Dec 16, 2020

\[ A x = b \]

\[ PA = LU \]
\[ A = LL^T \]

\[ k_2(A) = \sigma_{\text{max}}/\sigma_{\text{min}} \]

\[ \min \| Ax - b \|_2 \]

\[ A = QR \]
\[ A = U \Sigma V^T \]

\[ \text{orthog. matrices} \]

\[ k_2(A) \quad b, \text{range}(A) \]

\[ A x = r \]
\[ A = V \Lambda V^T \]
\[ A = QTQ^T \]
\[ A = UH\mathbf{U}^T \]
\[ A = U \Sigma V^T \]

\[ \text{orthog. matrices} \]

condition number \( \Rightarrow \) sensitivity to perturbations

\[ AQ_k = Q_{k+1} T_k \]
\[ AQ_k = Q_{k+1} H_k \]

\[ A V_k = U_{k+1} B_k \]
\[ A^T U_k = V_{k+1} B_{k+1}^T \]

condition number \( \Rightarrow \) convergence

structure matters!!!

sparse

symm

blocked

fast operators

diag dominant

symm

orthog

orthog
Other tools

- Optimization/analysis: CG (Krylov), $2x^T A^T A x = 2A^T b$
  \[ r(x) = \frac{x^T A x}{x^T x} \]
  Gauss-Seidel $\Rightarrow$ coord descent. $\Rightarrow A^T A x = A^T b$

- Graph theory: sparse direct
- Hardware: blocking / BLAS 3, layouts, floating point
- Statistics: truncated SVD $\Rightarrow$ reg LS
- Approx theory: convergence of iterative methods
Where to go from here

CS 6220: Data sparse matrix computations
  structure ⇒ super fast
  - rank-structured mats \( Ax = b \) in \( O(n) \) time
  - Rand NLA
  - FMM, FFT

CS 6241: Matrix comps in data science
  - NMF, other facts
  - CPs / kernel methods
  - Graph data
Thank you!