Oct 26, 2020

Double shifted QR

$T_0 = U^T A U = H$

$\lambda_k, \sigma_k = \text{evals } T_k(n-1:n, n-1:n)$

$Q_k R_k = (T_k - \lambda_k I)(T_k - \sigma_k I)$

$T_{k+1} = Q_k^T T_k Q_k$

check for convergence

$$\left(\begin{array}{c|c} T_{11} & t_{12} \\ \hline 0 & \lambda \end{array}\right) \qquad \left(\begin{array}{c|cc} T_{11} & & T_{12} \\ \hline 0 & a & b \\ 0 & c & d \end{array}\right)$$

"deflation"

$|h_{s+1,s}| = O(\varepsilon_{mach}(|h_{s,s}| + |h_{s+1,s+1}|))$

set $h_{s+1,s} = 0$

$Q_1 R_1 = T - \lambda I$

$T_1 = R_1 Q_1 + \lambda I \quad (T_1 = Q_1^H T Q_1)$

$Q_2 R_2 = T_1 - \sigma I$

$T_2 = R_2 Q_2 + \sigma I \quad (T_2 = Q_2^H T_1 Q_2)$

$(Q_1 Q_2)(R_2 R_1) = \underbrace{(T - \lambda I)(T - \sigma I)}_{M}$

if $\sigma = \bar{\lambda}$,

$M = T^2 - 2\,\text{Real}(\lambda)\,T + |\lambda|^2 I$

- one at a time, complex arith.
- form M explicitly $O(n^3)$ for T^2
- best of both?

Implicit Q theorem

$Q^T A Q = H$ unreduced $h_{k+1,k} \neq 0$

Given q_1

Proof: $AQ = QH$ $\quad Aq_1 = h_{11} q_1 + h$

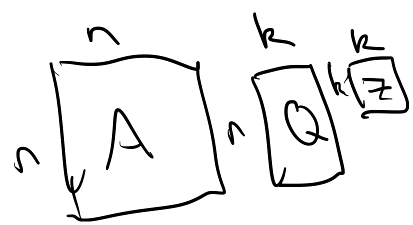
$\Rightarrow h_{21} q_2 = Aq_1 - h_{11} q_1$

q_j^T $\quad h_{ji} q_j$

$$Aq_i - \sum_{j=1}^{i} h_{ji} q_j = h_{i+1,i} q_{i+1}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$AQ = \lambda Q$$



$$z^T z = I$$

$$AQz = \lambda Qz$$

$$(Qz)^T Qz = \cancel{z^T}\overset{I}{\cancel{Q^T Q}}\cancel{z} = I$$