}(\mathbf{r}, \mathbf{n})$, and therefore the vector irradiance also, is in fact the same as the point source for all $\mathbf{n}$, for all $\mathbf{r}$ outside the sphere.

10. For what $\mathbf{r}$ and $\mathbf{n}$ does the net irradiance differ materially from the irradiance? Here "materially" means that flipping normal vectors doesn't suffice to make them the same.

11. By integrating the incident light at $\mathbf{r}$, conclude that the scalar irradiance is

$$\phi(\mathbf{r}) = 2\pi\, L \left(1 - \frac{\sqrt{\|\mathbf{r}\|^2 - R^2}}{\|\mathbf{r}\|} \right).$$

This does not match the point source, but show that it does agree in the limit of $\|\mathbf{r}\| \gg R$.

Finally, consider a rectangular source of width w and height h that is standing perpendicularly on a planar surface.

12. First consider the limit of large h. Draw the Nusselt analog for irradiance due to an infinitely tall rectangular source at a point a distance r away from the source along the source's line of symmetry. What is the irradiance for a point right at the foot of the source?

13. Write a formula for the irradiance as a function of r and w.

14. Now generalize to the case of finite h. Draw the Nusselt analog, write a formula for the irradiance as a function of r, h, and w, and plot it as a function of r for $w = 1$ and $h = 1$, 2, and 5. *Hint:* The shape whose area needs to be subtracted is a nonuniformly scaled copy of the same type of shape you are already dealing with.

You may be interested to investigate how the calculation you have done is a special case of Lambert's formula for the irradiance due to a polygonal source, which is described in the paper "The Irradiance Jacobian for Partially Occluded Polyhedral Sources" by Jim Arvo in SIGGRAPH 1994.