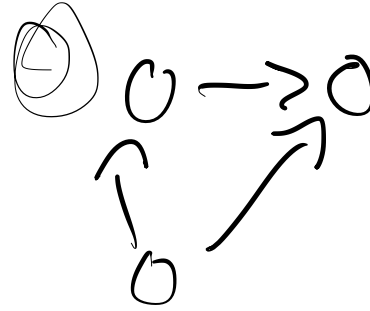
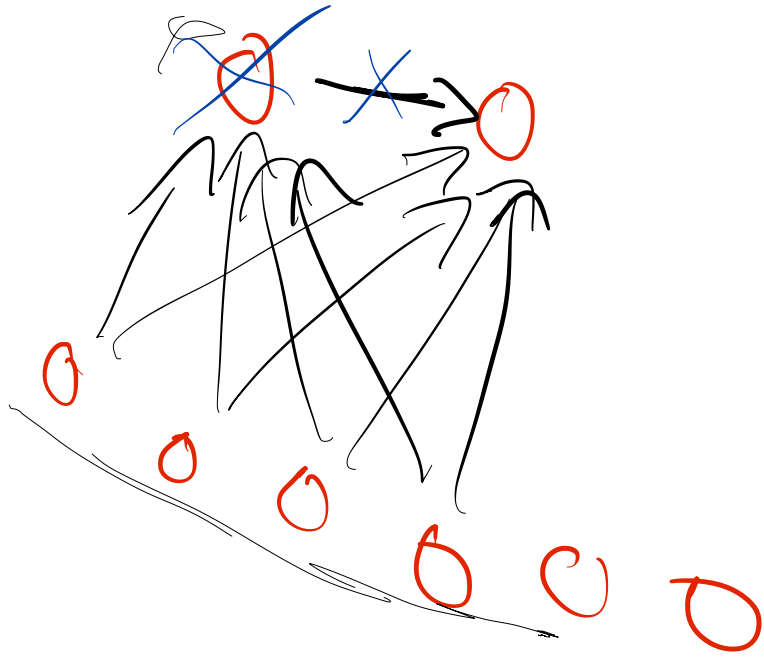
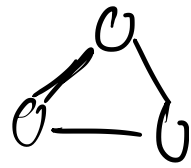


April 23, 2020



Counting triangles



Model: $O(1)$ check if $(i, j) \in G$

$O(d_i)$ to get neighbors of i

① Brute force

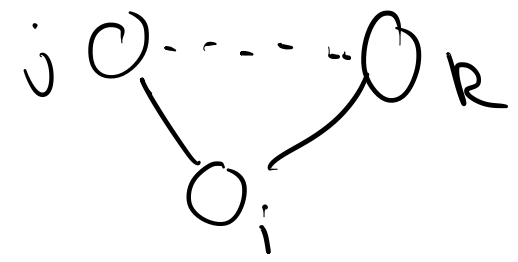
for $i, j, k \in V$

check: $(i, j), (i, k), (j, k)$

$\Theta(|V|^3)$

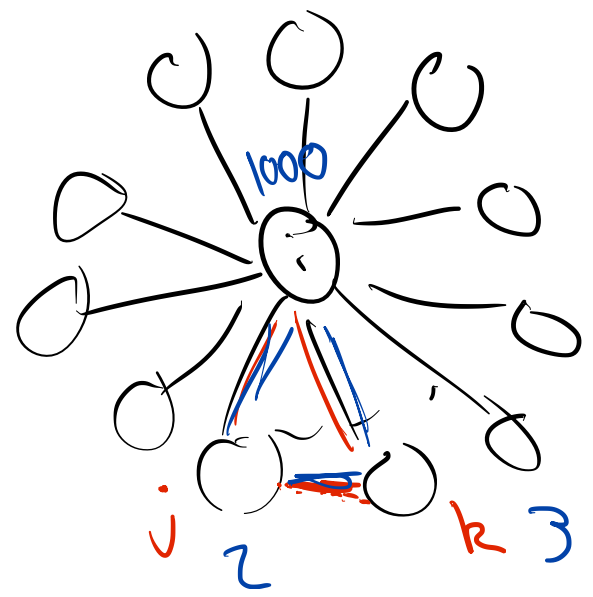
② Neighbors pairs

for $i \in V$



for j, k neighbor of i
check (j, k)

$$\Theta\left(\sum_i \binom{d_i}{2}\right)$$



$$c|V| \quad \Omega(|V|^2)$$

$$O(|V|) \text{ edges} \\ \Rightarrow \Omega(|V|^2)$$

③ How-degree center neighbor pairs

(A) sort nodes by degree σ

$$\sigma(i) < \sigma(j) \Rightarrow d_i \leq d_j$$

(B) for $i \in V$

for neighbors j, k

$$\sigma(i) < \sigma(j), \sigma(k)$$

check (j, k)

Claim: $O(m^{3/2})$ $m = |E|$

Proof: $B = \{i \in V \mid d_i > \sqrt{m}\}$

$S = \{i \in V \mid d_i \leq \sqrt{m}\}$

$$O\left(\sum_{i \in S} d_i^2 + \sum_{i \in B} d_i^2\right)$$

