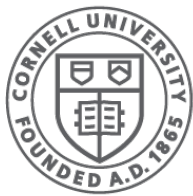


Concurrent Programming: Critical Sections

CS 6410



Cornell CIS
COMPUTING AND INFORMATION SCIENCE

[[Robbert van Renesse](#)]

Concurrent Programming is Hard

Why?

- Concurrent programs are *non-deterministic*
 - run them twice with same input, get two different answers
 - or worse, one time it works and the second time it fails
- Program statements are executed *non-atomically*
 - **`x += 1`** compiles to something like
 - **LOAD x**
 - **ADD 1**
 - **STORE x**

Enter *Harmony*

- A new concurrent programming language
 - heavily based on Python syntax to reduce learning curve for many
- A new underlying virtual machine

it tries *all* possible executions of a program (or rather, explores all possible reachable states) until it finds a problem, if any
(this is called “model checking”)

Non-Determinism

```
1      shared = True
2
3      def f(): assert shared
4      def g(): shared = False
5
6      f()
7      g()
```

(a) [[code/prog1.hny](#)] Sequential

```
1      shared = True
2
3      def f(): assert shared
4      def g(): shared = False
5
6      spawn f()
7      spawn g()
```

(b) [[code/prog2.hny](#)] Concurrent

Figure 3.1: A sequential and a concurrent program.

Non-Determinism

```
1      shared = True
2
3      def f(): assert shared
4      def g(): shared = False
5
6      f()
7      g()
```

(a) [[code/prog1.hny](#)] Sequential

```
1      shared = True
2
3      def f(): assert shared
4      def g(): shared = False
5
6      spawn f()
7      spawn g()
```

(b) [[code/prog2.hny](#)] Concurrent

Figure 3.1: A sequential and a concurrent program.

#states 2
2 components, 0 bad states
No issues

#states 11
Safety Violation
T0: `__init__()` [0-3,17-25] { *shared*: True }
T2: `g()` [13-16] { *shared*: False }
T1: `f()` [4-8] { *shared*: False }
Harmony assertion failed

Critical Sections in Harmony

```
def thread(self):  
    while True:  
        ...    # code outside critical section  
        ...    # code to enter the critical section  
        ...    # critical section itself  
        ...    # code to exit the critical section
```

```
spawn thread(1)  
spawn thread(2)  
...
```

- How do we check mutual exclusion?
- How do we check progress?

Critical Sections in Harmony

```
def thread(self):  
    while True:  
        ...    # code outside critical section  
        ...    # code to enter the critical section  
        @cs: assert atLabel(cs) == { (thread, self): 1 }  
        ...    # code to exit the critical section
```

```
spawn thread(1)  
spawn thread(2)  
...
```

- How do we check mutual exclusion?
- How do we check progress?

Critical Sections in Harmony

```
def thread(self):  
    while choose( { False, True } ):  
        ...    # code outside critical section  
        ...    # code to enter the critical section  
        @cs: assert atLabel(cs) == { (thread, self): 1 }  
        ...    # code to exit the critical section
```

```
spawn thread(1)  
spawn thread(2)  
...
```

- How do we check mutual exclusion?
- How do we check progress?
 - *if code to enter/exit the critical section cannot terminate, Harmony with balk*


First attempt: a naïve lock

```
1  lockTaken = False
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          await not lockTaken
7          lockTaken = True
8
9          # Critical section
10         @cs: assert atLabel(cs) == { (thread, self): 1 }
11
12         # Leave critical section
13         lockTaken = False
14
15  spawn thread(0)
16  spawn thread(1)
```

Figure 5.3: [[code/naiveLock.hny](#)] Naïve implementation of a shared lock.

First attempt: a naïve lock

```
1  lockTaken = False
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          await not lockTaken
7          lockTaken = True
8
9          # Critical section
10         @cs: assert atLabel(cs) == { (thread, self): 1 }
11
12         # Leave critical section
13         lockTaken = False
14
15     spawn thread(0)
16     spawn thread(1)
```



wait till lock is free, then take it

Figure 5.3: [[code/naiveLock.hny](#)] Naïve implementation of a shared lock.

First attempt: a naïve lock

```
1  lockTaken = False
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          await not lockTaken      ← wait till lock is free, then take it
7          lockTaken = True
8
9          # Critical section
10         @cs: assert atLabel(cs) == { (thread, self): 1 }
11
12         # Leave critical section
13         lockTaken = False      ← release the lock
14
15     spawn thread(0)
16     spawn thread(1)
```

Figure 5.3: [[code/naiveLock.hny](#)] Naïve implementation of a shared lock.

First attempt: a naïve lock

```
1  lockTaken = False
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          await not lockTaken
7          lockTaken = True
8
9          # Critical section
10         @cs: assert atLabel(cs) == { (thread, self): 1 }
11
12         # Leave
13         lockTaken = False
14
15     spawn thread(self)
16     spawn thread(self)
```

===== Safety violation =====

__init__ /() [0,26-36]	36 { lockTaken: False }
thread/0 [1-2,3(choose True),4-7]	8 { lockTaken: False }
thread/1 [1-2,3(choose True),4-8]	9 { lockTaken: True }
thread/0 [8-19]	19 { lockTaken: True }

>>> Harmony Assertion (file=code/naiveLock.hny, line=10) failed

Figure 5.3: [code/naiveLock.hny](#) Naive implementation of a shared lock.


Second attempt: *flags*

```
1  flags = [ False, False ]
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          flags[self] = True
7          await not flags[1 - self]
8
9          # Critical section
10         @cs: assert atLabel(cs) == { (thread, self): 1 }
11
12         # Leave critical section
13         flags[self] = False
14
15     spawn thread(0)
16     spawn thread(1)
```

Figure 5.5: [[code/naiveFlags.hny](#)] Naïve use of flags to solve mutual exclusion.

