

Feb 18, 2020

$$A \geq 0$$

Last lecture: NMF

$$\|A - LR\|_F^2 + \lambda (\|L\|_F^2 + \|R\|_F^2)$$

$$\text{s.t. } L, R \geq 0$$

separable NMF: $X = WH$, $W = X(:, K)$

$$K \subseteq \{1, \dots, n\}$$

scale X to \bar{X} : column sums to 1

$$\bar{X} = \bar{X}(:, K) H$$

$$\bar{X} \Pi = \bar{X}(:, K) [I \ G] \leftarrow \text{Example of an "interpolative decomposition"}$$

$$\text{ID: } A \Pi = C [I \ T]$$

C is a subset of columns of A

Pivoted QR

- finding representative data points for near-sep NMF (vertices of convex hull)
- LLS $\|Ax - b\|_2$ for feature selection

$${}^m \underset{\uparrow}{A} = {}^m \underset{\uparrow}{Q} \overset{\uparrow}{\cancel{R}} R \quad Q^T Q = I$$

- ① choose col t with largest 2-norm
- ② pivot t to front $q_1 = A(:, t) / \|A(:, t)\|_2$
- ③ Orthog. out $A \leftarrow (I - q_1 q_1^T) A$
- ④ "Update R" first row: first row of A
- ⑤ Repeat on $A(:, 2:n)$

A

$$A \bar{\Pi}_{:,2} = q_1 r_{12} + q_2 r_{22}$$

$$\|A \bar{\Pi}_{:,2}\|_2 = |r_{12}| + |r_{22}| \leq \|A \bar{\Pi}_{:,1}\| = |r_{11}|$$

$$|r_{22}| \leq |r_{11}| - |r_{12}| \leq |r_{11}|$$

$|r_{k+1,k+1}|$ small \Rightarrow can truncate at k for an approx

$$\text{Also, } |r_{12}| \leq |r_{11}|$$