CS633 Spring 06 — Problems 2

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These problems come from Chapter 3 of [L04].

1 Use of E-F Games

Use E-F games to show the following are not FO definable:

a) A graph is planar. (**Hint:** Recall Kuratowski's Theorem: the planar graphs are those containing a subgraph homeomorphic to K_5 or $K_{3,3}$).

b) A graph has a Hamiltonian circuit.

c) A graph is 2-colorable.

d) A graph is acyclic. (**Hint:** Tuesday's lecture.)

2 A Brain Teaser Related to Types

Consider the sets of the form

$$X_\phi \;=\; \{n \in \mathbb{N} \mid L_n \models \phi\}$$

where ϕ is a first-order sentence and L_n is a linear order with n elements. For example, if

$$\psi \;=\; \forall x \forall y \forall z (x = y \vee x = z \vee y = z)$$

then

$$X_\psi \;=\; \{1, 2\}$$

and if

$$\theta \;=\; \exists x \exists y \exists z (x < y \wedge y < z)$$

then

$$X_\theta \;=\; \{3, 4, 5, \ldots\}$$

Can you give a characterization of the sets X_ϕ in general?