Second attempt: *flags*

```
1  flags = [ False, False ]
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          flags[self] = True
7          await not flags[1 - self]
8
9          # Critical section
10         @cs: assert atLabel(cs) == { (thread, self): 1 }
11
12         # Leave critical section
13         flags[self] = False
14
15     spawn thread(0)
16     spawn thread(1)
```

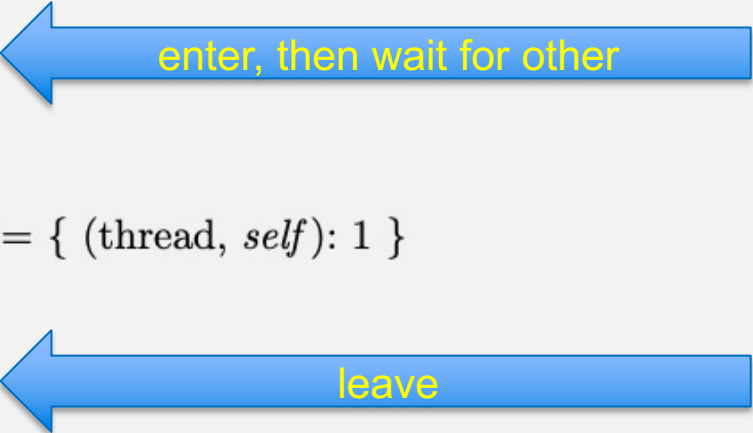


enter, then wait for other

Figure 5.5: [[code/naiveFlags.hny](#)] Naïve use of flags to solve mutual exclusion.

Second attempt: *flags*

```
1  flags = [ False, False ]
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          flags[self] = True
7          await not flags[1 - self]
8
9          # Critical section
10         @cs: assert atLabel(cs) == { (thread, self): 1 }
11
12         # Leave critical section
13         flags[self] = False
14
15     spawn thread(0)
16     spawn thread(1)
```



enter, then wait for other

leave

Figure 5.5: [[code/naiveFlags.hny](#)] Naïve use of flags to solve mutual exclusion.

Second attempt: *flags*

```
1  flags = [ False, False ]
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          flags[self] = True
7          await not flags[1 - self]
8
9          # Critical section
10         @cs: assert atLabel(cs) == { (thread, self): 1 }
11
12         # Leave critical section
13         flags[self] = False
14
15     spawn thread(0)
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Figure 5.5: [[code/naiveFlags.hny](#)] Naïve use of flags to solve mutual exclusion.

Second attempt: *flags*

```
1  flags = [ False, False ]
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          flags[self] = True
7          await not flags[1 - self]
8
9          # Critical section
10         @cs: assert atLabel(cs) == { (thread, self): 1 }
11
12         # Leave critical section
13         flags[self] = False
14
15     spawn thread(0)
16     spawn thread(1)
```

==== Non-terminating State ====

__init__ /() [0,36-46] 46 { flags: [False, False] }
thread/0 [1-2,3(choose True),4-12] 13 { flags: [True, False] }
thread/1 [1-2,3(choose True),4-12] 13 { flags: [True, True] }
blocked thread: thread/1 pc = 13
blocked thread: thread/0 pc = 13

Figure 5.5: [code/n](#)

Third attempt: *turn* variable

```
1  turn = 0
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          turn = 1 - self
7          await turn == self
8
9          # Critical section
10         @cs: assert atLabel(cs) == { (thread, self): 1 }
11
12         # Leave critical section
13
14     spawn thread(0)
15     spawn thread(1)
```

Figure 5.7: [[code/naiveTurn.hny](#)] Naïve use of turn variable to solve mutual exclusion.

Third attempt: *turn* variable

```
1  turn = 0
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          turn = 1 - self
7          await turn == self
8
9          # Critical section
10         @cs: assert atLabel(cs) == { (thread, self): 1 }
11
12         # Leave critical section
13
14     spawn thread(0)
15     spawn thread(1)
```

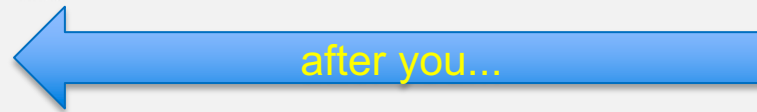


Figure 5.7: [[code/naiveTurn.hny](#)] Naïve use of turn variable to solve mutual exclusion.

Third attempt: *turn* variable

```
1  turn = 0
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          turn = 1 - self
7          await turn == self
8
9          # Critical section
10         @cs: assert atLabel(cs) == { (thread, self): 1 }
11
12         # Leave critical section
13
14     spawn thread(0)
15     spawn thread(1)
```

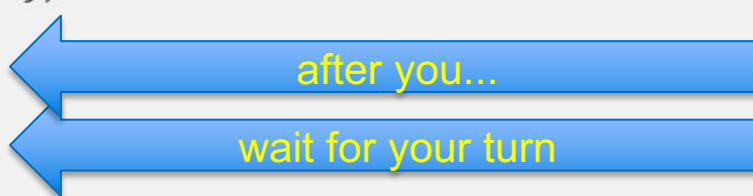


Figure 5.7: [[code/naiveTurn.hny](#)] Naïve use of turn variable to solve mutual exclusion.

Third attempt: *turn* variable

```
1  turn = 0
2
3  def thread(self):
4      while choose({ False, True }):
5          # Enter critical section
6          turn = 1 - self
7          await turn == self
8
9          # Critical section
10         @cs: assert atLabel(cs) == { (thread, self): 1 }
11
12         # Leave critical section
13
14     spawn thread(0)
15     spawn thread(1)
```

==== Non-terminating State ====

__init__/() [0,28-38]	38 { turn: 0 }
thread/0 [1-2,3(choose True),4-26,2,3(choose True),4]	5 { turn: 1 }
thread/1 [1-2,3(choose False),4,27]	27 { turn: 1 }
blocked thread: thread/0 pc = 5	

Figure 5.7: [coe

Peterson's Algorithm: *flags & turn*

```
1 sequential flags, turn
2
3 flags = [ False, False ]
4 turn = choose({0, 1})
5
6 def thread(self):
7     while choose({ False, True }):
8         # Enter critical section
9         flags[self] = True
10        turn = 1 - self
11        await (not flags[1 - self]) or (turn == self)
12
13        # critical section is here
14        @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16        # Leave critical section
17        flags[self] = False
18
19 spawn thread(0)
20 spawn thread(1)
```

Figure 6.1: [[code/Peterson.hny](#)] Peterson's Algorithm

Peterson's Algorithm: *flags & turn*

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1 sequential flags, turn
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3 flags = [ False, False ]
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6 def thread(self):
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8         # Enter critical section
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10        turn = 1 - self
11        await (not flags[1 - self]) or (turn == self)
12
13        # critical section is here
14        @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16        # Leave critical section
17        flags[self] = False
18
19 spawn thread(0)
20 spawn thread(1)
```

