CS6630 Homework 1

Analytic Shadows

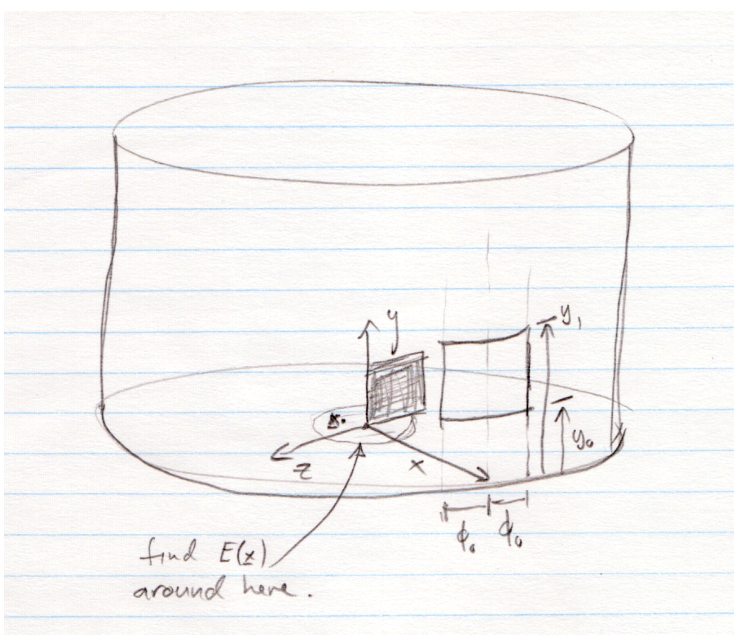
Problem 1.

For this problem we'll look at the shadow cast in a slightly special situation that I think you'll see the reason for. Consider a large cylindrical room with a single rectangular window looking out at an overcast sky (hence the window acts like an area source with uniform radiance). In the middle of the room there is an opaque vertical rectangle casting a shadow on a white sheet of paper. Everything but the paper is painted black, so that there are no inter-reflections. To describe the geometry, set up a coordinate system at the corner of the blocker, with y pointing up and ϕ increasing counterclockwise from the $+x$ axis; then the window goes from y_0 to y_1, and from $-\phi_0$ to ϕ_0. The radius of the room is R.

The blocker and the room are both very large compared to the area near the origin where we examine the shadow, so that the blocker effectively occupies the entire $z < 0$ halfspace of the y-z plane. Since the room is large, we can assume that all points are in the center of the room—that is, the solid angle subtended by the window does not change over the area of interest.

What is the irradiance on the paper near the origin? Write an expression for $E(\mathbf{x})$, and compute an image that shows the irradiance as a map over the surface, for a square region around the origin.

Once you have understood the situation this shouldn't take more than a few minutes; if you're making a lot of complicated calculations you're off track.



Problem 2.

Consider a point **x** on a planar surface that is illuminated by a spherical light source of radius R that has uniform radiance L. The center of the source is located a distance r from the illuminated point, and the line from **x** to the center of the source makes an angle θ with the surface normal.

(a) If $r >> R$ (so that the source can be approximated as a point source), what is the irradiance at **x**?

(b) By integrating the incident radiance at **x**, conclude that when $\theta = 0$ the same formula holds when r is not large (though still $r > R$).

(c) Draw the Nusselt analog for the two different cases that come up when $\theta \neq 0$.

(d) Show that the same formula for irradiance continues to hold under certain conditions on θ.

(e) Give a formula for $E(\theta)$ that works for all θ, r, and R (with $r > R$).