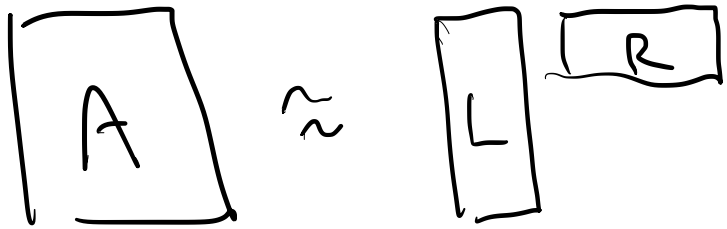
Feb 27, 2020

Lots of dimensionality reduction / latent factors

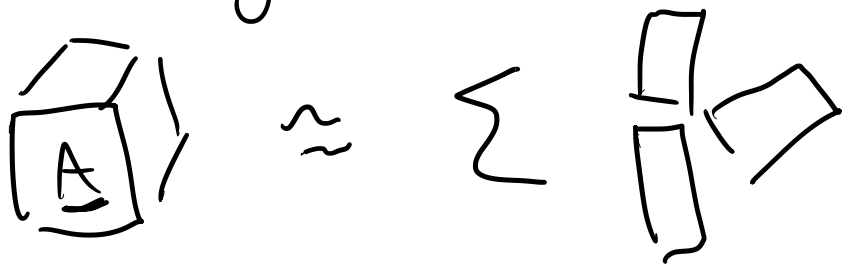
PCA, CUR, NMF, MC

Goal: reconstruct data

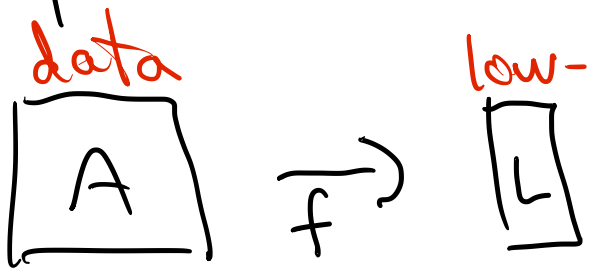


Last lecture: generalized to tensors

CP:

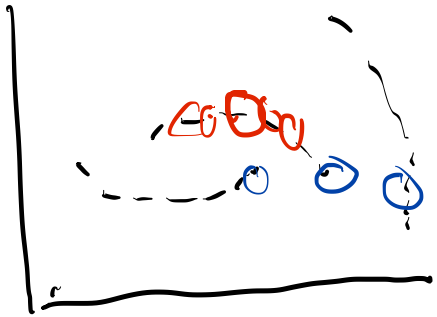


Today: "nonlinear dimensionality reduction"



Idea: look locally to find the manifold

Algorithm: Isometric feature mapping (ISOMAP)



1. form k-nearest neighbors graph
2. All-pairs shortest paths
3. normalize + TSVD

1. Approx who is a neighbor on manifold
2. Approx geodesic distance: D
3. $d_{ij}^2 \approx \|x_i - x_j\|_2^2$ (Multidimensional scaling)

   $\times$ low-dimensional

   $\simeq \|x_i\|_2^2 + \|x_j\|_2^2 - 2\bar{x}_i^T x_j$

$D^{(2)}$

$$\approx -2 H^T X^T X H +$$

$$\Big\} \; M \text{ SPD}$$

$$\min_B \|M - B\|_F \quad \text{s.t.} \quad B \text{ rank } k$$

$$B = U_k \Sigma_k V_k^T \Rightarrow U_k \Sigma_k U_k^T$$

$$Y = U_k \sqrt{\Sigma_k}$$