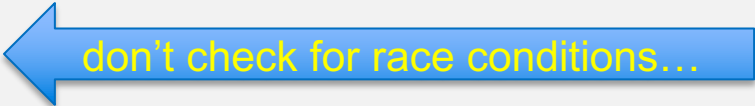


Figure 6.1: [[code/Peterson.hny](#)] Peterson's Algorithm

Peterson's Algorithm: *flags & turn*

```
1  sequential flags, turn
2
3  flags = [ False, False ]
4  turn = choose({0, 1})
5
6  def thread(self):
7      while choose({ False, True }):
8          # Enter critical section
9          flags[self] = True
10         turn = 1 - self
11         await (not flags[1 - self]) or (turn == self)
12
13         # critical section is here
14         @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16         # Leave critical section
17         flags[self] = False
18
19  spawn thread(0)
20  spawn thread(1)
```

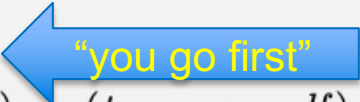


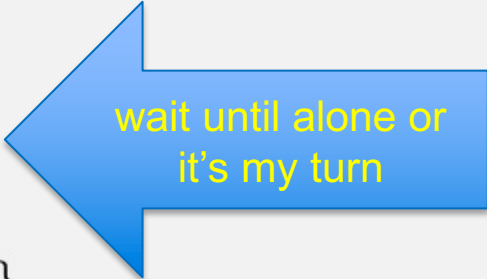
Figure 6.1: [[code/Peterson.hny](#)] Peterson's Algorithm

Peterson's Algorithm: *flags & turn*

```
1  sequential flags, turn
2
3  flags = [ False, False ]
4  turn = choose({0, 1})
5
6  def thread(self):
7      while choose({ False, True }):
8          # Enter critical section
9          flags[self] = True
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12
13         # critical section is here
14         @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16         # Leave critical section
17         flags[self] = False
18
19  spawn thread(0)
20  spawn thread(1)
```



“you go first”



wait until alone or
it's my turn

Figure 6.1: [[code/Peterson.hny](#)] Peterson's Algorithm

Peterson's Algorithm: *flags & turn*

```
1 sequential flags, turn
2
3 flags = [ False, False ]
4 turn = choose({0, 1})
5
6 def thread(self):
7     while choose({ False, True }):
8         # Enter critical section
9         flags[self] = True
10        turn = 1 - self
11        await (not flags[1 - self]) or (turn == self)
12
13        # critical section is here
14        @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16        # Leave critical section
17        flags[self] = False
18
19 spawn thread(0)
20 spawn thread(1)
```

"you go first"

wait until alone or
it's my turn

leave

Figure 6.1: [[code/Peterson.hny](#)] Peterson's Algorithm

Peterson's Algorithm: *flags & turn*

```
1 sequential flags, turn
2
3 flags = [ False, False ]
4 turn = choose({0, 1})
5
6 def thread(self):
7     while choose({ False, True }):
8         # Enter critical section
9         flags[self] = True
10        turn = 1 - self
11        await (not flags[1 - self]) or (turn == self)
12
13        # critical section is here
14        @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16        # Leave critical section
17        flags[self] = False
18
19 spawn thread(0)
20 spawn thread(1)
```

#states = 104 diameter = 5
#components: 37
no issues found

Figure 6.1: [[code/Peterson.hny](#)] Peterson's Algorithm

So, we proved Peterson's Algorithm correct by brute force, enumerating all possible executions. We now know *that* it works.

*But how does one prove it by deduction?
so one might understand *why* it works...*

What and how?

- Need to show that, for any execution, all states reached satisfy mutual exclusion
 - in other words, mutual exclusion is *invariant*
invariant = predicate that holds in every reachable state

How to prove an invariant?

- Need to show that, for any execution, all states reached satisfy the invariant
- Sounds similar to sorting:
 - Need to show that, for any list of numbers, the resulting list is ordered
- Let's try *proof by induction* on the length of an execution

Proof by induction

You want to prove that some *Induction Hypothesis* $IH(n)$ holds for any n :

- Base Case:

- show that $IH(0)$ holds

- Induction Step:

- show that if $IH(i)$ holds, then so does $IH(i+1)$

Proof by induction in our case

To show that some **IH** holds for an *execution* **E** of any number of *steps*:

- Base Case:

- show that **IH** holds in the initial state(s)

- Induction Step:

- show that if **IH** holds in a state produced by **E**, then for any possible next step **s**, **IH** also holds in the state produced by **E + [s]**



But there's a problem

- How do we characterize a “state produced by E”?
 - or how do we characterize a *reachable state*?
- Instead, it's much easier if we proved a so-called “inductive invariant”:
 - Base Case:
 - show that **IH** holds in the initial state(s)
 - Induction Step:
 - show that if **IH** holds in **any** state, then for any possible next step, **IH** also holds in the resulting state

First question: what should IH be?

- Obvious answer: mutual exclusion itself
 - if $T0$ is in the critical section, then $T1$ is not
 - without loss of generality...
 - Formally: $T0@cs \Rightarrow \neg T1@cs$
- Unfortunately, this won't work...

State before T1 takes a step:

```
1 sequential flags, turn
2
3 flags = [ False, False ]
4 turn = choose({0, 1})
5
6 def thread(self):
7     while choose({ False, True }):
8         # Enter critical section
9         flags[self] = True
10        turn = 1 - self
11  await (not flags[1 - self]) or (turn == self)
12
13 # critical section is here
14  @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16 # Leave critical section
17 flags[self] = False
18
19 spawn thread(0)
20 spawn thread(1)
```

flags = [True, True]
turn = 1

mutual exclusion holds

Figure 6.1: [code/Peterson.hny] Peterson's Algorithm

State after T1 takes a step:

```
1 sequential flags, turn
2
3 flags = [ False, False ]
4 turn = choose({0, 1})
5
6 def thread(self):
7     while choose({ False, True }):
8         # Enter critical section
9         flags[self] = True
10        turn = 1 - self
11        await (not flags[1 - self]) or (turn == self)
12
13        # critical section is here
14        @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16        # Leave critical section
17        flags[self] = False
18
19 spawn thread(0)
20 spawn thread(1)
```

flags = [True, True]
turn = 1

T0

T1

mutual exclusion violated

Figure 6.1: [<code/Peterson.hny>] Peterson's Algorithm

So, is Peterson's Algorithm broken?

