Nov 4, 2020

$\boxed{A}\,\boxed{x} = \boxed{b}$ $\qquad \min_x \| \underset{m \times n}{A}\,\underset{n \times 1}{x} - \underset{m \times 1}{b} \|_2 \qquad \underset{n \times n}{A}\,\underset{n \times 1}{x} = \lambda\, \underset{n \times 1}{x}$

$PA = LU$ $\qquad A = QR \qquad A = QTQ^H \quad T = $ upper triangular

$\lambda = T_{ii} \quad Ty = \lambda y$

$L(Ux) = P^T b \qquad Rx = Q^T b \qquad x = Qy$

direct methods

$A \to$ factorization $\to$ easy $\quad O(n^3) \quad O(mn^2)$

benefits:
1. can be fast (n small)
2. reliable: stability
3. reuse! $Ax = c \Rightarrow LUx = P^T c \quad O(n^2)$

Mostly "black box" performance

$A, b \to \boxed{Ax = b} \to x \qquad O(n^3)$

$A, b \to \boxed{\text{LLS}} \to \hat{x} \qquad O(mn^2)$

$A \to \boxed{\text{eval evec}} \to \lambda_1, \dots, \lambda_n;\ v_1, \dots, v_n \qquad O(n^3)$

Not totally black boxes

1. reuse
2. $k_2(A) = \|A\|_2 \|A^{-1}\|$ on sensitivity

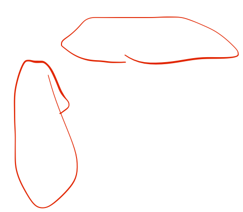
   $Ax=b$, LLS
3. whether or not LLS is "good"

   $A^T b \approx 0 \iff b \perp \mathrm{Range}(A) \Rightarrow$ no "good" solution

$$^2 + \lambda \|x\|_2$$

$O((m+n)$

A^+



PAQ

Iterative solvers

1. Splitting / stationary

2. Krylov subspace methods

$$\underbrace{b, \ Ab, \ A^2b, \ A^3b}, \ \ldots$$

3. Geometric methods (multigrid)