Dec 4, 2020

$Ax = b \quad K_k(A, b) = \{b, Ab, \dots, A^{k-1}b\}$

$\beta_0 = \|b\|_2$

$q_1 = b_1 \quad q_0 = 0$

$AQ_k = Q_{k+1}\bar{T}_k \quad \bar{T}_k = \begin{pmatrix} \alpha_1 & \beta_1 & & \\ \beta_1 & \ddots & \ddots & \\ & \ddots & \alpha_k & \beta_k \end{pmatrix}$

① $\beta_k q_{k+1} = Aq_k - \alpha_k q_k - \beta_{k-1} q_{k-1}$

② $Q_k^T A Q_k = T_k = \begin{pmatrix} \alpha_1 & \beta_1 & & \\ \beta_1 & \ddots & \ddots & \\ & \ddots & \alpha_{k-1} & \beta_{k-1} \\ & & \beta_{k-1} & \alpha_k \end{pmatrix}$

A SPD

CG

opt. problem	subproblem	factorization	recurrences
$\min \|Ax-b\|^2_{A^{-1}}$	$T_k y_k = \beta_0 e_1$	$T_k = L_k D_k L_k^T$	$W_k L_k^T = Q_k$
s.t. $x \in K_k$	$x_k = Q_k y_k$		$L_k D_k z_k = \beta_0 e_1$
			$x_k = x_{k-1} + \gamma_k w_k$

MINRES

$\min \|Ax-b\|_2^2$

s.t. $x \in K_k$

Error analysis

$$r_k = \min_x \|b - Ax\| \quad \text{s.t. } x \in K_k$$

$$x = p(A)b \quad p \in P_{k-1}$$

$$\Rightarrow \|b - Ap(A)b\|_2$$

$$= \|(I - p(A))b\|_2$$

$$\Rightarrow \min_q \|q(A)b\|_2 \le \|q(A)\|_2 \underbrace{\|b\|_2}_{\|r_0\|_2}$$

$$\text{s.t. } q \in P_k \quad q(0) = 1$$

$$\frac{\|r_k\|}{\|r_0\|} \le \|q(A)\|_2$$

$$A = V \Lambda V^T \quad \|q(A)\|_2 = \|q(\Lambda)\|_2 = \max_{\lambda_i} |q(\lambda_i)|$$

$$\min_q \max_{\lambda_i} |q(\lambda_i)|$$

$$\text{s.t. } q \in P_k$$

$$q(0) = 1$$