No, it'll turn out this prior state cannot be reached from the initial state (see later)

```
1 sequential flags, turn
2
3 flags = [ False, False ]
4 turn = choose({0, 1})
5
6 def thread(self):
7     while choose({ False, True }):
8         # Enter critical section
9         flags[self] = True
10        turn = 1 - self
11        await (not flags[1 - self]) or (turn == self)
12
13        # critical section is here
14        @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16        # Leave critical section
17        flags[self] = False
18
19 spawn thread(0)
20 spawn thread(1)
```

flags = [True, True]
turn = 1

T1

T0

mutual exclusion holds

Figure 6.1: [code/Peterson.hny] Peterson's Algorithm

Useful and obvious but insufficient invariant

```
1 sequential flags, turn
2
3 flags = [ False, False ]
4 turn = choose({0, 1})
5
6 def thread(self):
7     while choose({ False, True }):
8         # Enter critical section
9         flags[self] = True
10        turn = 1 - self
11        await (not flags[1 - self]) or (turn == self)
12
13        # critical section is here
14        @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16        # Leave critical section
17        flags[self] = False
18
19 spawn thread(0)
20 spawn thread(1)
```

$$Tx@cs \Rightarrow flags[x]$$

mutual exclusion holds

Figure 6.1: [[code/Peterson.hny](#)] Peterson's Algorithm

What else do we expect to hold @cs?

```
1 sequential flags, turn
2
3 flags = [ False, False ]
4 turn = choose({0, 1})
5
6 def thread(self):
7     while choose({ False, True }):
8         # Enter critical section
9         flags[self] = True
10        turn = 1 - self
11        await (not flags[1 - self]) or (turn == self)
12
13        # critical section is here
14        @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16        # Leave critical section
17        flags[self] = False
18
19 spawn thread(0)
20 spawn thread(1)
```

???

mutual exclusion holds

Figure 6.1: [[code/Peterson.hny](#)] Peterson's Algorithm

Another obvious IH to try

- Based on the **await** condition:

$$T0@cs \Rightarrow \neg flags[1] \vee turn = 0$$

- Promising because if $T0@cs \wedge T1@cs$ then

$$\left. \begin{array}{l} T0@cs \Rightarrow \neg flags[1] \vee turn = 0 \wedge \\ T1@cs \Rightarrow \neg flags[0] \vee turn = 1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} turn = 0 \wedge \\ turn = 1 \end{array} \right. \\ \Rightarrow \text{False (therefore mutual exclusion)}$$

- Unfortunately, this is not an invariant...

Another obvious IH to try

- Based on the **await** condition:

$$T0@cs \Rightarrow \neg flags[1] \vee turn = 0$$

-

Easy to check with Harmony
Just run it with the following:

@cs: assert (not flags[1 - self]) or (turn == self)

-

Unfortunately, this is not an invariant...

State before T1 takes a step:

```
1 sequential flags, turn
2
3 flags = [ False, False ]
4 turn = choose({0, 1})
5
6 def thread(self):
7     while choose({ False, True }):
8         # Enter critical section
9         flags[self] = True
10        turn = 1 - self
11        await (not flags[1 - self]) or (turn == self)
12
13        # critical section is here
14        @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16        # Leave critical section
17        flags[self] = False
18
19 spawn thread(0)
20 spawn thread(1)
```

flags = [True, False]
turn = 1

T1

T0

$T0@cs \Rightarrow \neg flags[1] \vee turn = 0$ holds

note: this is a reachable state

Figure 6.1: [[code/Peterson.hny](#)] Peterson's Algorithm

State after T1 takes a step:

```
1 sequential flags, turn
2
3 flags = [ False, False ]
4 turn = choose({0, 1})
5
6 def thread(self):
7     while choose({ False, True }):
8         # Enter critical section
9         flags[self] = True
10        turn = 1 - self
11        await (not flags[1 - self]) or (turn == self)
12
13        # critical section is here
14        @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16        # Leave critical section
17        flags[self] = False
18
19 spawn thread(0)
20 spawn thread(1)
```

flags = [True, True]
turn = 1

T1

T0

$T0@cs \Rightarrow \neg flags[1] \vee turn = 0$ violated

note: this is also a reachable state

Figure 6.1: [code/Peterson.hny] Peterson's Algorithm

But suggests an improved hypothesis

$$T0@cs \Rightarrow \neg flags[1] \vee turn = 0 \vee T1@gate$$

```
3  flags = [ False, False ]
4  turn = choose({0, 1})
5
6  def thread(self):
7      while choose({ False, True }):
8          # Enter critical section
9          flags[self] = True
10         T1 → @gate: turn = 1 - self
11         await (not flags[1 - self]) or (turn == self)
12
13         # Critical section
14         T0 → @cs: assert (not flags[1 - self]) or (turn == self) or
15                 (atLabel(gate) == {(thread, 1 - self): 1})
16
17         # Leave critical section
18         flags[self] = False
```

But suggests an improved hypothesis

$$T0@cs \Rightarrow \neg flags[1] \vee turn = 0 \vee T1@gate$$

3 *flags* = [False, False]

Also easy to check with Harmony

Proves that it is invariant, but not necessarily an
inductive invariant

10

@cs: assert (not *flags*[1 - *self*]) or (*turn* == *self*) or
(atLabel(gate) == {(thread, 1 - *self*): 1})

Leave critical section
flags[*self*] = False

Inductive Invariance Proof

Let I be the induction hypothesis:

$$I \triangleq T0@cs \Rightarrow \neg flags[1] \vee turn == 0 \vee T1@gate$$

I clearly holds in the initial state because $\neg T0@cs$ (false implies anything)

We are going to show: if I holds in a state (*reachable or not*), then I also holds in any state after either $T0$ or $T1$ takes a step

Tricky Case 1:

$\neg T0@cs$ and $T0$ takes a step so that $T0@cs$

This must mean that $\neg flags[1] \vee turn = 0$
before the step (see code line 11)

```
11      await (not flags[1 - self]) or (turn == self)
12
13      # critical section is here
14      @cs: assert atLabel(cs) == { (thread, self): 1 }
```

But then $\neg flags[1] \vee turn = 0$ still holds after
the step

So $T0@cs \Rightarrow \neg flags[1] \vee turn = 0 \vee T1@gate$



Tricky Case 2:

$T0@cs$ and $T1$ takes a step

This must mean that before the step

$\neg flags[1] \vee turn = 0 \vee T1@gate$ (by IH).

