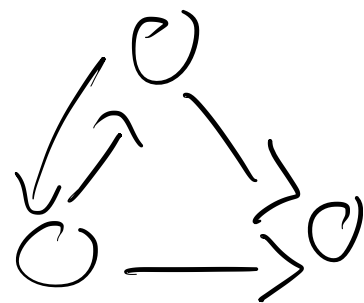
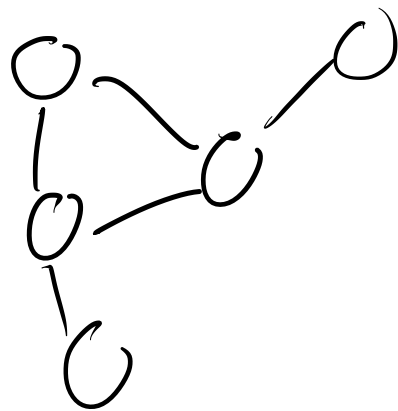


March 5, 2020

Last time: graphs

Fundamental matrices:



- $A$  adjacency

- $D$

- $P = A^T D^{-1}$  random walk matrix  $D^{-1} A$

- $L = D - A$  Laplacian (undir)

social (FB)

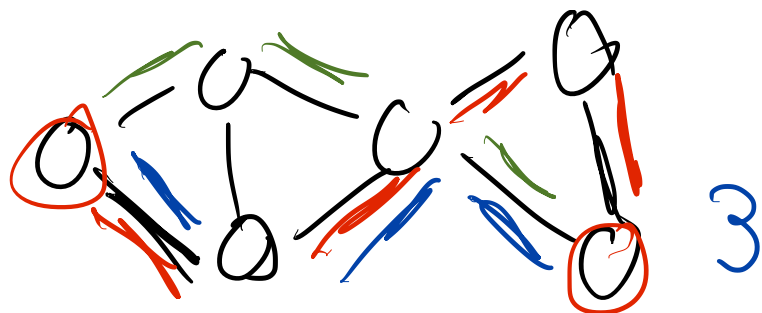
information (Web)

biological (PPI)

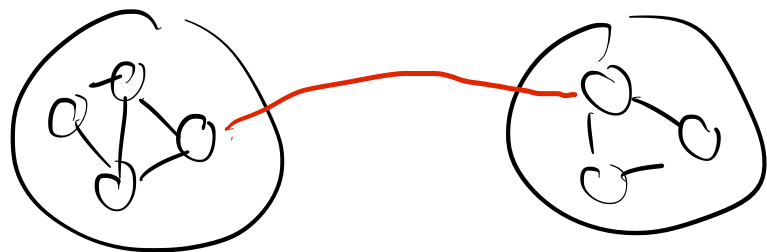
physical systems (Internet)  
commerce (co-purchasing)

# Common network properties

① Small "hop" distances



①a giant component - most nodes in one component



more general than social networks

(Watts /  
Strogatz  
98)

② sparse (not too many edges)

④ heavy-tailed degree dists

② sparsity  $\text{nnz}(A) \ll n^2$

Example: FB (2011)

$n \approx 721\text{M}$  users

$\sim 68.7\text{B}$  friendships

$\sim 95$  friends/person

$\text{Pr}(\text{edge}) \approx 3 \cdot 10^{-7}$

Matrix-vector products  $O(\#\text{edges})$

Linear systems

Eigenvectors