So 3 cases to consider:

- $\neg flags[1] \Rightarrow flags[1]$
→ this means $T1@gate$ after the step
- $turn = 0 \Rightarrow turn = 1$
→ can't happen (only $T0$ sets $turn$ to 1)
- $T1@gate \Rightarrow \neg T1@gate$
→ this means $turn = 0$ after step

```
# Enter critical section  
flags[self] = True  
@gate: turn = 1 - self
```

So $T0@cs \Rightarrow \neg flags[1] \vee turn = 0 \vee T1@gate$



Finally, prove mutual exclusion

$$T0@cs \wedge T1@cs \Rightarrow$$

$$\left\{ \begin{array}{l} \neg flags[1] \vee turn = 0 \vee T1@gate \\ \neg flags[0] \vee turn = 1 \vee T0@gate \end{array} \right. \wedge$$

$$\Rightarrow turn = 0 \wedge turn = 1$$

$$\Rightarrow \textit{False}$$



Finally, prove mutual exclusion

$$T0@cs \wedge T1@cs \Rightarrow$$



$$\left\{ \begin{array}{l} \neg flags[1] \vee turn = 0 \vee T1@gate \\ \neg flags[0] \vee turn = 1 \vee T0@gate \end{array} \right. \wedge$$

$$\Rightarrow turn = 0 \wedge turn = 1$$

$$\Rightarrow \textit{False}$$



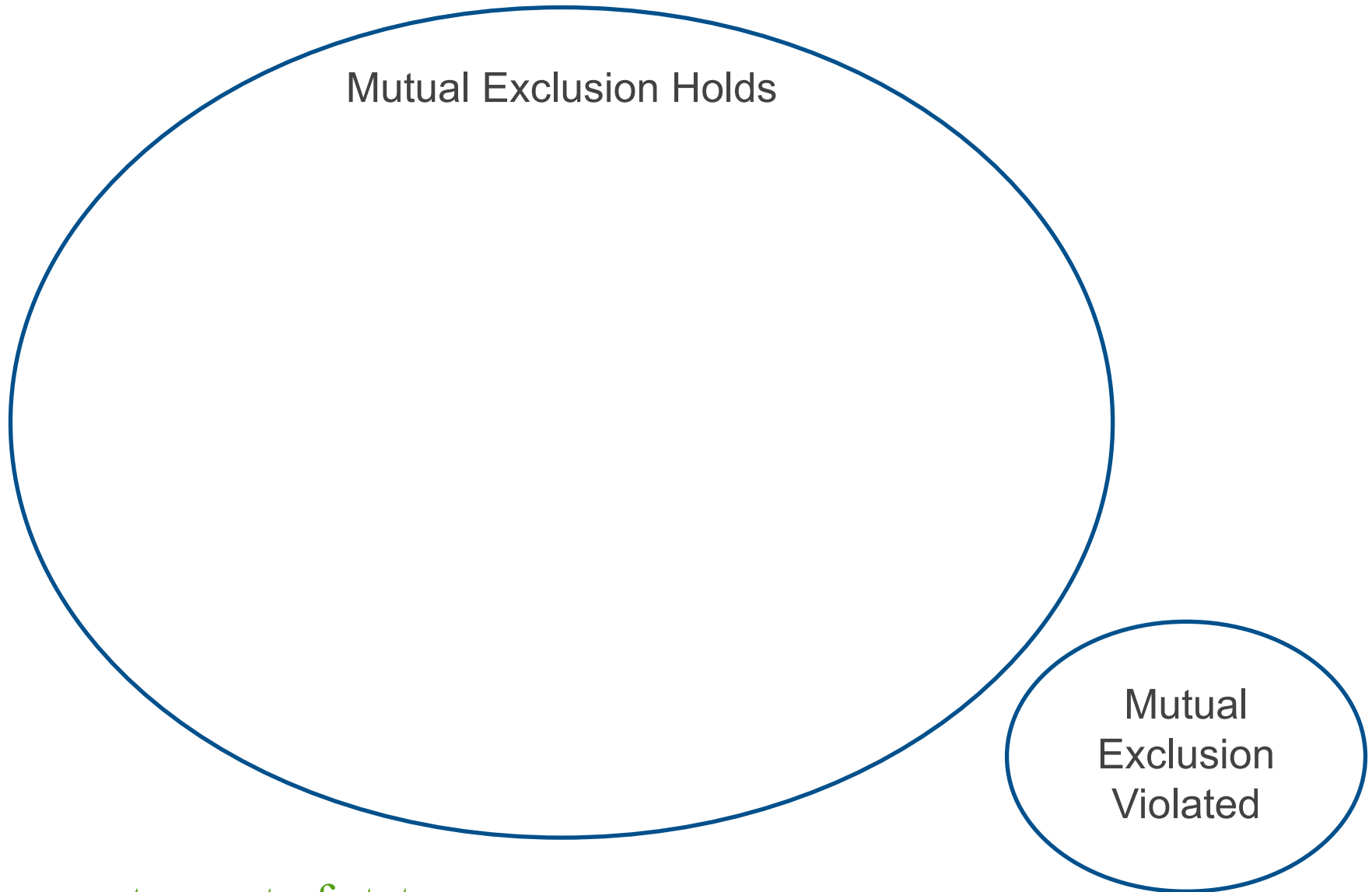
Now we can see why this state cannot be reached!

```
1 sequential flags, turn
2
3 flags = [ False, False ]
4 turn = choose({0, 1})
5
6 def thread(self):
7     while choose({ False, True }):
8         # Enter critical section
9         flags[self] = True
10        turn = 1 - self
11  await (not flags[1 - self]) or (turn == self)
12
13 # critical section is here
14  @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16 # Leave critical section
17 flags[self] = False
18
19 spawn thread(0)
20 spawn thread(1)
```

flags = [True, True]
turn = 1

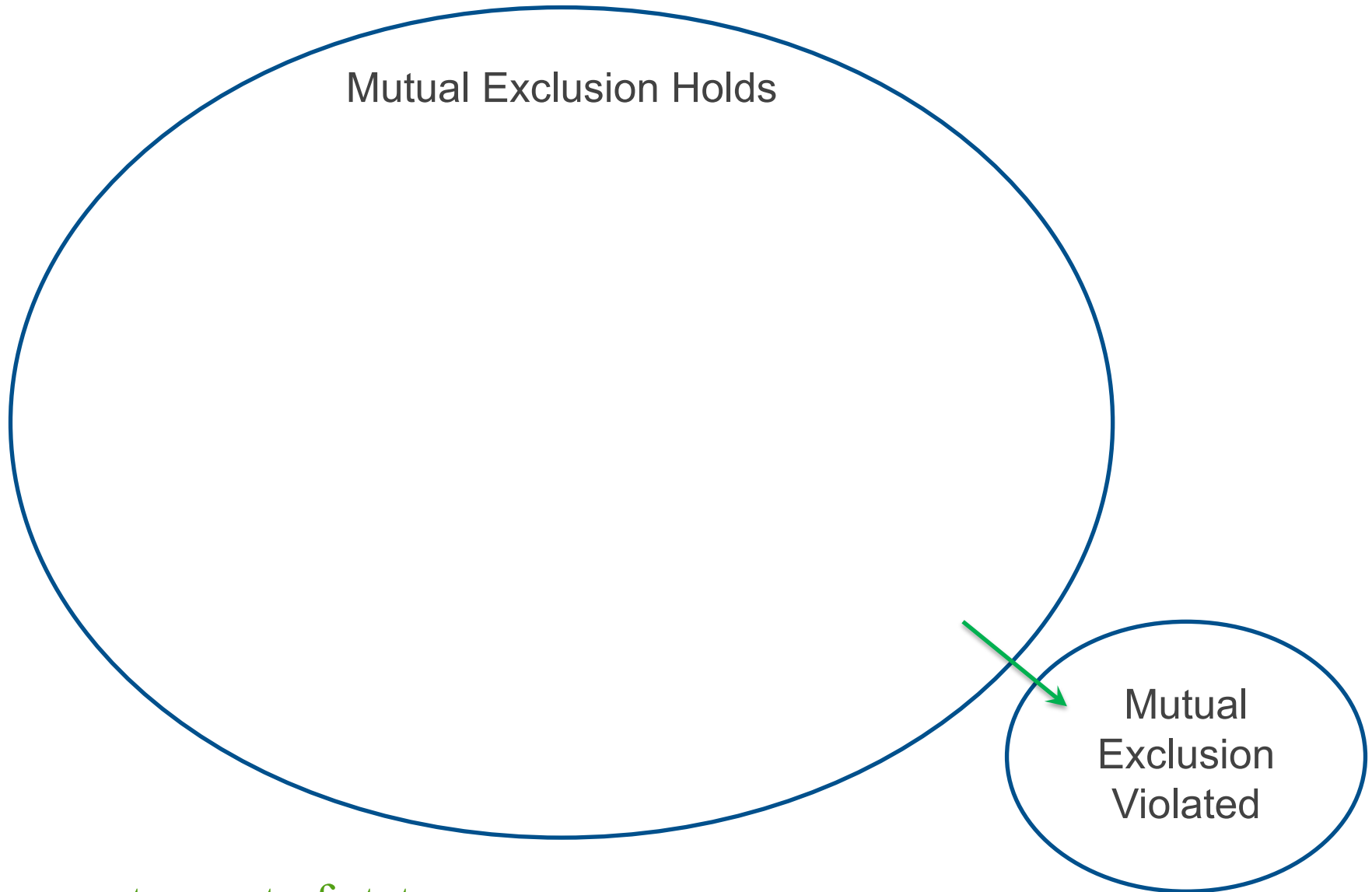
$T0@cs \Rightarrow \neg flags[1] \vee turn = 0 \vee T1@gate \quad \times$

Review in Pictures: State Space



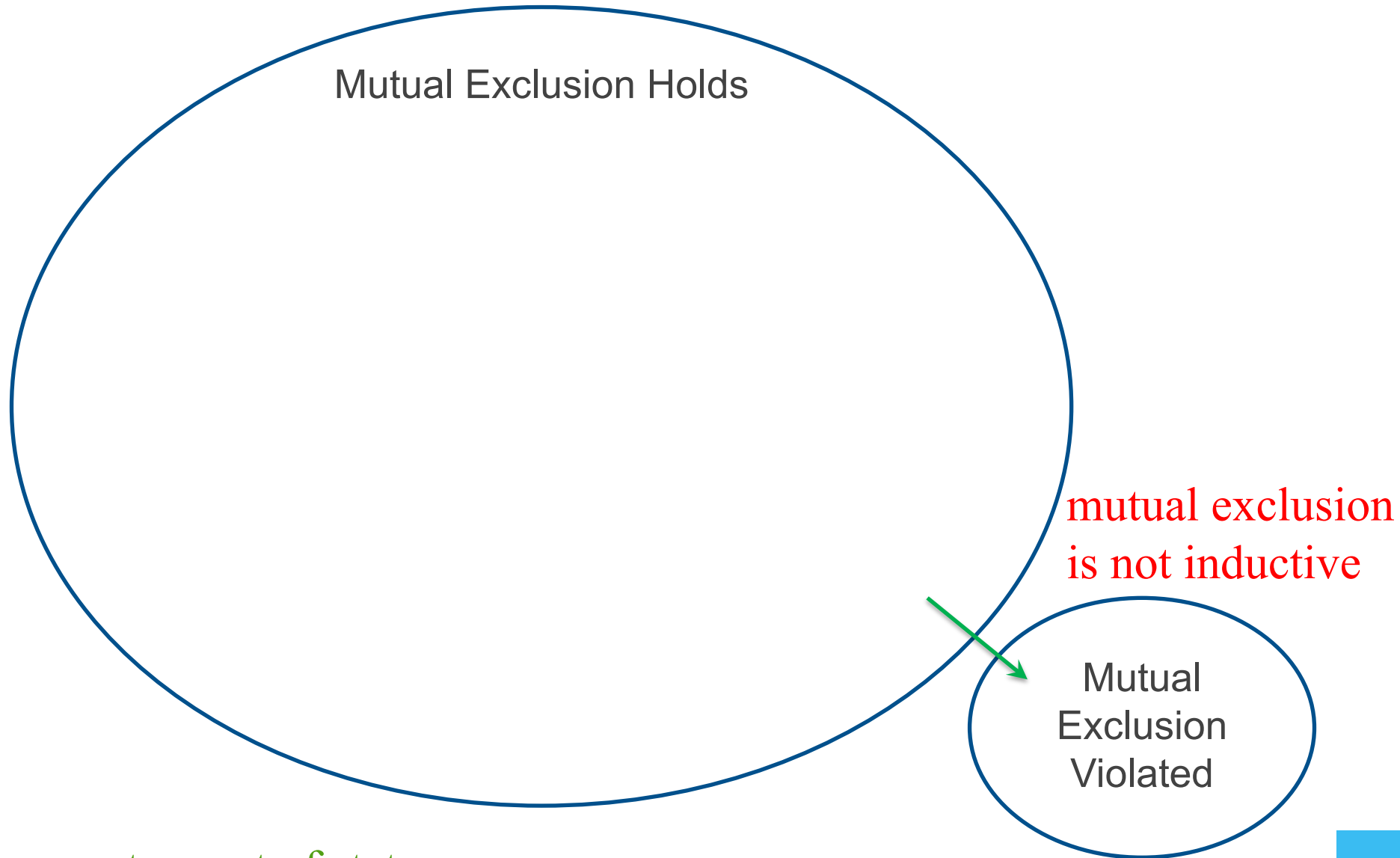
property = set of states

Review in Pictures: State Space



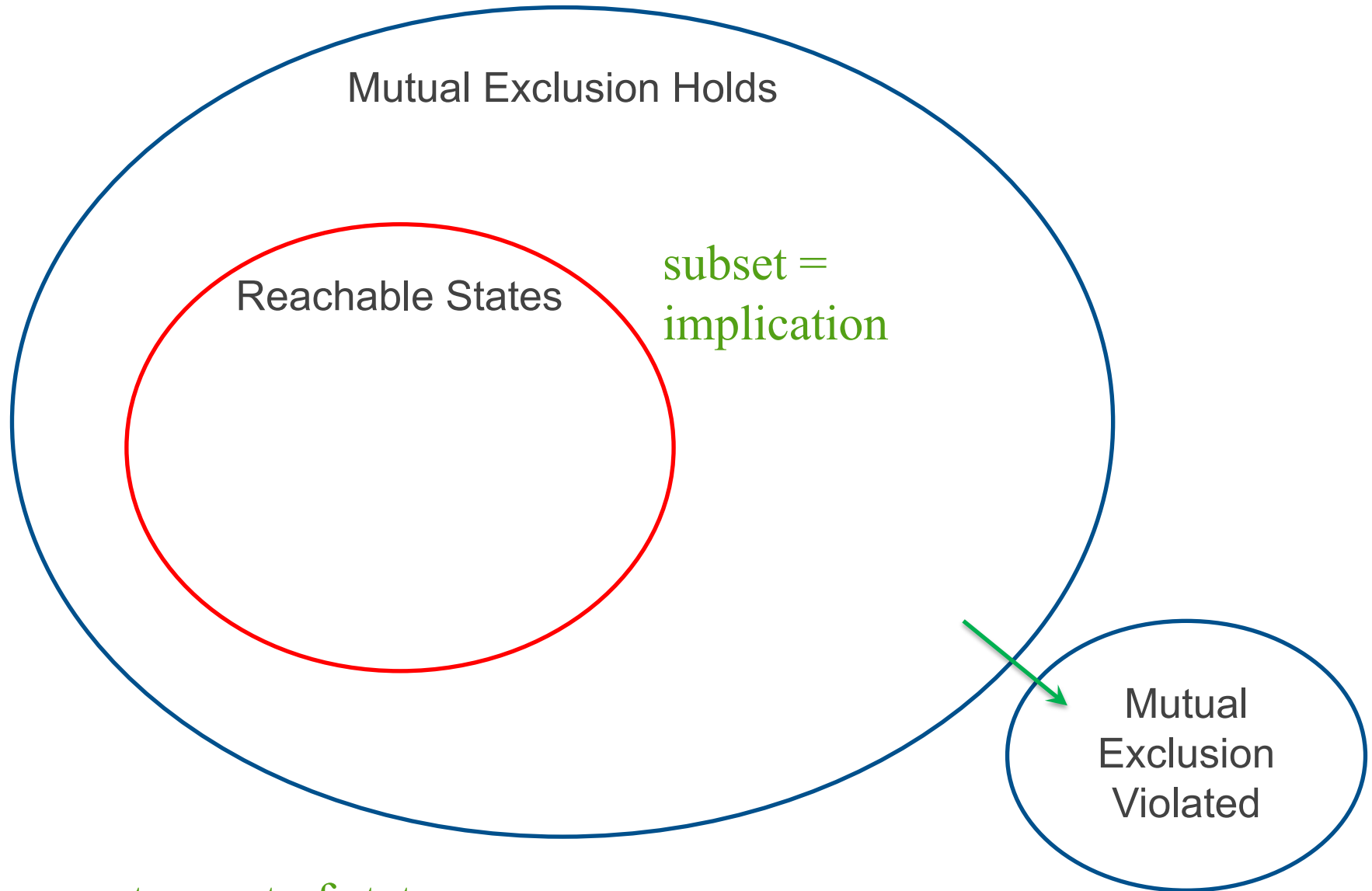
property = set of states

Review in Pictures: State Space



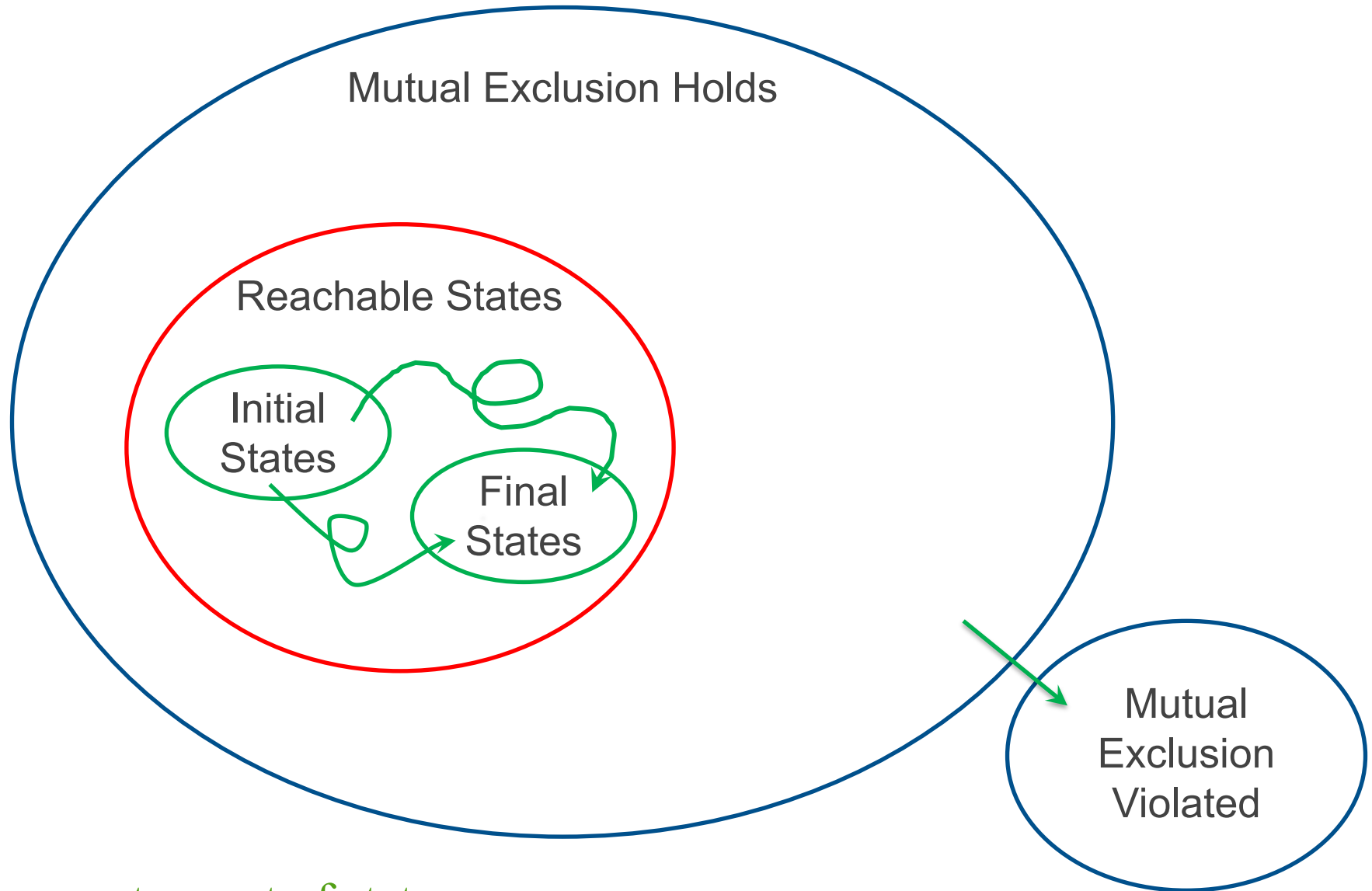
property = set of states

Review in Pictures: State Space



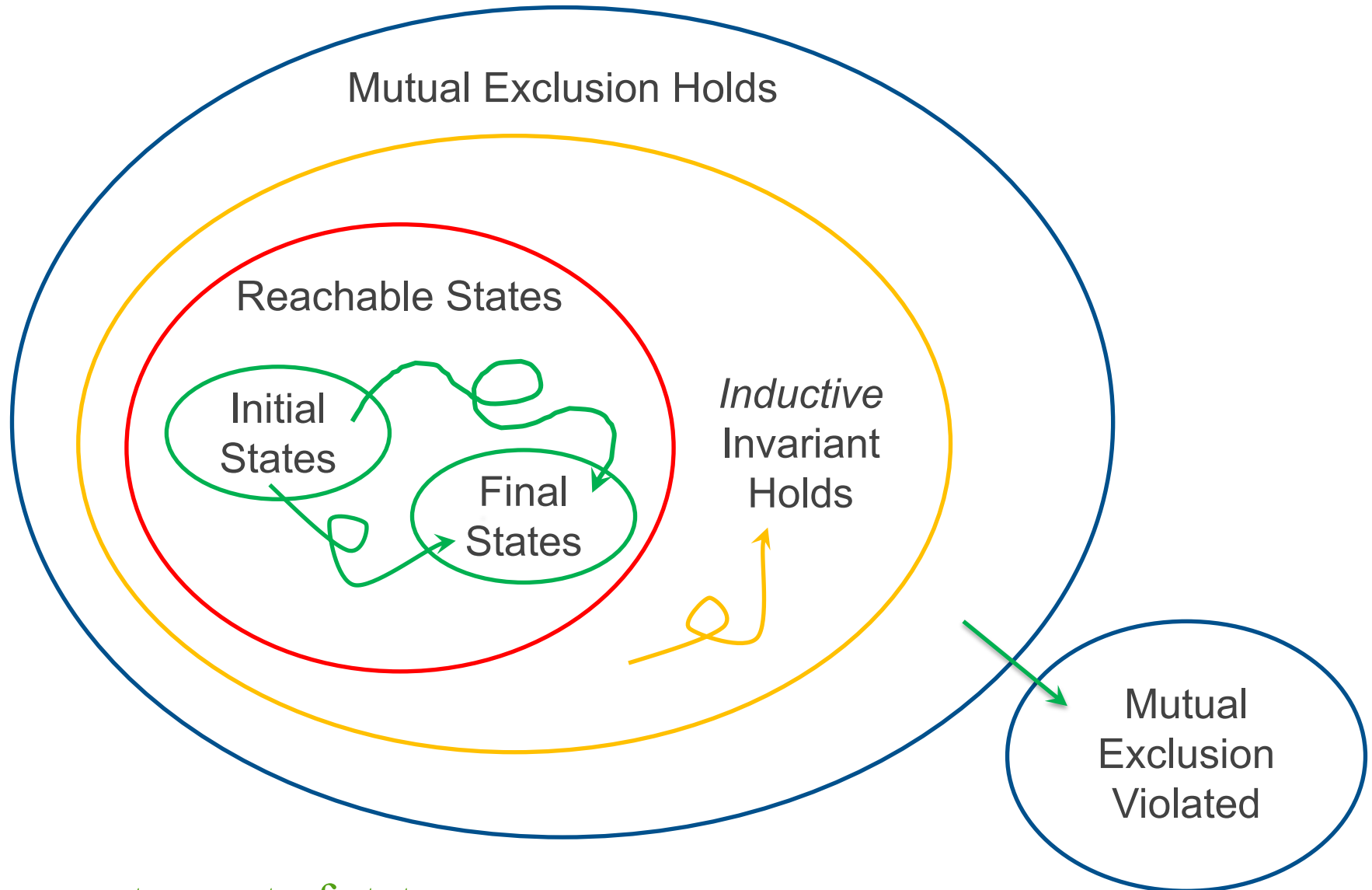
property = set of states

Review in Pictures: State Space



property = set of states

Review in Pictures: State Space



property = set of states

Swapping lines 9 and 10?

```
1 sequential flags, turn
2
3 flags = [ False, False ]
4 turn = choose({0, 1})
5
6 def thread(self):
7     while choose({ False, True }):
8         # Enter critical section
9         flags[self] = True
10        turn = 1 - self
11        await (not flags[1 - self]) or (turn == self)
12
13        # critical section is here
14        @cs: assert atLabel(cs) == { (thread, self): 1 }
15
16        # Leave critical section
17        flags[self] = False
18
19 spawn thread(0)
20 spawn thread(1)
```

Figure 6.1: [[code/Peterson.hny](#)] Peterson's Algorithm